Design and Multiplier-Free Realization of FIR Nyquist Filters with Coefficients Taking Only Discrete Values

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Abstract: This paper presents a design of FIR near-equiripple Nyquist filters having zero-intersymbol interference (ISI) and low sensitivity to timing jitter, with coefficients taking only discrete values. Using an affine scaling linear programming algorithm, an optimum discrete coefficient set can be obtained in a feasible computational time.

Also presented is a pipelined multiplier-free FIR filter realization with periodically time-varying (PTV) coefficients based on a hybrid form suitable for Nyquist filter. The realization exploits the coefficient symmetry to reduce the hardware by about one half. High speed computation and low power consumption are achieved by its pipelined and low fan-out structure.

1. Introduction

Nyquist or Mth-band filters are used in digital signal processing applications, such as filter banks, non-uniform sampling, and interpolation [1]. In communications, they are found in antenna array design and pulse shaping for zero intersymbol interference (ISI) [2]. The impulse responses of continuous-time Nyquist filters have zero-crossings at every T seconds, where T is the bit duration of the transmitted data. For digital Nyquist filters, the sampling rate ($1/T_s$) is chosen such that the zero-crossings are exactly at the sampling instants. That is, all coefficients at multiples of M are zero except the center coefficient, where $M = T/T_s$.

The challenge of Nyquist filter design is to find a finite set of coefficients which satisfies the zero ISI criterion and has low sensitivity to timing jitter (low sidelobe energy) [3]. In addition, it is desirable to have an equiripple frequency response [4]. Recently, Farhang-Boroujeny and Mathew [2] proposed a new technique for designing Nyquist filters with the above criteria. It is based on windowing method and Remez exchange algorithm. However, the exact zero-crossings may be lost after applying the Remez algorithm.

In this paper, we propose a Nyquist filter design based on the above three criteria, as well as having discrete-valued coefficients. The design is based on an affine scaling linear programming (LP) algorithm [5]. Unlike [2], exact zero crossings are obtained. By changing a weighting factor, a tradeoff between frequency-response ripples and impulse-response sidelobe energy is possible.

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2. Problem Formulation

2.1 *Minimizing Peak Error in the Frequency Domain*

Consider a linear-phase filter $H(\omega)$ of length N with symmetric (or antisymmetric) coefficients written as

$$H(\omega) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n} = \sum_{i=1}^{L} a(i)\mathrm{trig}(\omega, i), \quad (1)$$

where $\mathrm{trig}(\omega, i)$ denotes an appropriate trigonometric function, $a(i)$ is a function of the filter coefficients $h(j)$, and L is the number of coefficients after counting each pair of symmetric/antisymmetric coefficients as one. A Nyquist filter is a lowpass filter with odd length, with, $L = (N+1)/2$, $a(1) = h((N-1)/2)$, $a(i) = 2h((N+1)/2 - i)$ for $i \neq 1$, $\mathrm{trig}(\omega, i) = \cos(\omega(i-1))$, and $h(jM + (N-1)/2) = 0$ for $j \neq 0$. We shall use $\mathbf{1}_Q$ to denote a $Q \times 1$ vector with all elements equal to one, $\mathbf{0}_Q$ to denote a $Q \times 1$ zero vector, $\mathbf{0}_{PQ}$ to denote $P \times Q$ zero matrix, and $\boldsymbol{I}_P$ to denote $P \times P$ identity matrix.

Let $H_t(\omega)$ be the response of the ideal (target) filter. We wish to find $H(\omega)$ that minimizes the weighted peak error

$$E = \max_{0 \leq \omega \leq \pi} W(\omega)|H_t(\omega) - H(\omega)|, \quad (2)$$

where $W(\omega)$ is a non-negative weighting function which is zero in the transition band. Using Q frequency points, an LP optimization problem can be formulated as

$$\min_{\boldsymbol{a}} \epsilon_f \quad (3)$$

subject to

$$\boldsymbol{H}_t - \boldsymbol{Ta} \leq \epsilon_f \mathbf{1}_Q \text{ and } -\boldsymbol{H}_t + \boldsymbol{Ta} \leq \epsilon_f \mathbf{1}_Q, \quad (4)$$

where

$$H_t = [\ W(\omega_1)H_t(\omega_1)\ \ \dots\ \ W(\omega_Q)H_t(\omega_Q)\ ]^T$$

$$a = [\ a(1)\ \ a(2)\ \ \dots\ \ a(L)\ ]^T$$

$$T = \begin{bmatrix} W(\omega_1)\mathrm{trig}(\omega_1,1) & \cdots & W(\omega_1)\mathrm{trig}(\omega_1,\mathrm{L}) \\ W(\omega_2)\mathrm{trig}(\omega_2,1) & \cdots & W(\omega_2)\mathrm{trig}(\omega_2,\mathrm{L}) \\ \vdots & \vdots & \vdots \\ W(\omega_Q)\mathrm{trig}(\omega_Q,1) & \cdots & W(\omega_Q)\mathrm{trig}(\omega_Q,\mathrm{L}) \end{bmatrix}$$

2.2 Minimizing the Absolute Summation of Sidelobe in the Time Domain

To reduce the effect of timing jitter, the sidelobe energy of the impulse response should be minimized [3], subject to a constant total energy. This can be formulated as a least mean square (L_2) problem, but it is not suitable for LP algorithm. Therefore, we use L_1 error criterion instead. The objective is to minimize the absolute summation of the sidelobe of $\boldsymbol{a}$,

$$E_s = \sum_{i=M+1}^{L} |a(i)| \tag{5}$$

subject to the constant total absolute summation of $\boldsymbol{a}$,

$$E_o = \sum_{i=1}^{L} |a(i)|. \tag{6}$$

The L_1 error criterion can be formulated as

$$\min_{\boldsymbol{a}} \sum_{i=1}^{P} \epsilon_t(i) \tag{7}$$

subject to

$$|a(i+M)|/E_o \leq \epsilon_t(i),\ 1 \leq i \leq P, \tag{8}$$

where $P = L - M$ is the number of sidelobe coefficients and $\epsilon_t(i)$ denotes the upper bound of $|a(M+i)|/E_o$. E_o can be estimated from the coefficients $\boldsymbol{a}$ in the previous iteration. In matrix form, (7) can be written as

$$\min_{\boldsymbol{a}}\ \mathbf{1}_P\boldsymbol{\epsilon}_t \quad \text{subject to} \quad -\boldsymbol{D}\boldsymbol{a} \leq \boldsymbol{\epsilon}_t \text{ and } \boldsymbol{D}\boldsymbol{a} \leq \boldsymbol{\epsilon}_t, \tag{9}$$

where $\boldsymbol{\epsilon}_t = [\epsilon_t(1)\ \ \epsilon_t(2)\dots\epsilon_t(P)]^T$ and $\boldsymbol{D} = [\mathbf{0}_{PM}\ \ \boldsymbol{I}_P/E_o]$.

2.3 Nyquist Constraint

A Nyquist filter is usually designed as a lowpass FIR filter having cut-off frequency at π/M. Its impulse response must satisfy the constraint

$$a(n) = \begin{cases} 1/M, & n = 1 \\ 0 & n = kM+1, k = 1,2,\dots,l \end{cases} \tag{10}$$

where $l = \lfloor (L-1)/M \rfloor$, the interger part of $(L-1)/M$. The passband and stopband edges of a Nyquist filter are normally given by $(1-\beta)\pi/M$ and $(1+\beta)\pi/M$, where β is the roll-off factor. The constraint (10) can be written as

$$-\boldsymbol{C}\boldsymbol{a} \leq -\boldsymbol{l} \quad \text{and} \quad \boldsymbol{C}\boldsymbol{a} \leq \boldsymbol{l} \tag{11}$$

where $\boldsymbol{l} = [\frac{1}{M}\ 0\ \dots\ 0]^T$ and $\boldsymbol{C} = [\boldsymbol{i}_1\ \boldsymbol{i}_{M+1}\ \dots\ \boldsymbol{i}_{lM+1}]^T$, with $\boldsymbol{i}_k$ denoting the k-th column of the $L \times L$ identity matrix. The size of $\boldsymbol{C}$ is $(l+1) \times L$.

2.4 Combined Criterion

The design problem based on the three design constraints (3), (9), and (11) can be expressed as an LP problem as

$$\max_{\boldsymbol{w}}\ \boldsymbol{b}^T\boldsymbol{w} \quad \text{subject to} \quad \boldsymbol{A}^T\boldsymbol{w} \leq \boldsymbol{c}, \tag{12}$$

where

$$\begin{aligned} \boldsymbol{b} &= [\ \mathbf{0}_L^T\ \ 1\ \ \lambda\mathbf{1}_P^T\ ]^T \\ \boldsymbol{w} &= [\ -\boldsymbol{a}^T\ \ -\epsilon_f\ \ -\boldsymbol{\epsilon}_t^T\ ]^T \\ \boldsymbol{c} &= [\ -\boldsymbol{H}_t^T\ \ \boldsymbol{H}_t^T\ \ -\boldsymbol{l}^T\ \ \boldsymbol{l}^T\ \ \mathbf{0}_P^T\ \ \mathbf{0}_P^T\ ]^T \\ \boldsymbol{A} &= \begin{bmatrix} \boldsymbol{T}^T & -\boldsymbol{T}^T & \boldsymbol{C}^T & -\boldsymbol{C}^T & \boldsymbol{D}^T & -\boldsymbol{D}^T \\ \mathbf{1}_Q^T & \mathbf{1}_Q^T & \mathbf{0}_K^T & \mathbf{0}_K^T & \mathbf{0}_P^T & \mathbf{0}_P^T \\ \mathbf{0}_{PQ} & \mathbf{0}_{PQ} & \mathbf{0}_{PK} & \mathbf{0}_{PK} & \boldsymbol{I}_P & \boldsymbol{I}_P \end{bmatrix}. \end{aligned}$$

Here, λ is a positive weighting factor. A higher λ places less energy in the sidelobe coefficients, but it increases the stopband ripples. Therefore, the choice of λ provides a tradeoff.

The problem given by (12) is called the dual problem in LP optimization. Its associated minimization problem, called the primal problem, is

$$\min_{\boldsymbol{x}}\ \boldsymbol{c}^T\boldsymbol{x} \quad \text{subject to} \quad \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \text{ and } \boldsymbol{x} \geq \mathbf{0}_R \tag{13}$$

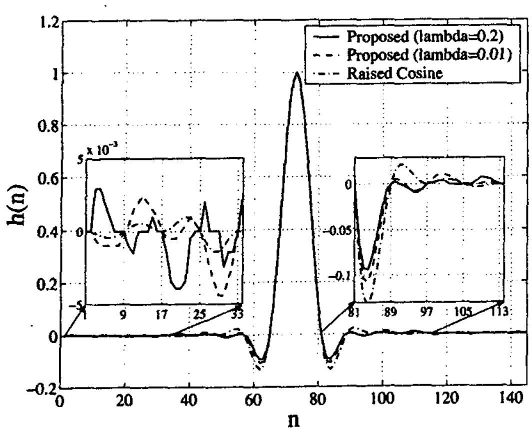
where $R = 2Q + 2(l+1) + 2P$ and $\boldsymbol{x} = [\boldsymbol{x}_{t1}\ \boldsymbol{x}_{t2}\ \boldsymbol{x}_{c1}\ \boldsymbol{x}_{c2}\ \boldsymbol{x}_{d1}\ \boldsymbol{x}_{d2}]^T$ is the vector of variables to be solved, with $\boldsymbol{x}_{t1}$ and $\boldsymbol{x}_{t2}$ being $1 \times Q$ vectors, $\boldsymbol{x}_{c1}$ and $\boldsymbol{x}_{c1}$ being $1 \times (l+1)$ vectors, and $\boldsymbol{x}_{d1}$ and $\boldsymbol{x}_{d1}$ being $1 \times P$ vectors.

In the primal-dual problem, solving (12) yields the same result as solving (13). Specifically, the solution $\boldsymbol{w}$ of (12) is related to the solution $\boldsymbol{x}$ of (13) by $\boldsymbol{c}^T\boldsymbol{x} = \boldsymbol{b}^T\boldsymbol{w}$. An affine scaling algorithm [5] can then be applied to solve the above primal-dual problem. Discrete coefficients can be obtained by rounding the step direction vector during each iteration in the algorithm. Detail of the algorithm is omitted in this paper due to the space limitation.

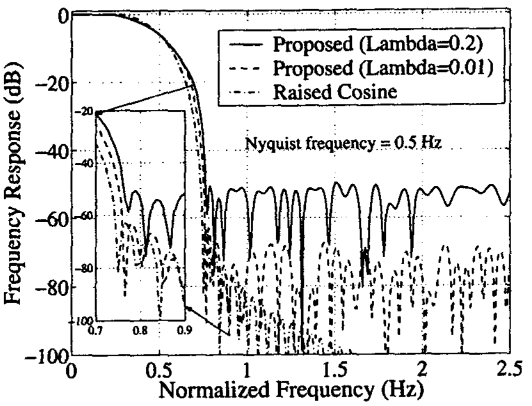
3. Example

The same design specification of a Nyquist filter as [2] is illustrated here. The filter length N is 145, the oversampling factor M is 8, and the roll-off factor β is 0.5. The results are compared to the conventional raised cosine Nyquist filter with real coefficients. Unlike the algorithm in [2], the proposed algorithm can obtain a filter with discrete coefficients and with exact zero ISI. Figure 1 shows the impulse and frequency responses of the resulting Nyquist filters whose coefficients are 6-digit radix-4 numbers. The ripples are near but not exactly equiripple due to the discrete values of the coefficients. Tradeoff between sidelobe energy in the time domain and peak ripple error in the frequency domain is observed as λ changes. We see from the magnification of the impulse responses that the two designs with $\lambda = 0.2$ and 0.01 have exact zero crossings, with less sidelobe energy than the raised cosine design.

Because the filter coefficients in [2] are not available, we cannot compare it with our designs.



(a)Impulse response



(b)Frequency response

Figure 1. Nyquist FIR filter designed using the proposed algorithm with discrete coefficients and raised cosine filter.

4. Hybrid Realizations

The two canonical structures of FIR filters, the direct and transposed forms, suffer from significant drawbacks. The direct form has a long critical delay path of successive additions which hinders the processing speed. The transposed form is good in terms of speed due to its pipelined architecture, but it consumes more power due to high signal fan-out.

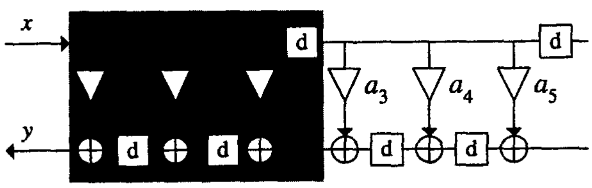


Figure 2. Hybrid form of FIR filter.

A hybrid form [6] of FIR filter is a compromise between the direct and transposed forms. The long critical delay path of the direct form is reduced by a modular and pipelined structure without introducing extra latency. Figure 2 shows a hybrid form comprising 3-tap modules.

It is observed that a hybrid form with M-tap modules is suitable for a Nyquist filter with over-sampling factor M. Since every M-th coefficient is zero, the critical path delay is minimized to the propagation delay of one multiplier and one adder, which is the same as that of the transposed form. Also, the maximum signal fan-out is $M-1$. Hence, a high-speed and low-power structure is achieved. Figure 3 shows a hybrid structure of a 17-tap Nyquist filter with symmetric coefficients and $M = 3$. It comprises 3-tap modules with the last coefficient of each module being zero. The delay d(K) denotes K delay units.

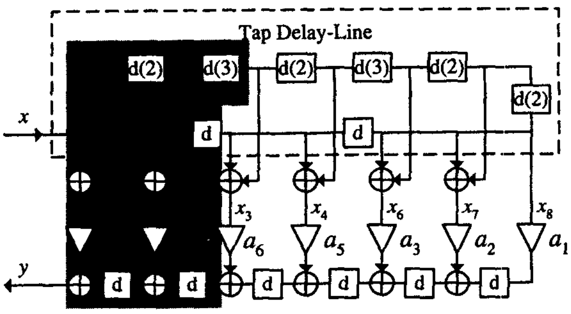


Figure 3. Hybrid form of Nyquist FIR filter.

Due to the discrete-valued coefficients with short word-length, we can obtain a multiplier-free realization for FIR Nyquist filters using the hybrid form with periodically time-varying (PTV) coefficients. Let the coefficient a_i be written in the signed-digit radix-r number as

$$a_i = \widehat{A}\sum_{l=0}^{q-1} \alpha_{il} r^{-l} \tag{14}$$

where $\alpha_{il} \in \{0, \pm 1\}$ for $r = 2$ and $\alpha_{il} \in \{0, \pm 1, \pm 2\}$ for $r = 4$. The constant $\widehat{A}$ is a power-of-two (POT) constant normalizing the range of the representation to cover the range of a_i. Letting $q = JK$, (14) can be written as

$$a_i = \widehat{A}\sum_{k=0}^{K-1}\sum_{j=0}^{J-1} \gamma_{ij}(k) r^{-j} r^{-(K-1-k)J} \tag{15}$$

where $\gamma_{ij}(k) = \alpha_{i(j+q-J-Jk)}$. For the example in Section 3, $r = 4$, $JK = q = 6$, and $\gamma_{ij}(k) \in \{0, \pm 1, \pm 2\}$, which can be implemented without hardware multiplier. We can use $J = 1$ and $K = 6$, or $J = 2$ and $K = 3$, or $J = 3$ and $K = 2$, or $J = 6$ and $K = 1$.

In (15), the filter coefficient a_i is expressed in 2 dimensions (j and k). The variable j represents distribution of the computation in space while the variable k represents distribution of the computation in time.

The proposed PTV structure is shown in Figure 4, where $< \cdot >_K$ denotes modulo K. The coefficient $\gamma_{ij}(m)$ is periodic with period K. ZOI(K) is a zeroth-order interpolator. Its output simply repeats each input value K times at K times faster signal rate. The binary-tree shift and sum (BTSS) unit collects the results that were distributively computed in space, while the scale-and-downsample (SAD) unit collects the results that were

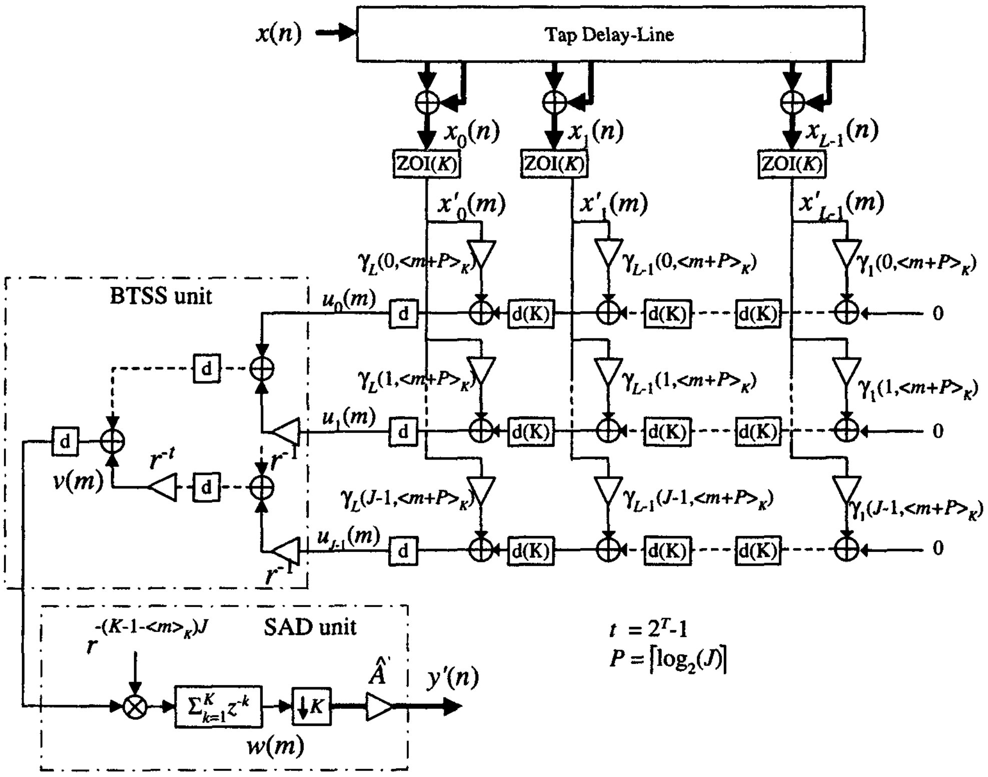


Figure 4. Proposed PTV multiplier-free realization.

distributively computed in time. The block of tap delay line is the same as that in Figure 3.

Besides exploiting the coefficient symmetry, it should be mentioned here that Figure 4 places most of the shifts followed by addition towards the back end of the structure (BTSS and SAD units), so that hardware can be reduced due to the shorter wordlength required by the adders.

5. Conclusion

A design of FIR Nyquist filters with zero-ISI and low sensitivity to timing jitter is presented. The design is based on an affine scaling LP algorithm. The problem is formulated based on three constraints: the Nyquist criterion, the L_∞ criterion of ripple error in the frequency domain, and the L_1 criterion of the sidelobe energy in the time domain. The algorithm provides flexible tradeoff between robustness to timing jitter and stopband attenuation. Also, discrete coefficients can be obtained in a feasible computational time.

In addition, an efficient PTV multiplier-free realization based on a hybrid form is proposed. Its hybrid structure yields high speed computation and low power consumption.

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