Structured Surface Grid Generation on Body Surfaces defined by NURBS

NURBS로 정의된 표면상에서의 정렬격자 생성 기법

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NURBS 곡면식으로 정의된 물체 표면상에서 표면 정렬 격자를 생성할 수 있는 방법을 소개하였다. 공학 응용분야에서의 물체 표면 정의는 여러 개의 패치들로 표현되는 것이 일반적이고, 여기서 소개하는 표면격자 생성기법은 이러한 여러 패치들에 걸쳐서 분포되는 정렬격자를 쉽게 생성할 수 있도록 한다. 이 기법은 매개변수 형태의 타원형 격자생성 방정식의 해를 구하되, 여러 NURBS 패치에 걸쳐서 투영/분포된 초기 격자계를 타원형 방정식 반복계산 과정의 매개변수형 표면 정의식으로 임시 활용한다. 매개변수형 타원형 방정식의 해가 얻어지고 나면, 그 결과 격자계를 다시 NURBS 패치에 투영을 시키고 타원형 방정식의 해를 구하는 과정이 반복된다. 이러한 반복과정이 전체적으로 수렴이 이루어질 때까지 반복된다. 이 방법에 의해서 얻어지는 표면 정렬 격자계들은 타원형 격자생성기법의 특징인 완만성을 가지면서 정의된 물체표면에서 벗어나지 않는 격자점들이 된다. 소개된 방법은 간단하면서도 하나의 NURBS 곡면만이 아니라 여러 개의 NURBS 곡면에 걸쳐있는 정렬격자계를 효율적으로 생성할 수 있도록 해주며, 그 기본적인 접근법은 NURBS 곡면식 만이 아니라 다른 형태의 매개변수형 형상 정의식에도 적용이 가능하다.

1. Introduction

In many engineering applications including CAD, Computer Graphics and shape optimization, body surfaces are required to be defined, expressed, and manipulated. In analytically representing surfaces, there are three major types: explicit, implicit, and parametric. The parametric form of surface representation is the most flexible and robust among them, especially in CAD system. NURBS are Non-Uniform Rational B-spline Surfaces and they are one of the most used parametric surface representations in engineering design, largely because they encompass all other types of representations. It is, therefore, most likely that surface representation given for some other engineering applications is in the NURBS form. One such situation is to solve flow fields by panel method when given a vehicle surface definition in terms of NURBS surfaces. To apply the panel method, the first step is to generate proper panels over the intended surface. Another situation is to apply computational fluid dynamics technique to solve flow fields around 3-dimensional bodies, and this case still needs some type of surface grid generation before generating volume grids and eventually flow field solutions.

In this presentation, a new approach is introduced for generating structured surface grids along a surface defined by a multiple number of NURBS patches. All the grid points of the final surface grid obtained by this approach exactly lie on the given surface, and has smooth distribution.

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2. Surface Grid Generation

2.1 NURBS theory

In representing surfaces, there are three major types: explicit, implicit, and parametric. According to the explicit representation, a surface requires two independent variables, and the representation might take the form $z=f(x,y)$ There is, however, no guarantee that an explicit representation form exists for a given surface.

The implicit form $f(x,y,z)=0$ also describes a surface. For example, any flat plane can be written as $ax+by+cz+d=0$ for constants a,b,c and d, and a sphere of radius r centered at the origin can be described by $x^2+y^2+z^2-r^2=0$

The parametric form, on the other hand, of a surface expresses the value of each spatial variable for points on the surface in terms of an independent variable, u and v: the parameters. In three dimensions, we have three explicit functions:

$$x=x(u,v) \quad y=y(u,v) \quad z=z(u,v) \tag{1}$$

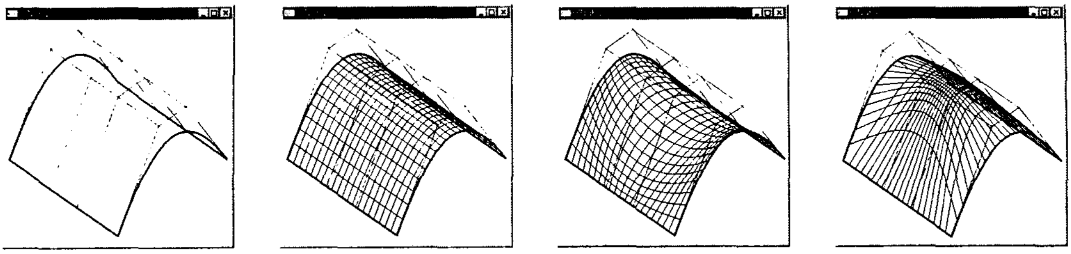
or we can use the column matrix: $p(u,v)=[\,x(u,v) \quad y(u,v) \quad z(u,v)]^T$ As u and v vary over some range, we generate all the points $p(u,v)$ on the surface. The parametric form of surfaces is the most flexible and robust one in representing surfaces.

NURBS are rational B-spline surfaces obtained with a nonuniform knot vector. They are one of the most used parametric surface representations in engineering design, largely because they encompass all other types of representations. The general expression for NURBS is

$$p(u,v)=\frac{\sum_{i=0}^{n}\sum_{j=0}^{m}N_{i,k}(u)N_{j,l}(v)\,w(i,j)\,V_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m}N_{i,k}(u)N_{j,l}(v)\,w(i,j)} \tag{2}$$

where $N_{i,k}(u)$, $N_{j,l}(v)$ are the B-spline blending functions and $V_{i,j}$ are the control points. $w(i,j)$ are the weights at each control point. NURBS are flexible enough to represent most of the analytic surfaces. NURBS can also express a variety of free-surfaces by adjusting the number(n,m) and the position($V_{i,j}$) of control points.

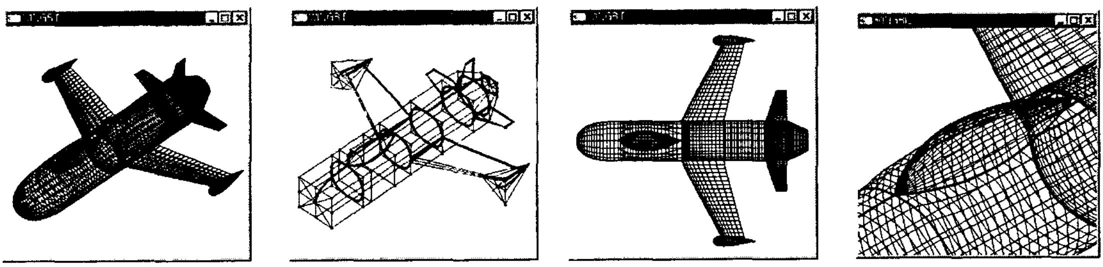
Furthermore, the weights $w(i,j)$ of NURBS provide an additional degree of freedom for the shaping of the surface. Fig. 1 demonstrates the flexibility of NURBS in shaping the surface by variations of the value of weights for a given set of control points. Fig. 1(a) shows the mesh of 4×4 control points and the patch boundary. Fig. 1(b) shows the NURBS surface by evenly-distributed weights for all the control points, while Fig. 1(c) shows the case of lower values of weight at the interior control points. Fig. 1(d) is the case of larger values of weight at the interior control points.



(a) (b) (c) (d)

Fig. 1 Effect of weight variation in NURBS

Even though NURBS offer many favorable characteristics and flexibility in shaping surfaces, it is not always possible to express a variety of surfaces encountered in engineering applications by a single NURBS equation. It is, therefore, common to use a multiple number of NURBS patches to model the surface shape of a complicated bodies, such as automobiles, airplanes, and ships. Such an example is shown in Fig. 2(a): a simplified airplane. Fig. 2(b) shows the control point meshes of multiple NURBS patches and the boundary edges of each patch. The top view of the airplane and the close-up view around the cockpit are shown in Fig. 2(c) and (d), respectively.



(a) Airplane surface **(b) Control points** **(c) Top view** **(d) Close-up view**

Fig. 2 Modeling of an Airplane with multiple NURBS patches

2.2 Parametric elliptic grid generation

Elliptic grid generation method is the most used scheme in generating grids. It solves elliptic partial differential equations to distribute points on a grid, and it generally provides smoothness to the grid and also provides control of point distribution by selecting proper source terms. The conventional elliptic partial differential equations for surface grid generation transformed into computational domain is as follows[1]:

$$\alpha(\vec{r}_{\xi\xi} + \Phi\vec{r}_{\xi}) - 2\beta\vec{r}_{\xi\eta} + \gamma(\vec{r}_{\eta\eta} + \Psi\vec{r}_{\eta}) = [(\alpha\vec{r}_{\xi\xi} - 2\beta\vec{r}_{\xi\eta} + \gamma\vec{r}_{\eta\eta})\cdot\vec{n}]\vec{n} \qquad (3)$$

where, (ξ, η) are computational domain coordinates, and $\vec{r} = [\,x \;\; y \;\; z\,]^T$ is the physical coordinate. Φ, Ψ are control functions and $\vec{n}$ is a unit vector normal to the given surface. Other terms are

$$\alpha = \vec{r}_\eta \cdot \vec{r}_\eta = x_\eta^2 + y_\eta^2 + z_\eta^2 , \quad \beta = \vec{r}_\xi \cdot \vec{r}_\eta = x_\xi x_\eta + y_\xi y_\eta + z_\xi z_\eta , \quad \gamma = \vec{r}_\xi \cdot \vec{r}_\xi = x_\xi^2 + y_\xi^2 + z_\xi^2 , \tag{4-1}$$

$$J^2 = [x_\xi y_\eta - x_\eta y_\xi]^2 + [y_\xi z_\eta - y_\eta z_\xi]^2 + [z_\xi x_\eta - z_\eta x_\xi]^2 \tag{4-2}$$

To apply this conventional elliptic solver, the prescribed surface is usually rotated such that one of the surface point coordinates is described in terms of the other two, for example $z = f(x, y)$ In general, however, there may be several z values for a given (x, y) point. In that case, the user needs to divide the prescribed surface into a number of smaller subfaces to make it single valued in z. Because of these problems associated with the conventional solvers, the elliptic solver that are not dependent on the grid's local orientation in three dimensional space is preferred, and one such method uses the parametric form of the elliptic solver.

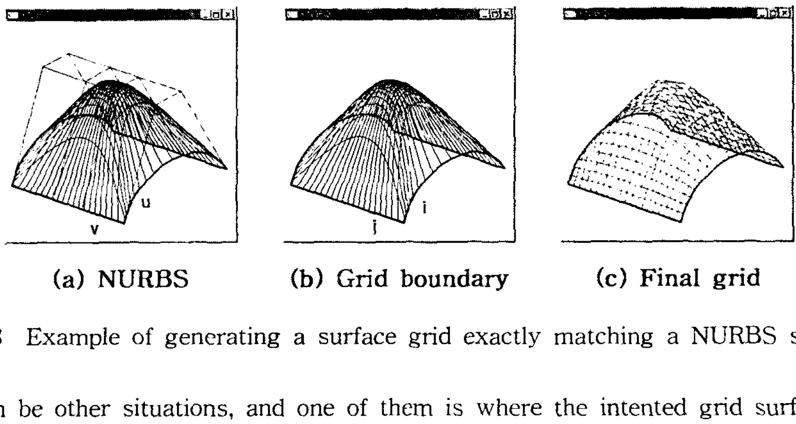
To use the parametric form of the elliptic surface grid generation method, the intended surface is described by two parametric coordinates u and v, such that

$$x = x(u, v) \quad y = y(u, v) \quad z = z(u, v) \tag{5}$$

which is the same equation as Eqn (1). The original surface elliptic PDE's, Eqn (3), are rearranged in terms of the parametric coordinates u and v, and the resulting equations are as follows[1]:

$$\alpha \vec{w}_{\xi\xi} - 2\beta \vec{w}_{\xi\eta} + \gamma \vec{w}_{\eta\eta} + \alpha \vec{w}_\xi \Phi + \gamma \vec{w}_\eta \Psi = J\left[\frac{\partial}{\partial \xi}\left[\frac{\alpha}{J} \vec{w}_\xi - \frac{\beta}{J} \vec{w}_\eta \right] + \frac{\partial}{\partial \eta}\left[-\frac{\beta}{J} \vec{w}_\xi + \frac{\gamma}{J} \vec{w}_\eta \right] \right] \tag{6}$$

where $\vec{w} = [\ u \quad v\ ]^T$ is the vector of the parametric coordinates u and v. In the original usage of the parametric elliptic solver, Equation (6) is solved iteratively for $\vec{w}$ with given values along the boundary, and the physical coordinate of each point is obtained from the parametric surface definition, Equation (5). This approach is applicable when the intended surface grid is confined in a single parametric surface. One such example is shown in Fig. 3, where the surface grid block matches a single NURBS surface. Fig. 3(a) shows the NURBS surface and parametric coordinates. Fig. 3(a) and (b) shows that the computational index (i, j) matches the parametric coordinates (u, v) The elliptically smoothed grid is shown in Fig. 3(c).

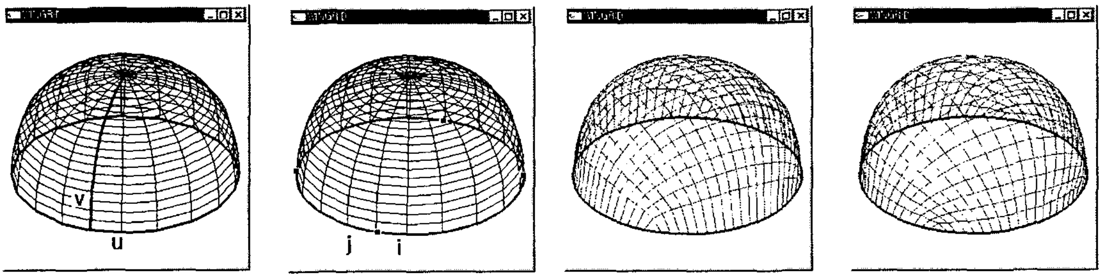


(a) NURBS **(b) Grid boundary** **(c) Final grid**

Fig. 3 Example of generating a surface grid exactly matching a NURBS surface

There can be other situations, and one of them is where the intented grid surface matches a

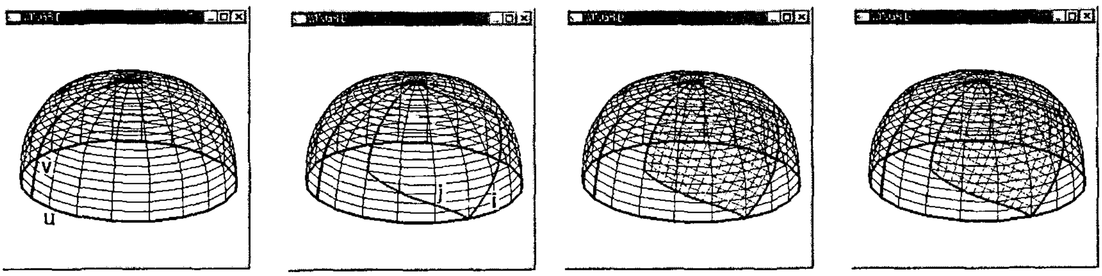
NURBS patch but the edges don't, as shown in Fig. 4: (a) shows a hemispheric surface defined by NURBS and (b) show the intended grid boundaries. This is an example of different topology between the surface definition and the surface grid. Fig. 4(c) shows an initial grid before smoothing, and Fig. 4(d) the resultant grid.



(a) NURBS surface **(b) Grid boundary** **(c) Initial grid** **(d) Smoothed grid**

Fig. 4 Example of a surface grid generation on a NURBS patch with different topology

Another situation is where the intended grid block lies partly on a single surface patch, as shown in Fig. 5. Fig. 5(b) shows the boundary of the surface grid, and Fig. 5(d) the resulting smoothed grid.



(a) NURBS surface **(b) Grid boundary** **(c) Initial grid** **(d) Smoothed grid**

Fig. 5 Example of a surface grid generation partly lying on a NURBS patch

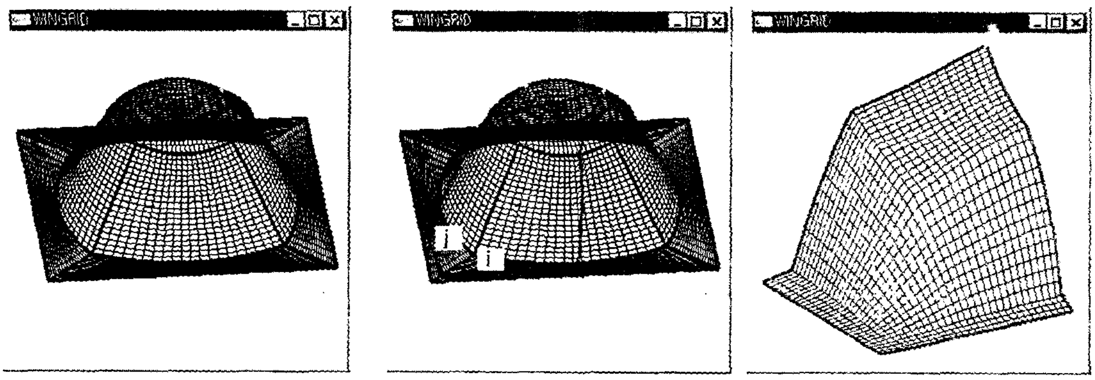
3. New Approach For Multiple NURBS Patches

In the previous examples, the parametric form of elliptic PDE's (6) is solved iteratively for (u, v) which is the parameters defining NURBS surface. On the other hand, if the intended surface grid block extends over more than one NURBS patch, the previous approach can not be used any more because there is no guarantee that we have global parameters (u, v) describing those patches in a single expression.

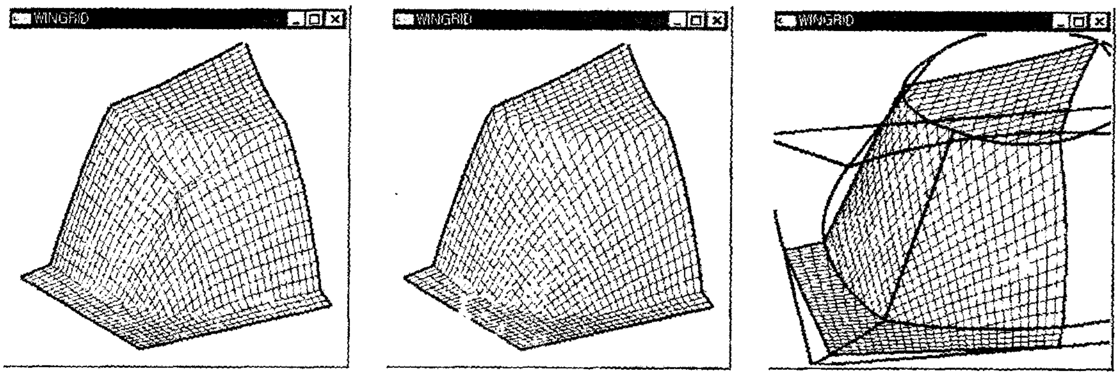
In this research, a new approach is introduced which enables elliptic grid generation in the parametric form for surface grids extending over multiple NURBS patches. This new approach still solves the parametric form of elliptic equations (6) as the original method does, but the parametric coordinates (u, v) are not the NURBS variables but the computational indices (i, j)

of a temporary grid. The steps to follow in this new method may be summarized as follows:

1) A surface defined by a multiple number of NURBS patches are prescribed.
2) The boundary of an intended surface grid is defined by the user, and the grid distribution along the boundary is specified.
3) An initial grid is generated by using a simple method such as TFI(transfinite interpolation).
4) The grid points are projected onto the given NURBS patches.
5) The parametric form of elliptic PDE's is solved iteratively until a convergence criteria is satisfied. In this step, the initial grid system is used as a prescribed surface along which new grid points are to move and its integral indices (i, j) are used as the dependent variables (u, v)
6) When step 5) is done, the resultant grid is constrained not by the intended NURBS surfaces but by the temporary grid surface. Therefore, step 4) and 5) are to be repeated until this outer iteration converges.



(a) Surface Definition (b) Surface grid boundary (c) Initial algebraic grid



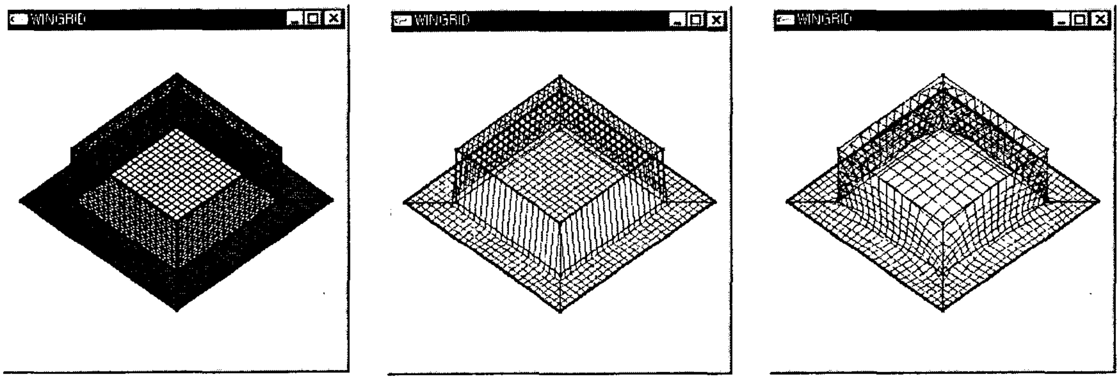
(d) Initial projected grid (e) Smoothed grid (f) Other view

Fig. 6 Surface grid generation extending over multiple NURBS patches

The first example of a surface grid extending over multiple patches is shown in Fig. 6. Fig.

6(a) shows a surface defined by 9 NURBS patches. Fig. 6(b) shows the intended grid boundary extending over 5 patches; flat base, conic side, and flat top plane. The algebraically-generated initial grid is shown in Fig. 6(c). The interior grid points of this grid is generated by using boundary points only, and therefore they usually are not constrained to the intended surface. Fig. 6(d) is the result of projection of points on to the NURBS patches. Once the projected grid is obtained, then it is smoothed by solving the parametric form of elliptic equations iteratively. The process of projection and parametric solution is repeated until it converges, and the final grid is shown in Fig. 6(e). Fig. 6(f) shows the resultant grid seen from another view.

The last example is generating a single block of structured surface grid on a surface defined by 9 flat NURBS patches, as shown in Fig. 7. In this example grid boundary is along the outermost boundary of the intended surface consisting of 9 patches. Fig. 7(b) shows the initial grid with which the present algorithm is to start the iterative process, and Fig. 7(c) the resultant surface grid after elliptic smoothing. As can be seen from the resultant grid, the present method successfully generates smooth structured grid, even on the side planes.



(a) NURBS patches **(b) Before Smoothing** **(c) After smoothing**

Fig. 7 Surface grid generation over a surface consisting of 9 NURBS patches

4. Conclusion

A new approach for surface grid generation along a prescribed surface defined by multiple NURBS patches is introduced. The new method solves the parametric form of elliptic PDE's while using a temporary grid as a prescribed surface along which new grid points are to move until the solution converges. Once the solution of elliptic equations are obtained the points are projected onto the NURBS patches, and the process is repeated until the overall process converges. Grid points of the final grid obtained by this method have smooth distribution which results from an elliptic solver, and grid points are guaranteed to be along the intended surfaces. Examples showed that the new approach can be applied to generate a surface grid extending over a surface consisting of any number of NURBS patches. The new method is simple but effective in generating surface grids on NURBS patches, and it can be easily extended to any other parametric surfaces.

Acknowledgement

This research was supported financially in part by a fund from the Agency for Defence Development. The author wishes to acknowledge the support.

References

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