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천연가스배관내 곡선 영역을 지나는 피그흐름의 동적모델링

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Dynamic Modeling of PIG Flow through Curved Section in Natural Gas Pipelines

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Key Words : Pipeline Inspection Gauge (PIG), Method Of Characteristic (MOC), Pipeline, Curved Pipe

Abstract

In this paper, dynamic modeling and its analysis for the PIG flow through 90^0 curved pipe with compressible and unsteady flow are studied. The PIG dynamics model is derived by using Lagrange equation under assumption that it passes through 3 different sections in the curved pipeline such that it moves into, inside and out of the curved section. The downstream and up stream flow dynamics including the curved sections are solved using MOC. The effectiveness of the derived mathematical models is estimated by simulation results for a low pressure natural gas pipeline including downward and upward curved sections. The simulation results show that the proposed model and solution can be used for estimating the PIG dynamics when we pig the pipeline including curved section.

Nomenclatures

A	pipe cross section	$[m^2]$
c	wave speed	$[m/s]$
C_C	convection heat transfer coefficient	$[m^2]$
C	linear damping coefficient of PIG	$[Ns/m]$
d	internal diameter of pipeline	$[m]$
f_c	friction coefficient in curved pipeline	
f_s	friction coefficient in straight pipeline	
F_f	friction force per unit pipe length	$[N/m]$
F_{fp}	friction force between the PIG and pipe's wall	$[N]$
F_p	force due to different pressure acting on the PIG	$[N]$
g	gravity acceleration	$[m/s^2]$
L	length of pipeline	$[m]$
L_{SR}	length of start riser	$[m]$
L_{ER}	length of end riser	$[m]$
L_{PIG}	length of the PIG	$[m]$
m	hydraulic mean radius of pipe	$[m]$
M	mass of the PIG	$[kg]$
p	flow pressure	$[N/m^2]$
q	compound rate of heat inflow per unit area of pipe's wall	$[W/m^2]$
R	gas constant	$[J/kgK]$
R_b	bending radius of curved section	$[m]$
S	perimeter of pipe	$[m]$
s	distance from inlet of curved section	$[m]$
T	flow temperature	$[^0C]$
T_{ext}	seabed temperature	$[^0C]$
u	flow velocity	$[m/s]$

Greeks:

γ	the ratio of specific heat	
κ	Dean number	
ν	kinetic viscosity of flow	$[m^2/s]$
ρ	flow density	$[kg/m^3]$

Subscripts:

n, t	denote the points at the nose and tail of the PIG
$0, L$	denote the points at inlet and outlet of pipeline

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1. Introduction

Pipelines are the most common way and the safest method to transport oil and gas products. During operation, the walls of pipelines suffer a deterioration process and pipeline conditions get worse. Pipelines can be failed with time if they are not properly maintained. One part of a maintenance procedure of pipelines is pigging them regularly to prevent the increase of their wall's roughness and the reduction of the internal diameter. The tool used for pigging is called Pipeline Inspection Gauge (PIG). There are different types of PIG and each type is designed for some different desired purposes. All of the PIGs are the most effective when they run at a near constant speed but will not be effective in case that they run at too high speed. The typical speeds for utility pigging are about 1-5m/s for on-stream liquids and 2-7m/s for on-stream gas. So, prediction and control of the PIG velocity are very important when we operate a pipeline system. Pigging of pipelines has become a standard procedure in gas and oil industry. One of the difficulties when we design a pigging operation is in fact that most of the available knowledge is based on experiment field. Hence, selecting the PIG and estimating its dynamics often involves some guesswork and, consequently, a high degree of uncertainty[7].

Results of research on the dynamics of the PIG in pipelines are scarcely found in the literature. Some works relating to this subject have been reported. J.M.M. Out[9], 1993, used Lax-Wendroff scheme for the integration of gas equations with adaptation of finite difference grid. The grid has to be continuously updated with the PIG position and the fluid values at the new grid points are estimated by interpolation. Azevedo et al.[7], 1996, simplified the solution with assumption of incompressible and steady state of flow in pipeline. P.C.R. Lima[6], 1999, solved the problem by using one-dimensional semi-implicit finite difference scheme. The nonlinear algebraic equations at each time step are solved adopting Newton's method. T.T. Nguyen et al.[1-3], 2000, treated the compressible, unsteady flow dynamic equations for flows in straight pipeline by using MOC and solved the PIG dynamic equation by using Runge-Kuta method.

The flow in the curved pipe is much more complex than the flow in straight pipe because of the centrifugally-induced secondary motions. Also the PIG dynamics in curved pipe are nonlinear. At this moment, there is no paper related to the dynamics of the PIG when it passes through the curved section of pipeline.

This paper deals with the PIG dynamics when it flows through a 90^0 curved section of pipeline with compressible and unsteady low pressure natural gas flow. The PIG dynamics model is obtained by using Lagrange equation under the condition that it passes through 3 different sections in the curved pipeline such that it moves into, inside and out of the curved section. The downstream and upstream flow dynamics including the curved sections are solved using MOC. The initial values of upstream and downstream are get from the analytical equation of general unsteady, compressible flow equations under assumption of steady state dynamics. The initial values of flows and the PIG dynamics are solved using Runge-Kuta method. The effectiveness of the derived mathematical models is estimated by simulation results for a natural gas pipeline including downward and upward curved sections.

2. Modeling

The scheme of PIG flow in curved pipe can be described in the Fig. 1.

1.1 Gas Flow Model

We assume as the following:

i. the natural gas is ideal,
ii. flow is one phase,
iii. the pipeline diameter is constant,
iv. the friction factor is a function of wall's roughness and Reynolds number. Steady state values are used in transient calculations,
v. the flow is quasi-steady heat flow.

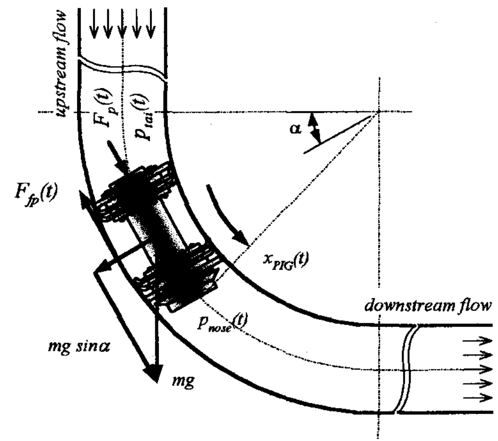


Fig. 1 PIG flow in the bending pipeline

The unsteady flow dynamic equations for flow in straight pipeline are given in the previous work[1-3]. In this paper, we consider the flow in curved section. The flows in curved pipe are much more complex than those in the straight pipe because of the centrifugally-induced secondary motions. There are rich analitical liturature related to the flow in curved pipe (Adler, 1934; Hasson, 1955; Barua,1963; Mori and Nakayama, 1965; Ito, 1975; Collin and Denis, 1975; VanDyke, 1978)[10].

It is well known that the flows in curved pipe depend on the Dean number κ :

$$\kappa = 2\delta^{1/2}\,\mathrm{Re} = \left(\frac{d}{R_b}\right)^{1/2}\frac{2du}{\nu} \tag{1}$$

The friction ratio between the curved pipe and the straight pipe under the same flow conditions are studied by Adler, 1934; Hasson, 1955; Ito, 1975; and Van Dyke, 1978[10]. They suggested the approximate function for the friction ratio f_c / f_s that is a function of Dean number. Among them, the model proposed by Hasson well fitted to the experiment data and the friction ratio is given as follows[10]:

$$\frac{f_c}{f_s} = 0.0969\kappa^{1/2} + 0.556 \tag{2}$$

Here we do not need to consider the behavior of the secondary flow. Our problem is to know the average flow values at each cross section in curved pipeline. Hence, in this paper we use the Hasson approximate function (2) to estimate the friction coefficient of the flow in the curved pipe. The friction coefficient in the straight pipe can be found in the references[1-3,5,9,12]

Consider the control volume as shown in Fig. 2. When $\delta\alpha$ is small enough, the four basic flow equations such as continuity, momentum, state and energy equations can be derived as the following:

$$\frac{\partial\rho}{\partial t} + \frac{u}{R_b}\frac{\partial\rho}{\partial\alpha} + \frac{\rho}{R_b}\frac{\partial u}{\partial\alpha} = 0 \tag{3}$$

$$\frac{1}{R_b}\frac{\partial p}{\partial\alpha} + \frac{\rho u}{R_b}\frac{\partial u}{\partial\alpha} + \rho\frac{\partial u}{\partial t} + \frac{F_f}{A} - \rho g\cos\alpha = 0 \tag{4}$$

$$\frac{p}{\rho}=(\gamma-1)C_V T \tag{5}$$

$$\frac{\partial}{\partial t}\left[\rho\left(C_V T+\frac{u^2}{2}\right)\right]+\frac{1}{R_b}\frac{\partial}{\partial \alpha}\left[\rho\left(C_V T+\frac{u^2}{2}\right)u\right]+\frac{1}{R_b}\frac{\partial}{\partial \alpha}(pu)-\frac{qS}{A}=0 \tag{6}$$

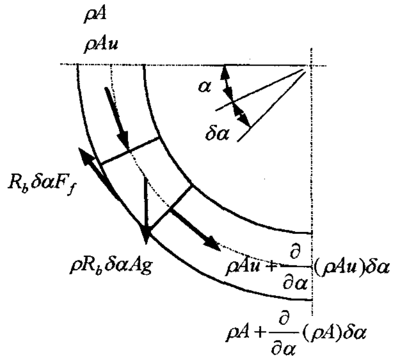


Fig. 2 Control volume for application of motion flow equation in curved pipe section

The mathematical description of the heat rate term, q in Eq. (6) depends on the problem assumptions. Because there is no heat producing in flow, q could be evaluated as a quasi-steady heat transfer from the surrounding environment to the gas:

$$q=C_C(T_{ext}-T) \tag{7}$$

Let us define

$$s \equiv R_b \alpha \qquad 0 \le s \le R_b \pi/2$$

After some rearrangement, the above equations can be rewritten as follows:

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial s}+\rho\frac{\partial u}{\partial s}=0 \tag{8}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial s}+\frac{1}{\rho}\frac{\partial p}{\partial s}=-\frac{F_f}{\rho A}+g\cos\left(\frac{s}{R_b}\right) \tag{9}$$

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial s}+\gamma p\frac{\partial u}{\partial s}=\frac{\gamma-1}{A}\left\{\left[F_f-A\rho g\cos\left(\frac{s}{R_b}\right)\right]u+qS\right\} \tag{10}$$

Using MOC[8,12] to transform the nonlinear hyperbolic partial differential equations (8)-(10) to the ordinary differential equations which can be integrated by finite differences in the form, we can obtain the following equations:

$$\frac{du}{dt}+\frac{c}{\rho p}\frac{dp}{dt}=E_1 \quad \text{along} \quad \frac{ds}{dt}=u+c \tag{11}$$

$$\frac{du}{dt}-\frac{c}{\rho p}\frac{dp}{dt}=E_2 \quad \text{along} \quad \frac{ds}{dt}=u-c \tag{12}$$

$$\frac{du}{dt}-c\frac{dp}{dt}=E_3 \quad \text{along} \quad \frac{ds}{dt}=u \tag{13}$$

where

$$E_1=\frac{\gamma-1}{c}\frac{q}{\rho m}+\left(\frac{F_f}{\rho A}-g\cos\alpha\right)\left(\frac{\gamma-1}{c}u-1\right) \tag{14}$$

$$E_2=-\frac{\gamma-1}{c}\frac{q}{\rho m}-\left(\frac{F_f}{\rho A}-g\cos\alpha\right)\left(\frac{\gamma-1}{c}u+1\right) \tag{15}$$

$$E_3=(\gamma-1)\frac{q}{m}+\left(\frac{F_f}{\rho A}-g\cos\alpha\right)(\gamma-1)u\rho \tag{16}$$

$$m=A/S$$

$$c=\sqrt{\frac{\gamma p}{\rho}}$$

The initial values of flow are given from steady state condition of flow. The governing steady flow equations can be derived from the analytical Eq.s (8)-(10) after some rearrangement as follows:

$$\frac{dp}{ds}=-\frac{F_f}{A}+\rho g\cos\left(\frac{s}{R_b}\right)+\Phi u \tag{17}$$

$$\frac{du}{dx}=-\frac{\Phi}{\rho} \tag{18}$$

$$\frac{d\rho}{dx}=\frac{\Phi}{u} \tag{19}$$

where

$$\Phi=\frac{1}{u^2-c^2}\left(\gamma\frac{F_f u}{A}-\gamma\rho g\cos\left(\frac{s}{R_b}\right)u+(\gamma-1)\frac{q}{m}\right)$$

1.2 PIG Dynamics Model

Dynamic equation for PIG flow in straight pipeline can be found in references[1-3,9]. In this paper we deal with the PIG moving in the curved section of pipeline. The PIG contacts pipe's wall through its front and tail cups. To model the PIG motion in curved pipe, we consider that the mass of the PIG is concentrated at its end parts as shown in Fig. 3. Hence, the mass of the PIG becomes 2 particle masses located at its nose and tail.

The PIG moves through curved pipe in three different sections: moving into, moving inside and moving out of the curved section as shown in the above figure. To derive its dynamic equation, we use Lagrange method[11]. Let us consider the system of 2 particles, whose positions are given by Cartesian coordinates (x_i, y_i), $(i=1,2)$. The system is one degree of freedom and α is chosen to be generalized coordinate. Lagrange equation is in the form:

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\alpha}}\right)-\frac{\partial K}{\partial \alpha}+\frac{\partial P}{\partial \alpha}=F \tag{20}$$

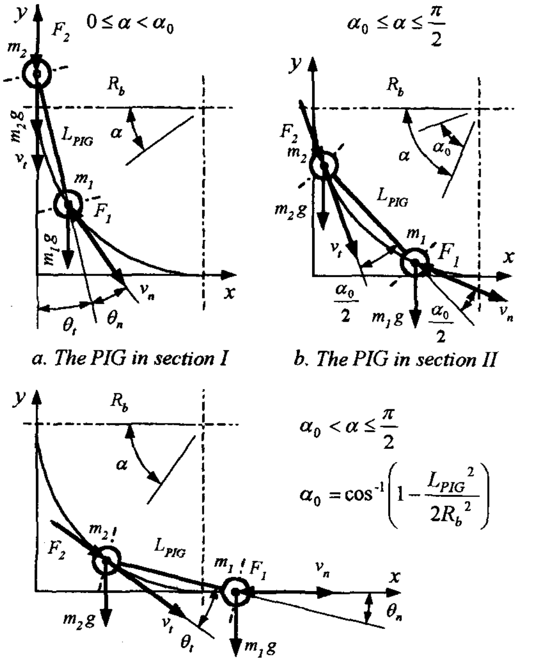
where K, P and F are kinetic energy, potential energy and generalized force, respectively:

$$K=\frac{1}{2}m_1\left(\dot{x}_1^2+\dot{y}_1^2\right)+\frac{1}{2}m_2\left(\dot{x}_2^2+\dot{y}_2^2\right) \tag{21}$$

$$P=m_1 g y_1+m_2 g y_2 \tag{22}$$

$$F=F_1^x\frac{\partial x_1}{\partial \alpha}+F_1^y\frac{\partial y_1}{\partial \alpha}+F_2^x\frac{\partial x_2}{\partial \alpha}+F_2^y\frac{\partial y_2}{\partial \alpha} \tag{23}$$

where m_i is the mass of the particle i; (x_i, y_i) is coordinate of the particle i; F_i^x, F_i^y is the force acting on the particle i in the direction of x and y coordinates $(i=1,2)$.



a. The PIG in section I
b. The PIG in section II
c. The PIG in section III

Fig. 3 The scheme of PIG flow in downward curved pipe

Using equations (20)-(23), after some calculations, we can get the dynamic equations of the PIG when it passes through each curved section I, II and III as shown in Fig. 3, respectively:

$$\ddot{\alpha} + \frac{m_2 Y Y_\alpha}{m_1 R_b^2 + m_2 Y^2}\dot{\alpha}^2 = \frac{m_1 g R_b \cos\alpha + m_2 g Y - F_1 R_b + F_2 Y}{m_1 R_b^2 + m_2 Y^2} \tag{24}$$

$$\ddot{\alpha} = \frac{m_1 g \cos\alpha + m_2 g \cos(\alpha - \alpha_0) - F_1 + F_2}{(m_1 + m_2) R_b} \tag{25}$$

$$\ddot{\alpha} + \frac{m_1 T T_\alpha}{m_1 T^2 + m_2 R_b^2}\dot{\alpha}^2 = \frac{m_2 g \cos\alpha - F_1 T + F_2 R_b}{m_1 T^2 + m_2 R_b^2} \tag{26}$$

where

$$X = \sqrt{L_{PIG}^2 - R_b^2 (1 - \cos\alpha)^2}$$

$$Y = R_b \cos\alpha + \frac{R_b^2}{X}\left(\sin\alpha - \frac{1}{2}\sin 2\alpha\right)$$

$$Y_\alpha = \frac{\partial Y}{\partial \alpha}$$

$$Z = \sqrt{L_{PIG}^2 - R_b^2 (1 - \sin\alpha)^2}$$

$$T = R_b \sin\alpha + \frac{R_b^2}{Z}\left(\cos\alpha - \frac{1}{2}\sin 2\alpha\right)$$

$$T_\alpha = \frac{\partial T}{\partial \alpha}$$

The forces acting on the PIG include gravity force of the PIG, the different pressure across its body and friction force. Generalized forces acting on the PIG in each section are given in the following:

In section I:

$$F_1 = -m_1 g \cos\alpha + \left(p_n A + \frac{1}{2}F_{fp}\right)\cos\theta_n$$

$$F_2 = m_2 g + \left(p_t A - \frac{1}{2}F_{fp}\right)\cos\theta_t$$

$$\theta_t = \sin^{-1}\left(\frac{R_b(1-\cos\alpha)}{L_{PIG}}\right), \; \theta_n = \alpha - \theta_t$$

In section II:

$$F_1 = -m_1 g \cos\alpha + \left(p_n A + \frac{1}{2}F_{fp}\right)\cos\frac{\alpha_0}{2}$$

$$F_2 = m_2 g \cos(\alpha - \alpha_0) + \left(p_t A - \frac{1}{2}F_{fp}\right)\cos\frac{\alpha_0}{2}$$

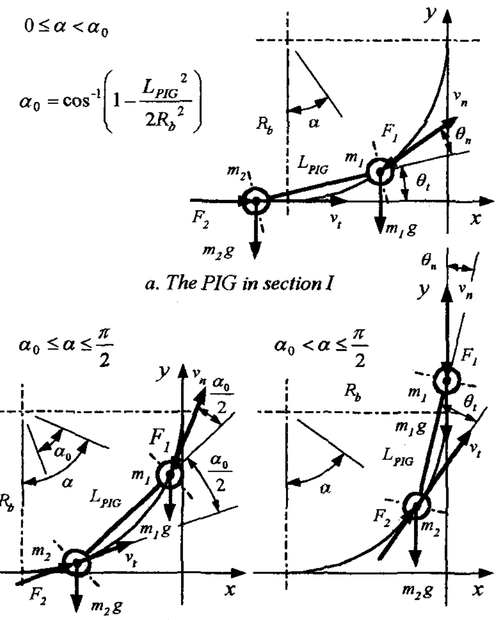
In section III:

$$F_1 = \left(p_n A + \frac{1}{2}F_{fp}\right)\cos\theta_n$$

$$F_2 = m_2 g \cos\alpha + \left(p_t A - \frac{1}{2}F_{fp}\right)\cos\theta_t$$

$$\theta_n = \sin^{-1}\left(\frac{R_b(1-\sin\alpha)}{L_{PIG}}\right), \; \theta_t = \frac{\pi}{2} - (\alpha + \theta_n)$$

The scheme for deriving the PIG dynamics when it moves in upward curved pipe is given in Fig. 4.



b. The PIG in section II
c. The PIG in section III

Fig. 4 The scheme of PIG flow in upward curved pipe

In the case of upward curved section, by the same way, we have the dynamic equations of the PIG as follows:

In section I:

$$F_1 = m_1 g \sin\alpha + \left(p_n A + \frac{1}{2}F_{fp}\right)\cos\theta_n$$

$$F_2 = \left(p_t A - \frac{1}{2}F_{fp}\right)\cos\theta_t$$

$$\theta_t = \sin^{-1}\left(\frac{R_b(1-\cos\alpha)}{L_{PIG}}\right),\ \theta_n = \alpha - \theta_t$$

$$\ddot{\alpha} + \frac{m_2 Y Y_\alpha}{m_1 R_b^2 + m_2 Y^2}\dot{\alpha}^2 = \frac{-m_1 g R_b \sin\alpha - F_1 R_b + F_2 Y}{m_1 R_b^2 + m_2 Y^2} \quad (27)$$

In section II:

$$F_1 = m_1 g \sin\alpha + \left(p_n A + \frac{1}{2}F_{fp}\right)\cos\frac{\alpha_0}{2}$$

$$F_2 = -m_2 g \sin(\alpha - \alpha_0) + \left(p_t A - \frac{1}{2}F_{fp}\right)\cos\frac{\alpha_0}{2}$$

$$\ddot{\alpha} = \frac{-m_1 g \sin\alpha - m_2 g \sin(\alpha - \alpha_0) - F_1 + F_2}{(m_1 + m_2)R_b} \quad (28)$$

In section III:

$$F_1 = m_1 g + \left(p_n A + \frac{1}{2}F_{fp}\right)\cos\theta_n$$

$$F_2 = -m_2 g \sin\alpha + \left(p_t A - \frac{1}{2}F_{fp}\right)\cos\theta_t$$

$$\theta_n = \sin^{-1}\left(\frac{R_b(1-\sin\alpha)}{L_{PIG}}\right),\ \theta_t = \frac{\pi}{2} - (\alpha + \theta_n)$$

$$\ddot{\alpha} + \frac{m_1 T T_\alpha}{m_1 T^2 + m_2 R_b^2}\dot{\alpha}^2 = \frac{-m_1 g T - m_2 g R_b \sin\alpha - F_1 T + F_2 R_b}{m_1 T^2 + m_2 R_b^2} \quad (29)$$

3. Simulation and results

The computational scheme for solving the PIG dynamics together with upstream and down stream flow dynamics by using MOC were presented in the previous work[1-3]. Three Eqs. (17)-(19) are solved using Runge-Kuta method to get the initial values of upstream and downstream flows in pipeline. Also the PIG dynamic equations are solved using Runge-Kuta method.

The simulation is done with a low pressure natural gas pipeline including start riser, downward curved, upward curved and end raiser sections using to transport gas from buoy to tank as shown in Figs. 5 and 6.

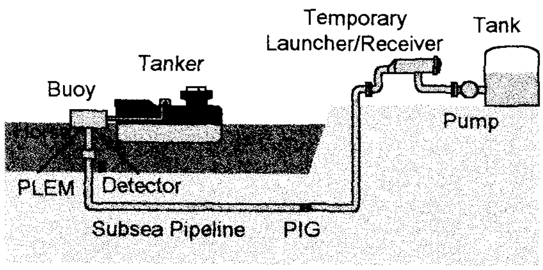


Fig. 5 The U-pipeline using to transport natural gas from buoy to tank

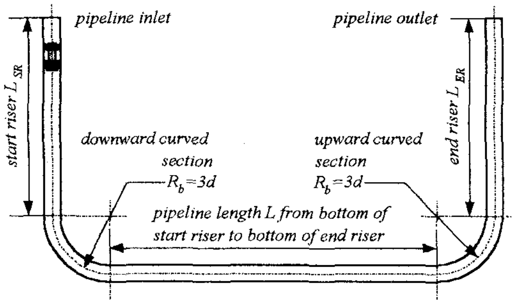


Fig. 6 Scheme of pipeline using in the simulation

The numerical values using in this simulation are given in the Table. 1.

Table. 1 Numerical values for simulation

Parameters	Values	Units	Parameters	Values	Units
L	12	m	ν	1.45e-5	m^2/s
L_{SR}	12	m	R	518.30	J/kgK
L_{ER}	12	m	γ	1.40	
L_{PIG}	1.1	m	M	750	kg
d	0.7366	m	C_C	2	W/m^2s
x^0_{PIG}	1.1	m	v^0_{PIG}	4	m/s
k	0.0450	mm	F_{fp}	0.33	bar
p_0	8	bar	p_L	7.65	bar
Q_0	1.16	m^3/s	Q_L	1.16	m^3/s
ρ_0	5.44	kg/m^3	ρ_L	5.20	kg/m^3
T_{ext}	15	^0C	T	15	^0C

We choose the sampling time $\Delta t = 0.001\,s$ and the sampling distance $\Delta x = 0.5785\,m$. And two following boundary conditions of interest are used:

i. constant flow rate at pipeline inlet $u_0(t,0) = u_0$, constant pressure at pipeline outlet $p_L(t,L) = p_L$.
ii. constant pressure at pipeline inlet $p_0(t,0) = p_0$, constant flow rate at pipeline outlet $u_L(t,L) = u_L$.

The different pressure acting on the PIG is shown in Fig. 7. It varies with different operational boundary conditions.

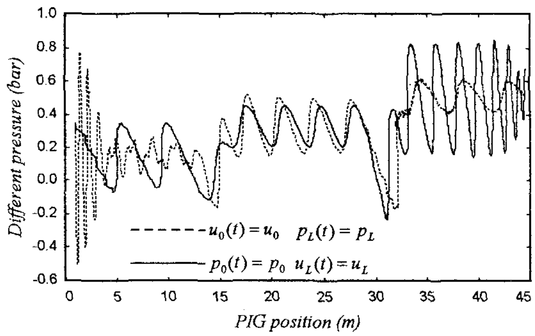


Fig. 7 Different pressure acting on the PIG

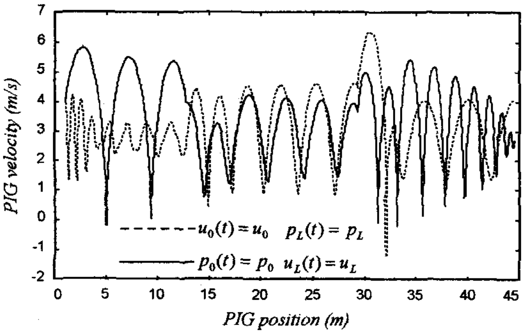


Fig. 8 PIG velocity vs. PIG position

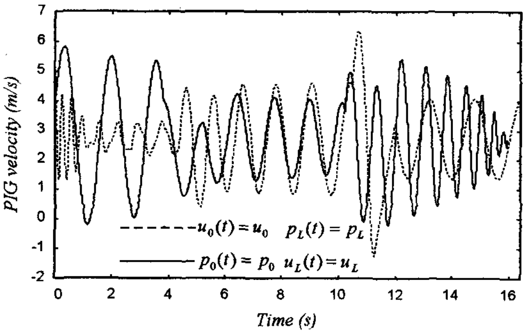


Fig. 9 PIG velocity vs. time

Fig. 8 shows the PIG velocity at each its position. In both cases, the maximum velocities seem to be the same but in case of the second boundary condition, the PIG arrives its trap barrel with lower velocity. Hence, we can choose appropriate boundary condition to get the PIG stop at it trap barrel without necessary of applying braking force which is harmful to pipelines. Fig. 9 shows the PIG velocity vs. time when it moves in the pipeline. The PIG velocity oscillates in the different manners in different parts of the pipeline.

4. Conclusion

This paper presents simple model for flow and the PIG dynamic equations when it passes through a 90^0 curved section of pipeline. The simulation has been done with two different operational boundary conditions. The solution for non-linear hyperbolic partial equations for flow is given by using MOC. The Runge-Kuta method is used to solve the initial condition equation for flow and the PIG dynamics equations. The simulation results show that the proposed model and solution can be used for estimating the PIG dynamics when the PIG runs in the pipeline including curved section.

Acknowledgment

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