

# Adaptive Tracking Control of Two-Wheeled Welding Mobile Robot -Dynamic Model Approach-

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**Abstract:** This paper proposes an adaptive control method of partially known system and shows its application result to control for two-wheeled WMR. The controlled system is stable in the sense of Lyapunov stability. To design a tracking controller for welding path reference, an error configuration is defined and the controller is designed to drive the error to zero as fast as desired. Moments of inertia of system are considered to be unknown system parameters. Their values are estimated using update laws in adaptive control scheme. The effectiveness of the proposed controller is shown through simulation results.

## 1. Introduction

A mobile robot is one of the well-known systems with non-holonomic constraints. There are many works on tracking control method for mobile robot in literatures[1-7]. Sarkar[6] proposed a nonlinear feedback that guarantees input-output stability and Lagrange stability for the overall system. Fierro[5] developed a combined kinematic/torque control law using back-stepping approach. Both of these papers did not consider to the system parameter uncertainties, which always exist in mobile robot control problem. Fukao[4] dealt with adaptive tracking control of two-wheeled mobile robot considering the model with unknown parameters in its kinematic part of overall system. An adaptive law was proposed for updating the error of unknown parameters. Applying the two-wheeled mobile robot for welding automation have been studied by Jeon[1-3] and Kam[2]. Jeon proposed a seam tracking and motion control of WMR for lattice type welding. Kam proposed a control algorithm for straight welding based on try and error each step time. Both of controllers proposed by Jeon and Kam have been successfully applied for the real systems. Yet there are still some problems: the controllers are complex and loss of generality when is applied for general smooth path tracking. This paper proposes an adaptive control of partially known system and shows its application result to control for two-wheeled WMR. The controlled system is stable in the sense of Lyapunov stability. To design a tracking controller for welding path reference, an error configuration is defined and the controller is designed to drive the error to zero as fast as desired.

Moments of inertia of system are considered to be unknown system parameters. Their values are estimated using update laws in adaptive control scheme.

## 2. Adaptive Control of Partially Known System

We discuss the following class of nonlinear systems that consists of two subsystems:

$$\dot{\xi} = f(\xi) + g(\xi)u \quad (1)$$

$$\Delta_1 \dot{\eta} = \Delta_2 h(\eta)\eta + k(\eta)u \quad (2)$$

where  $\xi \in R^n$ ,  $\eta, u \in R^m$ ,  $f \in R^n$ ,  $g \in R^{n \times m}$ ,  $\Delta_1, \Delta_2$  are diagonal matrices containing unknown parameters  $\theta_{1i}, \theta_{2i}$  respectively, and  $\Delta_1, \Delta_2, h, k \in R^{n \times m}$ . Moreover,  $k(\eta)$  is invertible and  $\theta_{1i} > 0$ . The  $\xi$ -subsystem represents the known part of the system and the  $\eta$ -subsystem is the unknown part of the system because of including unknown

parameters  $\theta_{1i}$  and  $\theta_{2i}$ . All the functions in the system are assumed to know.

**Theorem 2.1** The following controller stabilizes the system (1)-(2) and attains  $\xi \rightarrow 0$ .

$$u = k^{-1}(\eta) [-K_2(\eta - \alpha) - g^T(\xi)\xi + \widehat{\Delta}_1 \dot{\alpha} - \widehat{\Delta}_2 h(\eta)\eta] \quad (3)$$

with the update laws:

$$\dot{\widehat{\theta}}_{1i} = \gamma_{1i}(\eta_i - \alpha_i)\dot{\alpha}_i \quad (4)$$

$$\dot{\widehat{\theta}}_{2i} = -\gamma_{2i}(\eta_i - \alpha_i) \sum_{j=1}^m h_{ij}(\eta)\eta_j \quad (5)$$

where  $K_1 \in R^{n \times n}$ ,  $K_2 \in R^{m \times m}$  are positive definite diagonal matrices,  $\gamma_{1i}, \gamma_{2i} > 0, i = 1 \sim m$  are adaptive gains,  $\widehat{\Delta}_1, \widehat{\Delta}_2$  are the estimation values of unknown parameters  $\Delta_1, \Delta_2$  and the stabilizing function  $\alpha$  satisfies:

$$g(\eta)\alpha = -K_1\xi - f(\xi) \quad (6)$$

## 3. Model of Two-Wheeled Welding Mobile Robot

The WMR coordinate used in this paper is shown in Fig. 1.

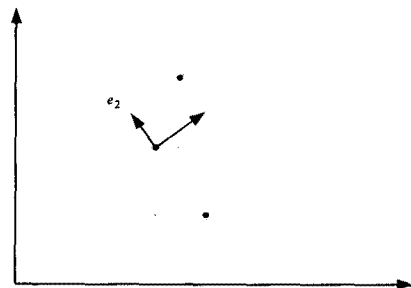


Fig. 1 WMR coordinate

There are three controlled motions in this model: two driving wheels and one torch-carrying slider. The non-holonomic mobile robot platform can be expressed in the following equations[4]:

$$\dot{q} = S(q)\nu \quad (7)$$

$$M(q)\dot{\nu} + V(q, \dot{q})\nu = B(q)\tau \quad (8)$$

The welding point  $W$  dynamics can be expressed as follows:

$$\begin{bmatrix} \dot{x}_w \\ \dot{y}_w \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos \phi - l \cos \phi \\ \sin \phi - l \sin \phi \\ 0 \quad 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} + \begin{bmatrix} -l \sin \phi \\ l \cos \phi \\ 0 \end{bmatrix} \quad (9)$$

A point  $R(x_r, y_r)$  moving with the constant velocity of  $v_r$  in the reference path has the coordinates and the heading angle  $\phi_r$  satisfies the following equation:

$$\dot{x}_r = v_r \cos \phi_r, \quad \dot{y}_r = v_r \sin \phi_r, \quad \dot{\phi}_r = \omega_r \quad (10)$$

where  $\omega_r$  is the rate of change of  $v_r$  direction.

Our purpose is to design a controller for the welding point  $W$  to track the reference point with a constant velocity. We define the tracking errors  $e = [e_1, e_2, e_3]^T$  as shown in Fig. 1. We will design a controller to achieve  $e_i \rightarrow 0$  when  $t \rightarrow 0$  and hence the welding point tracks its reference path. The dynamics of errors can be expressed as follows:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 - l \\ \omega_r \end{bmatrix} + \begin{bmatrix} -1 & e_2 + l \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (11)$$

The relationship between  $v, \omega$  and  $\omega_{rw}, \omega_{hw}$  is:

$$v = \begin{bmatrix} \omega_{rw} \\ \omega_{hw} \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & -\frac{b}{r} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (12)$$

Using Eq. (12), Eq. (8) can be rewritten in the form:

$$\begin{bmatrix} \frac{r}{2} m + \frac{1}{r} J_w & \frac{r}{2b} I + \frac{b}{r} J_w \\ \frac{r}{2} m + \frac{1}{r} J_w & -\frac{r}{2b} I - \frac{b}{r} J_w \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} + \frac{r}{2b} m_c d \omega \begin{bmatrix} 1 & -b \\ -1 & b \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \tau_{rw} \\ \tau_{hw} \end{bmatrix} \quad (13)$$

#### 4. Application to Control of WMR

##### 4.1 Control input for Kinematic Model

Kinematic model considers velocities  $v, \omega$  as control inputs.

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} l(\omega_r + k_{13} e_3) + v_r \cos e_3 + k_{11} e_1 \\ \omega_r + k_{13} e_3 \end{bmatrix} \quad (14)$$

with the control law for the torch:

$$l = v_r \sin e_3 + k_{12} e_2 \quad (15)$$

##### 4.2 Adaptive Control

Dynamic model considers the torques,  $\tau_{rw}, \tau_{hw}$ , acting on the driving wheels as control inputs. The control inputs:

$$\begin{cases} \tau_{rw} = \frac{1}{2} [-k_{21}(v - \alpha_1) - k_{22}(\omega - \alpha_2) - (1+l)e_1 + e_3 \\ \quad + \hat{\theta}_{11}\dot{\alpha}_1 + \hat{\theta}_{12}\dot{\alpha}_2 - \hat{\theta}_2\omega(b\omega - v)] \\ \tau_{hw} = \frac{1}{2} [-k_{21}(v - \alpha_1) + k_{22}(\omega - \alpha_2) - (1-l)e_1 - e_3 \\ \quad + \hat{\theta}_{11}\dot{\alpha}_1 - \hat{\theta}_{12}\dot{\alpha}_2 - \hat{\theta}_2\omega(b\omega + v)] \end{cases} \quad (16)$$

where  $\alpha$  follows Eq. (14) and

$$\begin{aligned} \dot{\alpha}_1 &= -k_{11}v + [k_{11}(e_2 + l) + k_{13}l + v_r \sin e_3]\omega + l\dot{\omega}_r \\ &\quad + k_{12}\omega_r e_2 + k_{11}v_r \cos e_3 + v_r k_{13} e_3 \sin e_3 \quad (17) \\ \dot{\alpha}_2 &= \dot{\omega}_r + k_{13}(\omega_r - \omega) \end{aligned}$$

The update laws:

$$\begin{aligned} \dot{\hat{\theta}}_{11} &= \gamma_{11}(v - \alpha_1)\dot{\alpha}_1 \\ \dot{\hat{\theta}}_{12} &= \gamma_{12}(\omega - \alpha_2)\dot{\alpha}_2 \\ \dot{\hat{\theta}}_2 &= -\gamma_2\omega[(v - \alpha_1)b\omega - (\omega - \alpha_2)v] \end{aligned} \quad (18)$$

#### 5. Simulation results

Table 1 The numerical values for simulation

Para.s	Values	Units	Para.s	Values	Units
$b$	0.105	$m$	$d$	0.01	$m$
$r$	0.025	$m$	$m_c$	16.9	$kg$
$m_w$	0.3	$kg$	$I_c$	0.2081	$kgm^2$
$J_w$	$3.75 \times 10^{-4}$	$kgm^2$	$I_m$	$4.96 \times 10^{-4}$	$kgm^2$

Table 2 The initial values for simulation

Para.s	Values	Units	Para.s	Values	Units
$x_r$	0.280	$m$	$y_r$	0.400	$m$
$x_w$	0.270	$m$	$y_w$	0.390	$m$
$v$	0	$mm/s$	$\omega$	0	$rad/s$
$\phi_r$	0	$deg$	$\phi$	15	$deg$
$l$	0.15	$m$	$\omega_r$	0	$rad/s$

$$k_{11} = 4.2, \quad k_{12} = 8, \quad k_{13} = 3.4, \quad k_{21} = k_{22} = 10, \\ \gamma_{11} = \gamma_{12} = \gamma_2 = 1.$$

The reference path is chosen as shown in Fig. 2. The welding speed is 7.5  $mm/s$ .

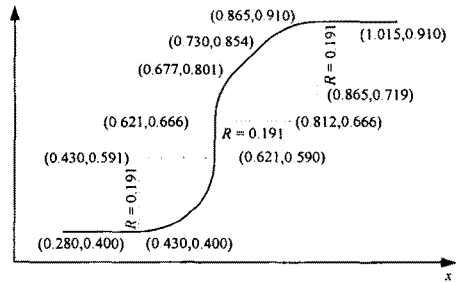


Fig. 2 Reference welding path

The movement of WMR when it tracks its reference weld path is given in Fig. 3. At beginning, the WMR adjusts its position very fast to reduce the initial error. These tracking errors can be seen in Fig. 4. After about 1.5 second, the welding torch attains its desired velocity as shown in Fig. 5.

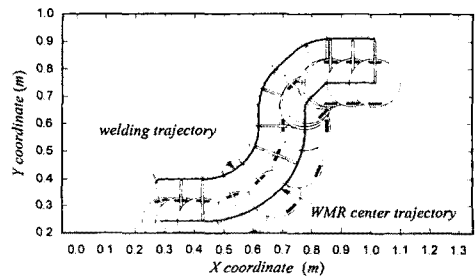


Fig. 3 WMR movement when tracking reference

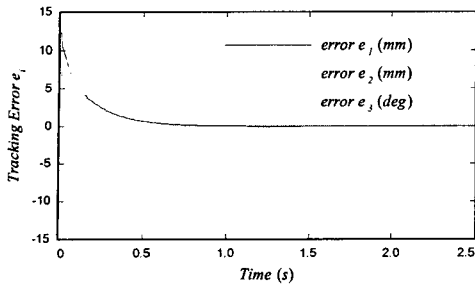


Fig. 4 Tracking errors at beginning

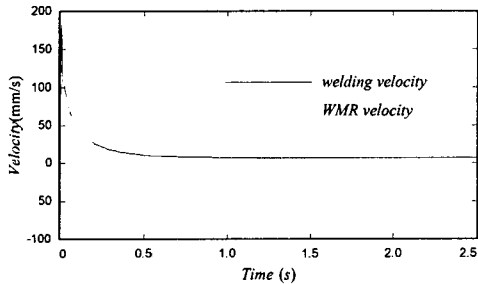


Fig. 5 Velocities of welding point and WMR at beginning

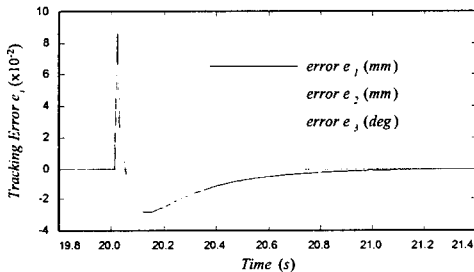


Fig. 6 Tracking errors at corner

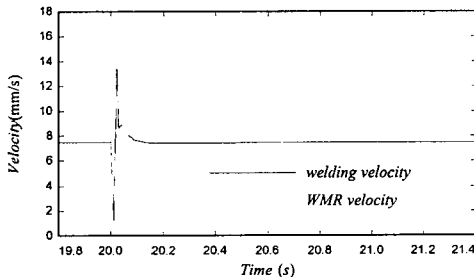


Fig. 7 Velocities of welding point and WMR at corner

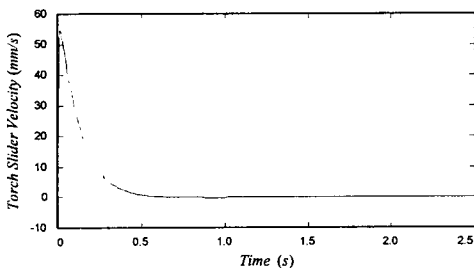


Fig. 8 Torch slider velocity

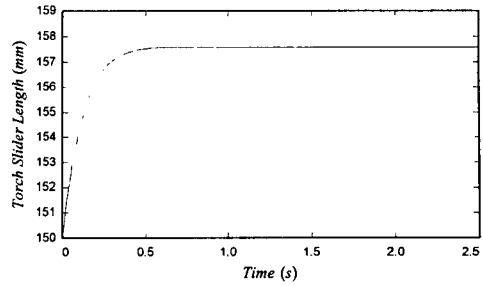


Fig. 9 Torch length

From straight line to curved line, there is a sudden change of  $\omega_r$  (from zero to a constant). Therefore there are errors as shown in Fig. 6 and the corresponding welding velocity is shown in Fig. 7. Torch slider speed is given in Fig. 8 and torch length is given in Fig. 9. As simulation results, the proposed controller can attain good tracking performance.

## 6. Conclusion

An adaptive control of partially known system based on Lyapunov control function is introduced and applied to control for two-wheeled WMR to enhance the tracking performances of MWR. The controlled system is stable in the sense of Lyapunov stability. Moments of inertia of system are considered to be unknown system parameters. The simulation of a WMR with a smooth curved reference-welding path has been done to show the effectiveness of the proposed controller. It is shown that the controller can be used for control of WMR with good performances.

## References

- [1] Y.B. Jeon, S.S. Park and S.B. Kim, Modeling and Motion Control of Mobile Robot for Lattice Type of Welding Line, *KSME International Journal*, Vol. 16, No. 1, pp. 1207-1216, 2001.
- [2] B.O. Kam, Y.B. Jeon and S.B. Kim, Motion Control of Two-Wheeled Welding Mobile Robot with Seam Tracking Sensor, *Proc. of the 6th IEEE Int. Symposium on Industrial Electronics*, Korea, Vol. 2, pp. 851-856, June 12-16, 2001.
- [3] Y.B. Jeon, B.O. Kam, S.S. Park and S.B. Kim, Seam Tracking and Welding Speed Control of Mobile Robot for Lattice Type of Welding, *Proc. of the 6th IEEE Int. Symposium on Industrial Electronics*, Korea, Vol. 2, pp. 857-862, June 12-16, 2001.
- [4] T. Fukao, H. Nakagawa and N. Adachi, Adaptive Tracking Control of a Nonholonomic Mobile Robot, *IEEE Trans. on Robotics and Automation*, Vol. 16, No. 5, pp. 609-615, October 2000.
- [5] R. Fierro and F.L. Lewis, Control of a Non-holonomic Mobile Robot: Backstepping Kinematics into Dynamics, *Proc. of the 34th Conf. on Decision & Control*, pp. 3805-3810, USA, Dec. 1995.
- [6] N. Sarkar, X. Yun and V. Kumar, Control of Mechanical Systems With Rolling Constraints: Application to Dynamic Control of Mobile Robots, *The Int. Journal of Robotics Research*, Vol. 13, No. 1, pp. 55-69, Feb. 1994.
- [7] X. Yun and Y. Yamamoto, Internal dynamics of a Wheeled Mobile Robot, *Proc. of the 1993 IEEE/RSJ Int. Conference on Intelligent Robots and Systems*, Japan, pp. 1288-1294, July 1993.
- [8] Jean-Jacques E. Slotine and Weiping Li, *Applied Nonlinear Control*, Prentice-Hall International, Inc., pp. 122-125.