Analysis of Propagation Properties in Junctions between Straight and Bent Waveguides Using Cylindrical Functions of Complex Order

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Abstract: This paper presents a method to determine the propagation properties in the junctions between straight and bent waveguides using cylindrical functions of complex order. The proposed method was succeeded by developing the method of numerical calculation of cylindrical functions of complex order. As a result, we numerically calculate the reflection and transmission coefficients in the junctions in various situations, and we compare these results with the results by the perturbation method.

1. Introduction

Propagation in rectangular waveguide bends has been investigated by many researchers [1]-[4]. Among them, Mahmoud [1] exactly calculated the propagation constants and the field distribution in waveguide bends, but he examined only the modes whose propagation constants are real. Cochran et al. [2] produced extensive numerical results for the phase factors. These results, however, are valid only for dominant modes. T. Kihara [3] and Lewin et al. [4] used the perturbation method for calculating phase constants and the reflection coefficients of the junctions between straight and curved waveguides. However, the perturbation method can not calculate the reflection coefficients in the junctions for higher order modes. It seems that we do not still have any good method to determine the propagation properties in waveguide bends and in junctions between straight and bent waveguide for arbitrary modes.

In this paper, the authors proposed a method to determine the propagation properties in the junctions between straight and bent waveguide using cylindrical functions of complex order. Since the methods of numerical calculation of cylindrical functions of complex order are not generally known, the authors have developed the methods of numerical calculation of cylindrical functions of complex order [5]-[8]. Accordingly, the proposed method was succeeded in analyzing the propagation properties in the junctions for arbitrary modes. The numerical results obtained by this method are compared with the results by the perturbation method. This proposed method using the cylindrical functions of complex order can also be applied to the bends of slab waveguides, the radiation from slots on conducting cylinders and so forth.

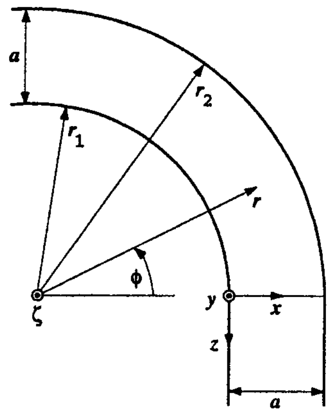


Fig. 1 Top view of a junction between straight and H-plane bend having rectangular cross section of width a and height b. The waveguide is in $0 \le y \le b$. $y = \zeta$.

2. Formulation

Figure 1 shows the configuration of a junction between straight and H-plane bend. All mettalic walls are assumed perfect conductor and all other losses are neglected. The rectangular coordinates are used for the straight waveguide in $z \ge 0$, and the cylindrical coordinates are used for the curved waveguide in $\phi \ge 0$.

The fields in the straight waveguide consist of TE_{n0} modes:

$$E_y = j\omega\mu \sum_{n=1}^{N} K_n^h \sin\frac{n\pi x}{a} \times \{A_n^o \exp(-j\beta_n z) + A_n^i \exp(j\beta_n z)\}, \quad (1a)$$

$$H_x = -\sum_{n=1}^{N} j\beta_n K_n^h \sin\frac{n\pi x}{a} \times \{A_n^o \exp(-j\beta_n z) - A_n^i \exp(j\beta_n z)\}, \quad (1b)$$

$$E_x = E_z = 0, \qquad H_y = 0, \quad (1c)$$

where A_n^i and A_n^o are input and output constants. Here we use the normalization factor K_n^h to satisfy the reciprocity [9], and K_n^h are determined so that the following equation must be satisfied at $z = 0$.

$$\int_0^a dx \int_0^b dy E_y(-H_x) = \sum_{n=1}^{N} \{(A_n^o)^2 - (A_n^i)^2\}.$$

Accordingly K_n^h is given by

$$K_n^h = \sqrt{-\frac{2}{ab\omega\mu\beta_n}}\,. \tag{2}$$

The phase constants β_n are given by

$$k^2 - \beta_n^2 - \frac{(n\pi)^2}{a^2} = 0\,, \qquad k = \omega\sqrt{\epsilon\mu},$$

$$\mathrm{Re}\,\beta_n \geq 0, \qquad \mathrm{Im}\,\beta_n \leq 0\,.$$

The description of H_z is omitted in Eqs. (1). Theoretically N in Eqs. (1) must be infinity, but we break off the series at the term N for numerical calculation.

Similarly, the fields in H-plane bend are

$$E_\zeta = j\omega\mu \sum_{n=1}^{N} L_n^h F_{\nu_n}(kr) \times \{B_n^o \exp(-j\nu_n\phi) + B_n^i \exp(j\nu_n\phi)\}\,, \tag{3a}$$

$$H_r = \sum_{n=1}^{N} \frac{j\nu_n L_n^h}{r} F_{\nu_n}(kr) \times \{B_n^o \exp(-j\nu_n\phi) - B_n^i \exp(j\nu_n\phi)\}\,, \tag{3b}$$

$$E_\phi = E_r = 0\,, \qquad H_\zeta = 0\,. \tag{3c}$$

where B_n^i and B_n^o are input and output constants. The normalization factors L_n^h are determined so that the following equation can be satisfied at $\phi = 0$.

$$\int_{r_1}^{r_2} dr \int_0^b d\zeta E_\zeta H_r = \sum_{n=1}^{N} \{(B_n^o)^2 - (B_n^i)^2\}\,.$$

Accordingly, L_n^h are given by

$$L_n^h = \frac{1}{\sqrt{-b\omega\mu\nu_n M_n}}\,, \tag{4}$$

where

$$M_n^h = \int_{r_1}^{r_2} \frac{\{F_{\nu_n}(kr)\}^2}{r} dr, \tag{5}$$

where r_1 and r_2 are shown in Fig. 1 and $a = r_2 - r_1$.

The functions $F_{\nu_n}(kr)$ are defined as

$$F_{\nu_n}(kr) = J_{\nu_n}(kr)N_{\nu_n}(kr_1) - J_{\nu_n}(kr_1)N_{\nu_n}(kr), \tag{6}$$

where $J_{\nu_n}(kr)$ are Bessel functions, and $N_{\nu_n}(kr)$ are Neumann functions. $F_{\nu_n}(kr)$ also satisfies Bessel's differential equation

$$\frac{d^2F_{\nu_n}(kr)}{dr^2} + \frac{1}{r}\frac{dF_{\nu_n}(kr)}{dr} + \left(k^2 - \frac{\nu_n^2}{r^2}\right) F_{\nu_n}(kr) = 0, \tag{7}$$

under the boundary conditions:

$$F_{\nu_n}(kr_1) = 0\,, \qquad F_{\nu_n}(kr_2) = 0\,. \tag{8}$$

The first equation of Eq. (8) is always satisfied. ν_n ($n = 1, 2, \cdots$) are the roots of the second equation of Eq. (8), and ν_n^2 are the eigenvalues of the eigenvalue problem. Since Eq. (7) is Sturm-Liouville's differential equation, the following equations are satisfied [10].

$$\mathrm{Im}\,\nu_n^2 = 0, \tag{9a}$$

$$\nu_1^2 \geq \nu_2^2 \geq \cdots \geq \nu_n^2 \geq \cdots\,, \tag{9b}$$

$$\nu_n^2 \to -\infty, \qquad (n \to \infty)\,, \tag{9c}$$

$$\mathrm{Re}\,\nu_n \geq 0, \qquad \mathrm{Im}\,\nu_n \geq 0\,. \tag{9d}$$

$F_{\nu_n}(kr)$ are real functions even when $\nu_n^2 < 0$. When $\nu_n^2 < 0$, $F_{\nu_n}(kr)$ becomes

$$F_{\nu_n}(kr) = -2\,\mathrm{Im}\{J_{-j\mu_n}(kr_1)J_{j\mu_n}(kr)\}/\sinh\mu_n\pi,$$

where $\nu_n = j\mu_n$. $F_{\nu_n}(kr)$ are orthogonal:

$$\int_{r_1}^{r_2} \frac{F_{\nu_m}(kr)\,F_{\nu_n}(kr)}{r} dr = 0 \qquad (m \neq n). \tag{10}$$

Let us now determine A_n^i, A_n^o, B_n^i and B_n^o from the continuity of fields at junction $z = 0$ or $\phi = 0$:

$$E_x|_{z=+0} = E_r|_{\phi=+0}, \tag{11a}$$
$$E_y|_{z=+0} = E_\zeta|_{\phi=+0}, \tag{11b}$$
$$H_x|_{z=+0} = H_r|_{\phi=+0}, \tag{11c}$$
$$H_y|_{z=+0} = H_\zeta|_{\phi=+0}. \tag{11d}$$

Equations (11a) and (11d) are already satisfied. We must determine A_n^i, A_n^o, B_n^i and B_n^o so that Eqs. (11b) and (11c) can be satisfied. Using Eqs. (1a), (3a) for (11b) and Eqs. (1b), (3b) for (11c), we get the following reduced equations.

$$A_m^o + A_m^i = \sum_{n=1}^{N}(B_n^o + B_n^i)R_{mn}^h\,, \tag{12}$$

$$\sum_{n=1}^{N}(A_n^o - A_n^i)R_{nm}^h = -B_m^o + B_m^i\,, \tag{13}$$

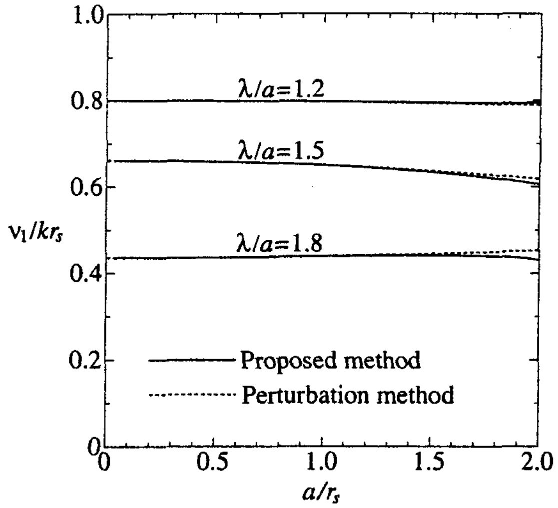
where

$$R_{mn}^h = \frac{P_{mn}^h}{ak}\sqrt{\frac{2\beta_m a}{\nu_n M_n^h}}\,, \tag{14a}$$

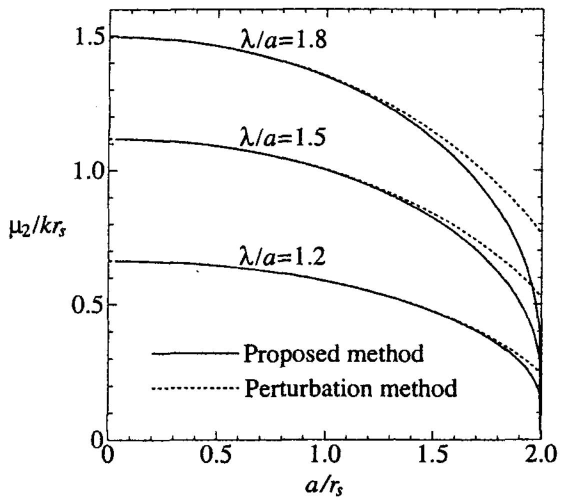
$$P_{mn}^h = \int_{r_1}^{r_2} \sin\frac{m\pi x}{a} F_{\nu_n}(kr) k dr\,. \tag{14b}$$

$$m = 1, 2, \cdots, N\,,$$

Equations (12) and (13) are the simultaneous linear equations to be solved for the unknown output constants $A_1^o, \cdots, A_N^o$ and $B_1^o, \cdots, B_N^o$ when the input constants $A_1^i, \cdots, A_N^i$ and $B_1^i, \cdots, B_N^i$ are known.



(a) The graph of ν_1/kr_s.



(b) The graph of μ_2/kr_s. $\nu_2 = j\mu_2$.

Fig. 2 The phase constants for the H-plane bends calculated by the proposed method and by the perturbation method. $r_s = (r_1 + r_2)/2$. $a = r_2 - r_1$. $\lambda = 2\pi/k$.

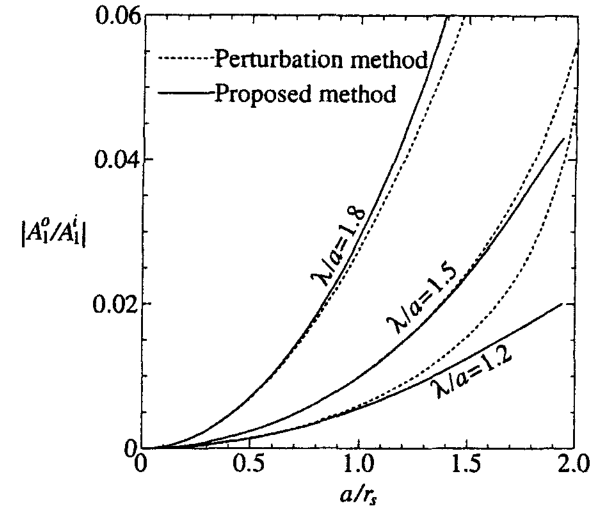


Fig. 3 The reflection coefficients $|A_1^o/A_1^i|$ calculated by the proposed method for $N = 3$, and $|A_1^o/A_1^i|$ by the perturbation method.

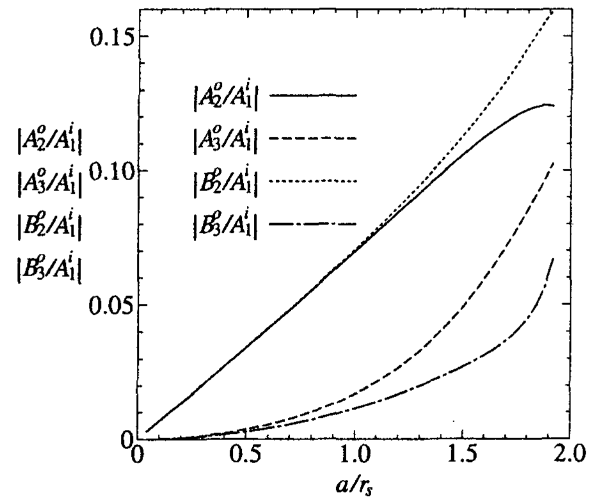


Fig. 4 The reflection and transmission coefficients of higher order mode calculated by the proposed method when $\lambda/a = 1.5$, and $N = 3$.

3. Results of Numerical Calculations

First, we calculate ν_n by solving the second equation of Eq. (8). At the same time, we calculate ν_n by the perturbation method [3]. Figure 2(a) shows ν_1/kr_s calculated by each method. Figure 2(b) shows the results of μ_2/kr_s, where $\nu_2 = j\mu_2$. From Fig. 2, it is seen that ν_n calculated by the perturbation method approaches ν_n by the proposed method as $a/r_s \to 0$.

Next, we calculate the reflection and transmission coefficients in the junctions, which are defined by $|A_m^o/A_1^i|$ and $|B_m^o/A_1^i|$ respectively. In our experiment, we use $A_1^i = 1$, $A_2^i = A_3^i = 0$, and $B_m^i = 0$. The output constants A_m^o and B_m^o are determined by solving the Eqs. (12) and (13) simultaneously. Figure 3 shows the reflection coefficient $|A_1^o/A_1^i|$ calculated at $N = 3$, and it also shows $|A_1^o/A_1^i|$ calculated by the perturbation method. It seems from Fig. 3 that the differences between these two results increase as a/r_s increases. Figure 4 shows $|A_n^o/A_1^i|$ and $|B_n^o/A_1^i|$ for $n \geq 2$ calculated by the proposed method at $N = 3$.

By the numerical experiments, it has been ascertained that $|A_n^o/B_1^i| \fallingdotseq |B_n^o/A_1^i|$, and $|B_n^o/B_1^i| \fallingdotseq |A_n^o/A_1^i|$ $(n \geq 2)$. Because of the the normalization factors K_n^h and L_n^h, we can prove numerically that the reciprocity is satisfied:

$$|B_1^o/A_1^i| = |A_1^o/B_1^i|.$$

Since $|B_1^o/, A_1^i| \fallingdotseq 1$ then figures of $|B_1^o/, A_1^i|$ are not illustrated.

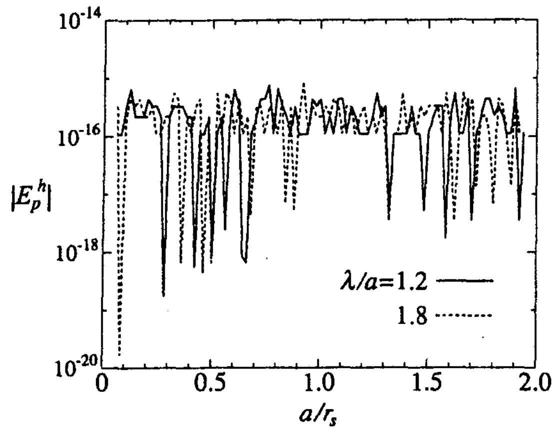


Fig. 5 The graph of $|E_p^h|$ for $N = 3$.

Finally, we examine the law of consevation of energy at the junction $z = 0$ by calculating the error power E_p^h. We define E_p^h as

$$E_p^h = \frac{P_{out,s} + P_{out,c}}{|P_{in}|} \tag{15}$$

where $P_{out,s}$ and $P_{out,c}$ are the output power in straight and bent waveguide respectively. P_{in} is the input power. $P_{out,s}$, $P_{out,c}$ and P_{in} can be obtained by

$$P_{out,s} = \frac{1}{2}\sum_{n=1}^{N} \frac{\beta_n^*}{|\beta_n|}(A_n^o + A_n^i)(A_n^o - A_n^i)^*, \tag{16a}$$

$$P_{out,c} = \frac{1}{2}\sum_{n=1}^{N} \frac{\nu_n^*}{|\nu_n|}(B_n^o + B_n^i)(B_n^o - B_n^i)^*, \tag{16b}$$

$$P_{in} = \frac{1}{2}\sum_{n=1}^{N} \frac{\beta_n^*}{|\beta_n|}|A_n^i|^2 + \frac{1}{2}\sum_{n=1}^{N} \frac{\nu_n^*}{|\nu_n|}|B_n^i|^2 \tag{16c}$$

where the asterisk $*$ denotes the complex conjugate. It is ascertained by numerical experiments that the error power $|E_p^h| < 1 \times 10^{-15}$ as shown in Fig. 5.

4. Conclusions

The proposed method can be used well for determining the propagation properties in junctions between straight and H-plane bends of the rectangular waveguides for arbitrary modes. In future, we intend to apply the proposed method to the analysis of the fields in bends of nonradiative dielectric waveguides and slab waveguides.

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