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전자소자의 형상최적화를 위한 3차원 요소의 재생성법

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A New 3D Mesh Regeneration Method in the Shape Optimal Design of

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Abstract - A novel and simple method, which can be used to automatically regenerate 3D finite element meshes, is presented in the paper. This technique based on the structural deformation analysis. It is problem independent and can be used to renew the mesh of any kind of 3D shape design system whether the geometric surface is parameterized or not. The mesh deformation degree can be adjusted by choosing suitable subregion and giving proper parameters. It is sufficient to obtain a smooth contour with proper mesh quality. Application to the optimum design of shielding plate shows the effectiveness of the proposed technique.

1. Introduction

The geometric finite element model must be modified during the shape optimal design procedure. It is necessary to integrate geometric modeler, finite element mesh regenerator and optimization algorithms into a general system for achieving a variety of designs automatically and problem independently. Furthermore a good method, which describes the changing geometry with a preserved mesh topology and smooth outline shape, has to be developed for the successful application of efficient gradient methods[1]. The mesh distortion must be minimized in order to maintain accuracy of the finite element analysis results. This is still a quite new research for 3D shape optimization of electromagnetic devices[2]. In this paper, a topologically constant 3D mesh regeneration method is presented. It based on the elastic stress equations. The perturbation of the boundary can be mapped onto body with deformed meshes. The parameter selection and subregion technique are suggested. Some conclusions for its applications are presented by the implementations of two test examples.

2. New mesh regeneration method

For the F.E. analysis of 3D stress problem, the matrix equation can be obtained as following:

$$K\Delta x = f \tag{1}$$

where K is the stiffness matrix determined by the geometry and material parameters of the problem; Δx is nodal displacement vector; f is the forcing load vector[2,3]. It can be seen from appendix that equation (1) is much similar to the one of 3D static magnetic analysis by nodal element. The consistent and interrelated property of the deformations in a elastic body guarantee smooth shape contours as the elastic body deforming. If the structural deformation of the shape is obtained by a finite element solution using a certain discretization, the deformation of the surface results in an evenly distorted mesh of the body. In this paper it is used to the mesh regeneration during the optimal design of electromagnetic device by specifying f as a fictitious load force to control mesh density and the amount of relocation in different domains. Rewrite (1) in segmented form

$$\begin{bmatrix} K_{bb} & K_{bd} \\ K_{db} & K_{dd} \end{bmatrix} \begin{pmatrix} \Delta x_b \\ \Delta x_d \end{pmatrix} = \begin{bmatrix} f_b \\ f_d \end{bmatrix} \tag{2}$$

where $\{\Delta x_b\}$ is the known perturbation of nodes on the boundary, $\{\Delta x_d\}$ is the unknown nodal displacement vector in the interior of the domain, $\{f_b\}$ and $\{f_d\}$ are the fictitious force acting on the boundary and domain respectively. The unknown interior nodal displacement vector can be obtained from

$$[K_{dd}]\{\Delta x_d\} = \{f_d\} - [K_{db}]\{\Delta x_b\} \tag{3}$$

which defines a linear relation between the boundary and domain deformations. To evaluate $\{\Delta x_d\}$, it is necessary to suppress all the degrees of freedom that represent the fixed shape contour of a domain in the finite element analysis. The flow chart of the algorithm is shown in Fig.1. The mesh quality of a element is defined as

$$q = 8.47967 \cdot Volume\left(\frac{1}{6}\sum_{i=1}^{6} L_i^2\right)^{-1.5} \tag{4}$$

where L_i is the edge length of a tetrahedron. If the coefficient q approaches 0, the mesh quality is low.

Since this structural analysis is merely used as a tool to map the coordinates relocation from surface to interior nodes, the material parameters related to (1) could be free to chose. However mesh deforming degree and mesh quality depend on the selection of parameters and solving region. If there is no load force, mesh deformation is independent of E and its quality could be improved a little with smaller ν. Appropriate quantity of f might improve mesh quality and at this time E will affect the mesh deformation, which can be seen in next section. To limit the computation efforts of remeshing, only part region of the electromagnetic solution domain is defined as remesh region in the vicinity of the shape to be optimized. If the difference between the initial and last shape configurations is not much great, it is sufficient to obtain good mesh quality with smooth shape outline. However, if the design change becomes large, element distortion might exist. The parameters should be adjusted or mesh smoothing scheme is needed[2].

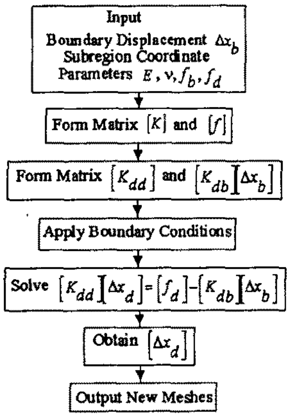


Fig.1 The flow chart of the 3D mesh regenerating technique based on structural stress analysis

3. Test Examples

3.1 Example 1: A Block with Deformed Surface

Suppose some nodes of the up-surface moves after a fictitious force applied on a line in different position. The displacements in Fig.2 is 45% edge length of the block. Table.1 and Table.2 show the relations between parameters and mesh quality. From Table.1, if load force exists, larger E and smaller ν result in better meshes. Fig.3 shows the meshes of some cases. It can be seen from Fig.2, Table.2 and Fig.4 that the bigger the solving region of structural analysis the better is the mesh quality. However if the region gets bigger the computational efforts will be greater while quality is improved very little. It is sufficient to settle the solving region surround the deformed shape. Giving each subregion a fictitious force can control the mesh quality. Select different E and ν for each region, proper mesh quality under $f=0$ is also obtained.

From the implementation, following conclusions can be obtained. The first, the interior mesh can be relocated by the nodal moving on the boundary. The second, the deformed meshes are constructed in a smooth contour automatically. Last, the mesh quality can be guaranteed by properly choosing the parameters and regenerating region.

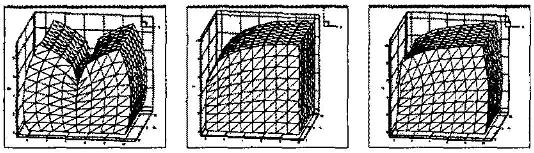
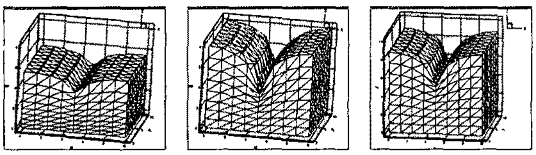


Fig.2 The deformed block with forces applied on up surface



(a) $E=0.5, f=-8\times10^{-3}$ (b) $E=5 f=-8\times10^{-3}$ (c) $E=0.5, f=0$

Fig.3 The deformed mesh with different parameters ($\nu=0.3$)

Table 1. The relations between mesh quality and parameters

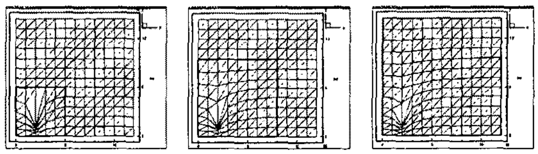
ν	E	f	q_{worst}	Element Num. Ratio
				$q\leq0.3$
0.3	0.5	0	3.1956e-2	4.9479%
0.1			2.5289e-2	3.7109%
0.03			2.3568e-2	3.7109%
0.3	0.1	-8e-3	2.6571e-2	75.0325%
	0.5		3.2474e-1	0
	5.0		4.4653e-2	4.4271%
	50		3.3050e-2	4.8828%

Table 2. The mesh quality under different solving region, f, and elected case

	Equ. Num.	q_{worst}	Element Num Ratio (%)		
			$q\leq0.3$	$0.3<q<0.6$	$q\geq0.6$
Small	360	1.98e-2	1.109	2.091	96.79
Middle	2172	3.09e-2	0.877	3.443	95.68
Large	6576	3.46e-2	0.819	3.771	95.41
A	2172	2.37e-2	0.964	22.216	76.82
B	$E=0.5$	4.51e-2	0.752	5.058	94.19
C	$\nu=0.3$	1.88e-2	0.424	22.356	77.22
Elected	2172	5.58e-2	0.672	5.678	93.65

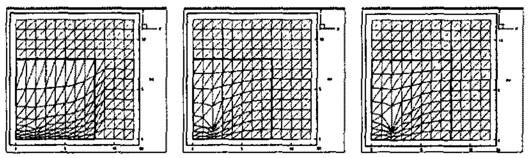
A: $f=\begin{pmatrix}-2e-2\\-2e-2\end{pmatrix}$, B: $f=\begin{pmatrix}-2e-2\\0\end{pmatrix}$, C: $f=\begin{pmatrix}0\\-2e-2\end{pmatrix}$

Elected: $f_1=f_2=0$, $E_1=50$, $\nu_1=0.003$, $E_2=0.1$, $\nu_2=0.3$



Small Middle Large

Fig.4 The deformed mesh computed by different structural analysis domain



A. B. Elected

Fig.5 Deformed mesh with different f

3.2 Example 2: The Surface Shape Optimal Design of an Electromagnetic Shield

3D electromagnetic shielding model is shown in Fig.6. The steel plate is 10mm thick, whose maximum and minimum thickness is restricted to 30mm and 5mm respectively, $\sigma=7\times10^{6}[(\Omega m)^{-1}]$, $\mu_r=800$, is energized by 40000AT at power frequency. The solving regions of electromagnetic and structural analysis are shown in Fig.7. The objective function is $F(p)=\sum_{k=1}^{nb} B\cdot B^{*}$. The z coordinate of 91 nodes on the plate surface are chosen as design variables to mostly decrease the flux density in the observation area. After 13 iterations, objective function becomes 51.16% of the initial value, shown in Fig.8. The optimized plate shape is shown in Fig.9.

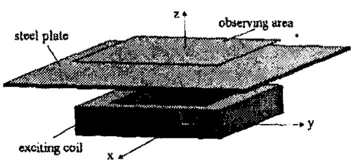


Fig.6. The electromagnetic shield

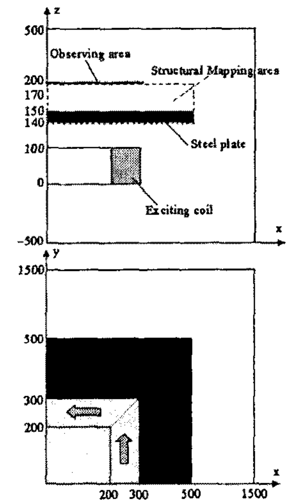


Fig.7 Solving regions of electromagnetic and structural analysis

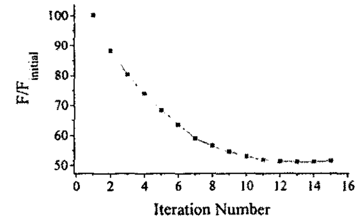


Fig.8. Convergence of objective function.

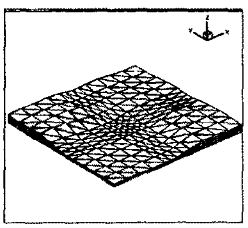


Fig.9. The optimized plate shape of electromagnetic shield

4. Conclusions

A mesh regenerating method is proposed in the paper. By using structural analysis, the surface displacements of the varying geometry could be mapped onto the finite element mesh of the body, while maintaining the mesh topology. This technique is very easy in the program development. By using subregion of the electromagnetic analysis as the structural solving area and reduced equations, the computational efforts are very small. If the design change from the initial to the final is not great, it is sufficient to obtain a smooth contour with proper mesh quality. The mesh deformation degree and quality can be adjusted by choosing suitable subregion and parameters. Whether the geometric surface is parameterized or not, this method can be used to renew the finite element meshes.

References

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[2] Y.Y. Yao, C.S. Koh, "A Novel Mesh Regeneration Technique for 3D Shape Optimization of Electromagnetic Device Based on Structural Deformation Analysis", 대한전기학회 충북지부 춘계학술대회 논문지, pp.49-53, 2002

[3] O.C.Zienkiewicz, The Finite Element Method (third edition), 1977 McGraw-Hill Book Company (UK).

Appendix

The governing equation for 3D stress problem is

$$\varepsilon=\left[\frac{\partial u}{\partial x}\ \frac{\partial v}{\partial y}\ \frac{\partial w}{\partial z}\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\ \frac{\partial v}{\partial v}+\frac{\partial w}{\partial y}\ \frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right]^T \quad \text{(A-1)}$$

$$\sigma=D\varepsilon \quad \text{(A-2)}$$

$$\frac{\partial \sigma_{ix}}{\partial x}+\frac{\partial \sigma_{iy}}{\partial y}+\frac{\partial \sigma_{iz}}{\partial z}+X_i=0 \quad i=x,y,z \quad \text{(A-3)}$$

where, σ is stress; ε is the corresponding strain. Elasticity matrix D gives the relationship between stresses and strains. For the electromagnetic system, there is

$$\nabla\times(\nabla\times A)-\nabla(\nu\nabla\cdot A)=J_s \quad \text{(A-4)}$$

$$\nu=\begin{bmatrix}\nu_{xx} & 0 & 0\\ 0 & \nu_{yy} & 0\\ 0 & 0 & \nu_{zz}\end{bmatrix} \quad \text{(A-5)}$$

From (A-1) to (A-3), following equations can be obtained

$$\begin{aligned}&\frac{\partial^2 u}{\partial x^2}+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 u}{\partial y^2}+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 u}{\partial z^2}+\frac{\nu}{1-\nu}\frac{\partial^2 v}{\partial x\partial y}\\&+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 v}{\partial y\partial x}+\frac{\nu}{1-\nu}\frac{\partial^2 w}{\partial x\partial z}+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 w}{\partial z\partial x}+X_x=0\end{aligned} \quad \text{(A-6)}$$

$$\begin{aligned}&\frac{\partial^2 v}{\partial x^2}+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 v}{\partial y^2}+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 v}{\partial z^2}+\frac{\nu}{1-\nu}\frac{\partial^2 u}{\partial y\partial x}\\&+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 u}{\partial x\partial y}+\frac{\nu}{1-\nu}\frac{\partial^2 w}{\partial y\partial z}+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 w}{\partial z\partial y}+X_y=0\end{aligned} \quad \text{(A-7)}$$

$$\begin{aligned}&\frac{\partial^2 w}{\partial x^2}+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 w}{\partial y^2}+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 w}{\partial z^2}+\frac{\nu}{1-\nu}\frac{\partial^2 u}{\partial z\partial x}\\&+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 u}{\partial x\partial z}+\frac{\nu}{1-\nu}\frac{\partial^2 v}{\partial z\partial y}+\frac{1-2\nu}{2(1-\nu)}\frac{\partial^2 v}{\partial y\partial z}+X_z=0\end{aligned} \quad \text{(A-8)}$$

From (A-1) to (A-3), following equations can be obtained

$$\begin{aligned}&\nu_{xx}\frac{\partial^2 A_x}{\partial x^2}+\nu_{zz}\frac{\partial^2 A_x}{\partial y^2}+\nu_{yy}\frac{\partial^2 A_x}{\partial z^2}+\nu_{xx}\frac{\partial^2 A_y}{\partial x\partial y}\\&-\nu_{zz}\frac{\partial^2 A_y}{\partial y\partial x}+\nu_{xx}\frac{\partial^2 A_z}{\partial x\partial z}-\nu_{yy}\frac{\partial^2 A_z}{\partial z\partial x}=-J_{sx}\end{aligned} \quad \text{(A-9)}$$

$$\begin{aligned}&\nu_{zz}\frac{\partial^2 A_y}{\partial x^2}+\nu_{yy}\frac{\partial^2 A_y}{\partial y^2}+\nu_{xx}\frac{\partial^2 A_y}{\partial z^2}+\nu_{yy}\frac{\partial^2 A_x}{\partial y\partial x}\\&-\nu_{zz}\frac{\partial^2 A_x}{\partial x\partial y}+\nu_{yy}\frac{\partial^2 A_z}{\partial y\partial z}-\nu_{xx}\frac{\partial^2 A_z}{\partial z\partial y}=-J_{sy}\end{aligned} \quad \text{(A-10)}$$

$$\begin{aligned}&\nu_{yy}\frac{\partial^2 A_z}{\partial x^2}+\nu_{xx}\frac{\partial^2 A_z}{\partial y^2}+\nu_{zz}\frac{\partial^2 A_z}{\partial z^2}+\nu_{zz}\frac{\partial^2 A_x}{\partial z\partial x}\\&-\nu_{yy}\frac{\partial^2 A_x}{\partial x\partial z}+\nu_{zz}\frac{\partial^2 A_y}{\partial z\partial y}-\nu_{xx}\frac{\partial^2 A_y}{\partial y\partial z}=-J_{sz}\end{aligned} \quad \text{(A-11)}$$

By comparing (A-6)~(A-8) with (A-9)~(A-11), it indicates that two system have similar governing equations.