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# Finite Element Analysis for Three Dimensional Welding Processes

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# 3 차원 용접과정의 유한요소해석

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**Abstract**

We propose an implicit numerical implementation for the Leblond's transformation plasticity constitutive equations, which are widely used in welded steel structure. We apply generalized trapezoidal rule to integrate the equations and determine the consistent tangent moduli. The implementation may be used with updated Lagrangian formulation. We test a simple butt-welding process to compare with SYSWELD and discuss the accuracy.

Nomenclator

$\dot{\sigma}^J$ : Jaumann stress rate

$\dot{\varepsilon}^{th}$ : thermal strain rate

$I$ : 2-nd order identity tensor

$II$ : 4-th order identity tensor

$\hat{\varepsilon}^{th}$ : thermal strain given

^ : the rotation neutralized variable

$\omega$ : the factor for the memory effect

$\sigma^y$ : Yield stress

$L^t$ : tangent stiffness

$F_{n+1}$ : deformation gradient

$\hat{\sigma}_{n+1}$ : rotation neutralized stress

$L$ : isotropic elastic modulus

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## 1. Introduction

Leblond[1] proposed a transformation plasticity model on the theoretical foundation, transformation plasticity is a consequence of the homogenization process without postulating arbitrary extra plastic strain. Since it has more reasonable than other empirical models, SYSWELD selected this model for finite element analysis of welding process in spite of its complexity. SYSWELD uses *explicit* multi-step method to integrate the constitutive model[2]. We propose a new *implicit* integration method and corresponding stiffness calculation method for the Leblond constitutive model. More detailed procedure are omitted because of the limitation of space available.

## 2. Numerical implementation of Leblond's transformation plasticity constitutive equations

We use the following isotropic thermo-elastoplastic rate form constitutive equation.

$$\dot{\sigma}^J = \boldsymbol{L} : \left(\hat{\varepsilon} - \hat{\varepsilon}^p - \hat{\varepsilon}^{th}\right) \quad (1)$$

where

$$\boldsymbol{L} = 2\mu \boldsymbol{II} + \left(\kappa - \frac{2}{3}\mu\right)\boldsymbol{I} \otimes \boldsymbol{I}$$
$$(L_{ijkl} = \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \left(\kappa - \frac{2}{3}\mu\right)\delta_{ij}\delta_{kl}) \quad (2)$$

Leblond[1] proposed a transformation plasticity constitutive model for isotropic hardening case as follows

$$\bar{\sigma} = \left(\frac{3}{2}\sigma' : \sigma'\right) \quad (3)$$

$$\dot{\bar{\varepsilon}}^p = \left(\frac{2}{3}\dot{\varepsilon}^p : \dot{\varepsilon}^p\right) \quad (4)$$

$$\sigma^y(\bar{\varepsilon}_1^{eff}, \bar{\varepsilon}_2^{eff}) = [1 - f(z)]\sigma_1^y(\bar{\varepsilon}_1^{eff}) + f(z)\sigma_2^y(\bar{\varepsilon}_2^{eff}) \quad (5)$$

If $\bar{\sigma} < \sigma^y$:

$$\dot{\varepsilon}^p = \dot{\varepsilon}^{tp} + \dot{\varepsilon}^{cp} \quad (6.a)$$

$$\dot{\varepsilon}^{tp} = (-)\frac{3\Delta\varepsilon_{1\to 2}^{th}}{\sigma_1^y(\bar{\varepsilon}_1^{eff})} h\left(\frac{\bar{\sigma}}{\sigma^y}\right)\sigma'(\ln z)\dot{z} \quad (6.b)$$

$$\dot{\varepsilon}^{cp} = \frac{3(1-z)}{2\sigma_1^y(\varepsilon_1^{eff})}\frac{g(z)}{E}\sigma'\dot{\bar{\sigma}} + \frac{3(\alpha_1 - \alpha_2)}{\sigma_1^y(\varepsilon_1^{eff})} z(\ln z)\sigma'\dot{\theta} \quad (6.c)$$

$$\dot{\bar{\varepsilon}}_1^{eff} = (-)\frac{2\Delta\varepsilon_{1\to 2}^{th}}{1-z} h\left(\frac{\bar{\sigma}}{\sigma^y}\right)(\ln z)\dot{z} + \frac{g(z)}{E}\dot{\bar{\sigma}} + \frac{2(\alpha_1 - \alpha_2)z\ln z}{1-z}\dot{\theta} \quad (6.d)$$

where the first term of the right-hand side is to be replaced by 0 if $z < 0.03$;

$$\dot{\bar{\varepsilon}}_2^{eff} = (-)\frac{\dot{z}}{z}\bar{\varepsilon}_2^{eff} + \omega\frac{\dot{z}}{z}\bar{\varepsilon}_1^{eff} \quad (6.e)$$

If $\bar{\sigma} = \sigma^y$:

$$\dot{\varepsilon}^p = \frac{3}{2}\frac{\dot{\bar{\varepsilon}}^p}{\bar{\sigma}}\sigma' \quad (7.a)$$

$$\dot{\bar{\varepsilon}}_1^{eff} = \dot{\bar{\varepsilon}}^p \quad (7.b)$$

$$\dot{\bar{\varepsilon}}_2^{eff} = \dot{\bar{\varepsilon}}^p - \frac{\dot{z}}{z}\bar{\varepsilon}_2^{eff} + \omega\frac{\dot{z}}{z}\bar{\varepsilon}_1^{eff} \quad (7.c)$$

where f(z), g(z), $h\left(\frac{\bar{\sigma}}{\sigma^y}\right)$ are correction functions with respect to experimental result. Yield stress, $\sigma^y$ depends upon yield stress of each phase and proportion of each phase ($\sigma_1^y, \sigma_2^y, z$). We can rewrite (6.a~c) as follows.

$$\dot{\varepsilon}^p = \frac{3}{2}\frac{\dot{\bar{\varepsilon}}^p}{\bar{\sigma}}\sigma' \quad (8.a)$$

where $\dot{\bar{\varepsilon}}^p = \frac{(1-z)\dot{\bar{\varepsilon}}_1^{eff}}{\sigma_1^y(\bar{\varepsilon}_1^{eff})}\dot{\bar{\sigma}}$ (8.b)

Linearized updated Lagrangian weak formulation can be expressed as in the following equation for the negligible inertia case.

$$\int_\Omega (L_{ijkl}^t d\varepsilon_{kl}\frac{\partial w_j}{\partial x_i} - (d\varepsilon_{im}\sigma_{mj} + \sigma_{im}d\varepsilon_{mj})\frac{\partial w_j}{\partial x_i} + \sigma_{ik}\frac{\partial du_j}{\partial x_k}\frac{\partial w_j}{\partial x_i})d\Omega \quad (9)$$

where

$$dT_{ij} \approx d\sigma_{ij}^J - d\varepsilon_{im}\sigma_{mj} - \sigma_{im}d\varepsilon_{mj} + \sigma_{ik}\frac{\partial du_j}{\partial x_k}$$

and $d\sigma_{ij}^J = L_{ijkl}^t d\varepsilon_{kl}$ (10)

To implement (10) we must determine how to calculate the tangent stiffness, $L^t$ and how to integrate the rate form constitutive equations and update other state variables.

First, we explain the integration and update state variable. We separate the deformation gradient, $F_{n+1}$ by Hoger and Carlson method[3] and calculate the rotation neutralized strain increment by [4] and [5] in the integration of constitutive equations. If we set $\hat{N} = \sqrt{\frac{3}{2}}\frac{\hat{\sigma}'}{\bar{\sigma}}$, rotation neutralized plastic strain rate can be expressed as $\hat{\dot{\varepsilon}}^p = \sqrt{\frac{3}{2}}\dot{\bar{\varepsilon}}^p\hat{N}$. Using trapezoidal time integration rule, rotation neutralized stress, $\hat{\sigma}_{n+1}$ is integrated as

$$\hat{\sigma}_{n+1} = \hat{\sigma}^E_{n+1} - \sqrt{6}\mu\Delta t[(1-\beta)\dot{\bar{\varepsilon}}^p_n\hat{N}_n + \beta\dot{\bar{\varepsilon}}^p_{n+1}\hat{N}_{n+1}] \quad (11)$$

where

$$\hat{\sigma}^E_{n+1} = \sigma_n + L : lnU_{n+1} - \frac{E}{1-2v}[(1-z_{n+1})\alpha_1 + z_{n+1}\alpha_2]\Delta\theta I \quad (12)$$

$$\Delta\theta = \theta_{n+1} - \theta_n \quad (13.a)$$

$$\Delta\hat{\varepsilon} \cong lnU_{n+1} \cong 2(U-I)(U+I)^{-1} \quad (13.b)$$

Other state variables are also integrated by the same trapezoidal rule. We can find out a non-linear equation which has only one unknown state variable, effective plastic strain of phase 1, $(\bar{\varepsilon}^{eff}_1)_{n+1}$ after some mathematical treatments. Equation (14) is the equation for the $\bar{\sigma} < \bar{\sigma}^y$ and z<0.03 case.

$$\begin{aligned} G_1((\bar{\varepsilon}^{eff}_1)_{n+1}) = \{ & \frac{E}{g(z_{n+1})}(\bar{\varepsilon}^{eff}_1)_{n+1} \\ & -\frac{\beta E}{g(z_{n+1})}(A_1 + A_3) + A_2\} \\ & \times[1 + 3\mu\frac{(1-z_{n+1})\{(\bar{\varepsilon}^{eff}_1)_{n+1} - \beta A_3\}}{\sigma^y_1((\bar{\varepsilon}^{eff}_1)_{n+1})}] \\ & = \bar{\sigma}^{E*}_{n+1} \end{aligned} \quad (14)$$

where

$$\begin{aligned} A_1 &= \frac{2(\alpha_1 - \alpha_2)z_{n+1}\ln z_{n+1}}{\beta(1-z_{n+1})} \times \\ & \quad \{\theta_{n+1} - \theta_n - (1-\beta)\Delta t\dot{\theta}_n\}, \\ A_2 &= \bar{\sigma}_n + (1-\beta)\Delta t\dot{\bar{\sigma}}_n, \\ \text{and} \quad A_3 &= \frac{(\bar{\varepsilon}^{eff}_1)_n + (1-\beta)\Delta t(\dot{\bar{\varepsilon}}^{eff}_1)_n}{\beta}. \end{aligned} \quad (15.a,b,c)$$

The problem of integration of objective stress rate reduces to solving problem of the non-linear equation. Now, we can find out effective strain increment of phase 1 by solving the nonlinear equation, (14) applying Newton's method. And then we update other state variables.

Second, we explain how to determine the tangent stiffness matrix corresponding to the stress integration method. We set the time integration coefficient $\beta$ equal to 1 for the unconditional convergence.

$$d\hat{\sigma} = \hat{L}^t : d\hat{\varepsilon} \quad (16)$$

where

$$\hat{L}^t = \frac{\partial\hat{\sigma}^E_{n+1}}{\partial\Delta\hat{\varepsilon}} - \sqrt{6}\mu\Delta t\hat{N} \otimes \frac{\partial\dot{\bar{\varepsilon}}^p}{\partial\Delta\hat{\varepsilon}} - \sqrt{6}\mu\Delta t\dot{\bar{\varepsilon}}^p\frac{\partial\hat{N}}{\partial\Delta\hat{\varepsilon}} \quad (17)$$

After some mathematical treatments, for the $\bar{\sigma} < \bar{\sigma}^y$ and z<0.03 case, we can write the derivative of equivalent plastic strain as

$$\frac{\partial\bar{\varepsilon}^p_{n+1}}{\partial\Delta\hat{\varepsilon}} = \sqrt{6}\mu\xi\hat{N} \quad (18)$$

where

$$\xi = \left(\frac{C_1 + C_2C_3}{1 + 3\mu C_1}\right) \quad (19)$$

$$C_1 = \frac{(1-z)\{(\bar{\varepsilon}_I^{eff})_{n+1} - (\bar{\varepsilon}_I^{eff})_n\}}{\sigma_I^y((\bar{\varepsilon}_I^{eff})_{n+1})},$$

$$C_2 = (1-z)\bar{\sigma}_{n+1} \times [\frac{1}{\sigma_I^y} - \frac{\{(\bar{\varepsilon}_I^{eff})_{n+1} - (\bar{\varepsilon}_I^{eff})_n\}}{(\sigma_I^y)^2} \frac{\partial \sigma_I^y}{\partial (\bar{\varepsilon}_I^{eff})_{n+1}}]$$

and $$C_3 = \frac{1}{\frac{\partial G_I}{\partial (\bar{\varepsilon}_I^{eff})_{n+1}}} \qquad (20.a,b,c)$$

The rotation neutralized consistent tangent moduli can be expressed by

$$\hat{L}^t = \hat{L}^* - \frac{3}{2}(\xi - \eta)\hat{M} \otimes \hat{M} \qquad (21)$$

where

$$\hat{L}^* = 2\mu^* II + \left(\kappa - \frac{2}{3}\mu^*\right) I \otimes I \qquad (22)$$

$$\mu^* = (1 - 3\eta)\mu \qquad (23)$$

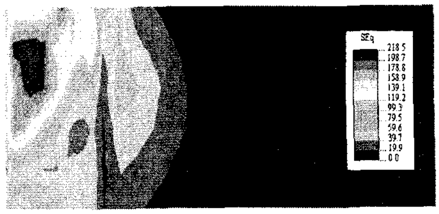
$$\eta = \frac{\{\bar{\varepsilon}_{n+1}^p - \bar{\varepsilon}_n^p\}}{\bar{\sigma}_{n+1}^{E^*}} \qquad (24)$$

$$\hat{M} = 2\mu\hat{N} \qquad (25)$$

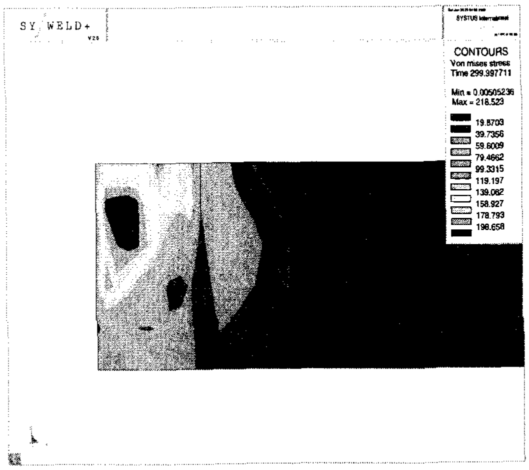
Finally, stress increment and tangent stiffness matrix can be calculated by transformation of the rotation neutralized variables. We can also apply this procedure to other cases.

## 3. Numerical example

We choose a simple butt-welding process of steel structure for comparing the accuracy and the efficiency of our implementation with SYSWELD. Assuming two-dimensional plane strain state. A finite element analysis was conducted and Fig. 1 show the Von-Mises stress distribution when the structure are fully cooled. We see that our result is in an excellent agreement with SYSWELD result. We have shown the accuracy by comparing the results but not compare the efficiency because of the difficulty of computation time estimation. Three-dimensional finite element analysis will be discussed at the Conference.



(a) The present result




(b) SYSWELD result

**Fig. 1** Von-Mises stress distribution of the test example.

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