Multiple-Criteria Decision-Making Using Sectional Supermatrix

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Abstract: Presently, Analytic Network Process which evaluates man's intention and offers decision-making support is capturing the spotlight. It originates in the ability of Analytic Network Process to treat various decision-making support system. However, no detailed reference is available when dealing with the group case. This paper examines the technique, which can also cope with the group decision-making support system, and describes the validity of the technique.

A characteristic feature of the proposed technique is that it can detect a group's intention in a given section, and it decomposes the sectional supermatrix into a small supermatrix and a large one. A general supermatrix treats the convergence value by taking the limiting process method of the power of an evaluation value. On the other hand, when a supermatrix has nonnegative value, it can easily be solved by the eigenvector method. The decomposition of the supermatrix has been considered in this work.

1. Introduction

Construction of a system which offers the effective information for man's decision-making, i.e., a "decision support system", attracts attention from progress of many technologies, such as the latest workstation, GUI, AI, and multimedia [1]. There are various scenes where decision-making using a computer is expected. For example, the importance of the system that supports everyday decision-making in an individual, decision-making in planning by a lot of people and the management of a company is recognized. Therefore, it is important to determine an algorithm for deriving the weight of evaluation criteria that the decision-maker thinks of, when he makes decision-making support system.

As an algorithm that evaluates weight of the evaluation criteria of which man thinks, there is Analytic Hierarchy Process (AHP) that Saaty proposed [2]. The feature of this technique is in the point of evaluating alternative based on pairwise comparison, using the principal eigenvector for the analyzing method and evaluating structure as a hierarchy structure. Analytic Network Process(ANP) is regarded as an extension of AHP of which this hierarchy structure is extended to network structure[3]. The application field has spread to great widths since ANP was proposed. However, ANP is not mentioned in detail about the decision-making support system for a group intention. Therefore, this paper examines the technique to treat group decision-making in ANP. Various decision-making support can be offered by applying this technique.

2. ANP

ANP analysis is used to evaluate the weights of objects by so called supermatrix. The hierarchy structure of ANP is shown in Figure 2.1.

When there are criteria $C_1,..,C_i$ and alternatives $A_1,...,A_j$, Saaty called the matrix that is composed of evaluation values W_{ji} of alternative and evaluation values V_{ij} of criteria as shown in (2.1), a supermatrix.

$$S = \begin{array}{c} \\ C_1 \\ \vdots \\ C_i \\ A_1 \\ \vdots \\ A_j \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & \cdots & C_i & A_1 & \cdots & A_j \end{array} \\ \begin{pmatrix} 0 & \cdots & 0 & V_{11} & \cdots & V_{1j} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & V_{i1} & \cdots & V_{ij} \\ W_{11} & \cdots & W_{1i} & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ W_{j1} & \cdots & W_{ji} & 0 & \cdots & 0 \end{pmatrix} \end{array} \quad (2.1)$$

A supermatrix can deal with both criteria and alternatives as an quivalent. The purpose of the analysis of ANP is calculating the comprehensive target evaluation value of alternatives and criteria, when this supermatrix is given. It derives the limiting process method of the power of a supermatrix in ANP. Each sequence vectors of all of this limit supermatrix turn into the same sequence vector, and treat each element of a sequence vector as a comprehensive evaluation value.

When a supermatrix has nonnegative, irreducible, and stochastic natures, an evaluation matrix can be obtained by taking the limiting process method of the power of a supermatrix in ANP. However, when it is neither irreducible nor stochastic, reference is not made in detail. On the other hand, Sekitani and Takahashi [4] showed that the evaluation vector obtained by the solution method of the usual ANP and the principal eigenvector of a supermatrix are in agreement.

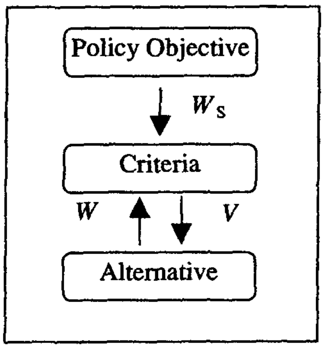


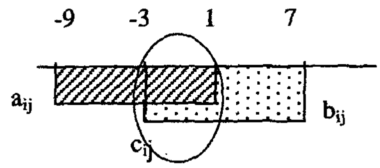
Figure 2.1 Structure of ANP

3. Sectional ANP

In order to express a group intention, a section is used by this technique. In this technique, individuals in the group (decision-maker) are asked to provide pairwise comparison form of answer, in which case, a reply is obtained not with one value but with a sectional value. The reply is totaled and concentration of the range accepted without each people's resistance in a group intention shows the section.

1) When a common section exists in the section

When a common section exists between the sections that each person gave, then let the maximum section of the common section be the group section.



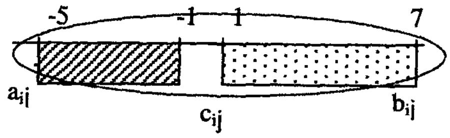
a_{ij}=[-9,1] The sectional value of which A gives
b_{ij}=[-3,7] The sectional value of which B gives
c_{ij}=[-3,1] The group sectional value

Figure 3.1 When a common section exists

2) When a common section does not exist in the section

When a common section does not exist between the sections which each parson gave, then let the minimum section be the pairwise comparison section for the group.

Next, although a sectional supermatrix is created from the grouped section, it is impossible to take the limiting process method of power in the sectional supermatrix,



a_{ij}=[-5,-1] The sectional value of which A gives
b_{ij}=[1,7] The sectional value of which B gives
c_{ij}=[-5,7] The group sectional value

Figure 3.2 When a common section does not exist

Table 3.1 Algorithm of Sectional ANP

Step1: Ask the people in a pairwise comparison form, and show with a section value.
Step2: The group section is created based on the shown result.
Step3: A sectional supermatrix is created from the group section.
Step4: Divides the sectional supermatrix into a small portion and a large portion. A small supermatrix and a large supermatrix are created.
Step5: Normalization is performed in each sectional supermatrix.
Step6: The principal eigenvectors are computed from each sectional supermatrix.
Step7: A sectional principal eigenvector is created by combining a small sectional principal eigenvector and a large principal sectional eigenvector.
Step8: When a reversal takes place of a small portion and a large portion in a sectional principal eigenvector, exchange them.
Step9: A sectional principal eigenvector is treated as an evaluation weight, and it is taken as the evaluation value.

neither the principal eigenvector be used. Then, by this technique, each sectional value which forms the sectional supermatrix is divided into a small portion and a large portion, and a smallsupermatrix and a large supermatrix are created. By the proposed technique of Saaty, an evaluation value is acquired by taking the limiting process method of power of a supermatrix to the supermatrix with nonnegativity, irreducibility and stochastic. However, in that technique, reference is not made in detail about the general solution method of the supermatrix. On the other hand, by Sekitani and Takahashi, a solution technique is proposed in the case that the supermatrix is nonnegative. It is the technique of using the principal eigenvector, and is generally treated in AHP. Step4 of Table 3.1 considers calculation of the supermatrix using the principal eigenvector.

Next, by computing the principal eigenvector in the small supermatrix and the large supermatrix, and unifying the small principal eigenvector and the large principal eigenvector, the sectional principal eigenvector is formed. There is a case where the size relation is reversed by the small principal eigenvector and the large principal eigenvector. However, for the sectional principal eigenvector, since all the principal eigenvectors are contained, it is solvable by replacing them in order that the size relation between the minimum value and maximum may be reversed.

4. Numerical Example

A questionnaire of selecting one considered to be superior between two Web contents for education purposes to five subjects survey is conducted, and the numerical example that applies this technique is shown. In this example, the Web contents 1 consist of a slide and a sound, and Web contents 2 consist of slides, sounds, and images.

Example

Policy Objective : Superior web contents selection

Criteria : The play heart, Intelligibility, Conspicuousness, Feeling of presence

Alternatives : The Web contents 1, Web contents 2

First, the 5 college student objects are asked in pairwise comparison form according to step1.

$$\begin{bmatrix} 0.0 & 0.0 & [0.127\ 0.290] & [0.080\ 0.790] & [0.060\ 0.873] & [0.060\ 0.290] \\ 0.0 & 0.0 & [0.710\ 0.873] & [0.210\ 0.920] & [0.127\ 0.920] & [0.710\ 0.920] \\ [0.040\ 0.333] & [0.055\ 0.060] & 0.0 & 0.0 & 0.0 & 0.0 \\ [0.254\ 0.294] & [0.270\ 0.310] & 0.0 & 0.0 & 0.0 & 0.0 \\ [0.214\ 0.556] & [0.078\ 0.528] & 0.0 & 0.0 & 0.0 & 0.0 \\ [0.192\ 0.495] & [0.276\ 0.661] & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{4.1}$$

In Step4, by dividing the elements of the sectional supermatrix into small portion and large portion, the small supermatrix and the large supermatrix are created as shown below.

Normalization is performed in Step5 and the principal vector is computed in Step7.

Finally, the part that the reversal has generated of the small portion and the large portion in the sectional principal eigenvector in Step8 is corrected.

The small supermatrix

$$\begin{bmatrix} 0.0 & 0.0 & 0.127 & 0.080 & 0.060 & 0.060 \\ 0.0 & 0.0 & 0.710 & 0.210 & 0.127 & 0.710 \\ 0.040 & 0.055 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.254 & 0.270 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.214 & 0.078 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.192 & 0.276 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{4.2}$$

The large supermatrix

$$\begin{bmatrix} 0.0 & 0.0 & 0.290 & 0.790 & 0.873 & 0.290 \\ 0.0 & 0.0 & 0.873 & 0.920 & 0.920 & 0.920 \\ 0.333 & 0.060 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.294 & 0.310 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.556 & 0.528 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.495 & 0.661 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{4.3}$$

$$\begin{bmatrix} 0.145 & 0.388 \\ 0.855 & 0.612 \\ 0.077 & 0.103 \\ 0.392 & 0.189 \\ 0.143 & 0.336 \\ 0.387 & 0.372 \end{bmatrix} \tag{4.4}$$

$$\begin{bmatrix} 0.145 & 0.388 \\ 0.612 & 0.855 \\ 0.077 & 0.103 \\ 0.189 & 0.392 \\ 0.143 & 0.336 \\ 0.372 & 0.387 \end{bmatrix} \tag{4.5}$$

This section weight vector is five college students' group evaluation value. The result gives the general thinking of the object group with respect to the contents of a web site. The object group thinks that presence is important in web contents.It is estimated that the web contents 2 are superior than the first one.

5. Conclusion and Future Extensions

In the conventional ANP, the grouped decision-making method has not been made in detail. This paper examined the technique to treat the evaluation for the group. It is possible by the proposed technique to offer various decision-making support qualities. However, when the common section does not exist in the group intention, and when deviation and spread arise to the group intention, it is difficult to obtain the evaluation which safisfies all groups. Also, the common sections seldom exist when the number constituting the group increase. This problem remains as a future subject.

References

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