A Packet Scheduling for Input-Queued Router with Deadline Constraints

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Abstract: This paper considers a scheduling problem of routers with VOQ(Virtual Output Queue)s, where the router has an $N \times N$ port input-queued switch and each input queue is composed of N VOQs. The objective of the paper is to develope scheduling algorithms which minimize mean tardiness under a common due date. The paper characterizes the optimal solution properties. Based upon the characterization, a integer programming is formulated for the optimal solution and two optimal solution algorithms are developed for two special cases of 2×2 switch and $N \times N$ switch with identical traffic.

1. *Introduction*

A high speed router is required to have capability of good forwarding and packet scheduling with a buffering mechanism. This paper considers a packet scheduling algorithm on a router with VOQ(Virtual Output Queue). We assume that a communication is organized into frames. Each frame is divided into equal length time slots, representing the synchronous transmission. Three types of conflicts can be occurred at a router for packet switching. The input and output port conflicts confine that each input and output port cannot be involved in more than one transmission at a time, respectively. The third conflict is head-of-line(HOL) blocking. The HOL blocking arises when the input buffer is arranged as a single FIFO queue and a packet destined to an output that is free may be held up in line behind a packet that is waiting for an output that is busy. However, this paper considers the input and output port conflicts on the packet scheduling since the HOL blocking does not arise on a router with VOQ. Thus, we can have concurrent transmissions, provided that packets do not involve the same port more than once. When this is not the case, all the transmissions with common ports are damaged, since a collision occurs. Collided transmissions are useless and must be discarded and repeated later. This is a source of performance degradation and collision avoidance can be done by a proper scheduling of the messages. The scheduling problems can be classified into static and dynamic scheduling ones according to the arrival pattern of packets.

As previous researches on the static scheduling, [1], [2], and [4] have developed scheduling algorithms for minimization the schedule length. Even though the schedule length is usually used as a measure of whole throughput of the router, it does not guarantee the QoS(Quality-of-Service) of individual packet. For the individual packet QoS, [2] and [3] have considered individual due date(deadline) of each packet. However, their objectives are to develop algorithms for finding the feasible schedule and for minimizing the schedule length, respectively. This paper considers the static scheduling problem on a packet switched router which minimizes mean tardiness for QoS under a common due date. Even though the tardiness is one of popular measure for evaluating the conformity of the schedule to a desired packet connection time, there is no previous result on the scheduling algorithm of the packet routing problems in our knowledge.

2. Problem Description and Analysis

Consider a router with a $N \times N$ cross-bar switch. The packets are waited for routing service at each VOQ as depicted in Figure 1. The waiting packets can be represented as a traffic matrix $P = (p_{ij})$, where p_{ij} represents the number of packets to be transmitted from input port i to output port j. Let d_{ij} denotes the due date(deadline) of the packets p_{ij} and C_{ij} represents the transmission(service) completion time of the packet p_{ij}. And we denote by $T_{ij} = \max\{0, C_{ij}-d_{ij}\}$ the tardiness representing degree of unsatisfaction of the QoS. The objective of the paper is to develop scheduling algorithms which minimize mean tardiness $\frac{\sum_{i=1}^{N}\sum_{j=1}^{N} T_{ij}}{\sum_{i=1}^{N}\sum_{j=1}^{N} p_{ij}}$. However, the problem with general due date d_{ij} is NP-complete as shown by [5].

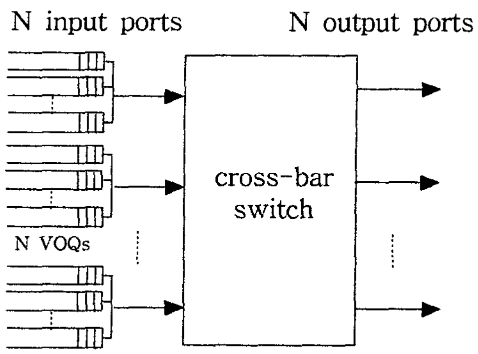

Figure 1. Architecture of the router

Accordingly, this paper assumes that all the due date are equal, i.e., $d_{ij}=d$, $\forall i,j$. Then, the problem becomes to finding the schedule minimizing $\sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij}$ since the tardiness becomes to T_{ij} = max{0, C_{ij}- d} and $\sum_{i=1}^{N}\sum_{j=1}^{N} p_{ij}$ is constant for a given traffic matrix P.

Let S_k be a switching matrix having at most one element of positive value at each row and column. And if all the positive value of S_k are 1, then the switching matrix is denoted by M_k. Then, the traffic matrix P can be serviced as forms of the switching matrix{ S_k} and packets composing a switching matrix S_k will be switched as the forms of { M_k}, $S_k = M_k^1 + M_k^2 + \cdots + M_k^{L(S_k)}$, where $K \le N!$ and $L(S_k)$ denotes the maximum value of all the elements of the matrix S_k. Accordingly, the objective of the paper can be rephrased as finding a schedule of { S_k} and { M_k} which minimize $\sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij}$ for the given traffic matrix P.

The following property characterizes that a scheduling for the switching matrix is sufficient for the optimal schedule.

Property 1. It is optimal that each input port i switches the packets p_{ij} without preemption.

Proof: Suppose that two switching matrix S_a and S_b include packets p_{ij_a} and p_{ij_b} of an input port i, respectively. Then, we can easily show that a schedule S in Figure 2 having sub-sequence M_a^1-> M_a^2-> ⋯-> M_a^i-> ⋯-> $M_a^{L(S_a)}$-> M_b^1-> M_b^2-> ⋯-> M_b^i-> ⋯-> $M_b^{L(S_b)}$ is not worse than the other schedules S[1] and S[2] with sub-sequences M_a^1-> M_a^2-> ⋯-> M_a^i-> M_b^i-> M_a^{i+1}-> ⋯-> $M_a^{L(S_a)}$-> M_b^1-> M_b^2-> ⋯-> $M_b^{L(S_b)}$ and

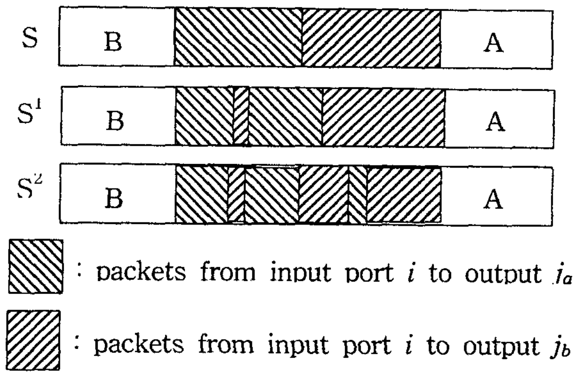

Figure 2. Schedules S, S^1, and S^2

M_b^1 M_a^1-> M_a^2-> ⋯-> M_a^i-> M_b^i-> M_a^{i+1}-> ⋯-> $M_a^{L(S_a)}$- >-> M_b^2-> ⋯-> M_b^{j-1}-> M_a^i-> M_b^i-> ⋯-> $M_b^{L(S_b)}$, respectively.

Property 1 implies that the "burst-based" and "packet-mode" scheduling of [6] and [7] are optimal for minimizing mean tardiness, respectively.

3. Solution Algorithms

3.1 Mathematical Formulation for the Optimal Solution

For the formulation, let us define two additional notations as follows : X_{ijt} = 1 if a packet is switched from input port i to out port j at time slot t, and 0 otherwise. And $T_{\max}$ denotes the maximum required time for all the traffic, where Max { $\lceil \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} p_{ij}}{N} \rceil$, $\sum_{i=1}^{N} p_{ij}$, $\sum_{j=1}^{N} p_{ij}$} $\le T_{\max} \le \sum_{i=1}^{N}\sum_{j=1}^{N} p_{ij}$. Then, the optimal solution is obtained by applying the following problem MIP to a commercial package for an integer programming.

$$\text{Min.} \quad \sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij} \qquad \text{(MIP)}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} X_{ijt} \le 1, \quad \forall j,\ t \qquad (1)$$

$$\sum_{j=1}^{N} X_{ijt} \le 1, \quad \forall i,\ t \qquad (2)$$

$$\sum_{t=1}^{T_{\max}} X_{ijt} = p_{ij}, \quad \forall I,\ j \qquad (3)$$

$$p_{ij}(X_{ijt} - X_{ij,t+1}) + \sum_{k=t+2}^{T_{\max}} X_{ijk} \le p_{ij} \qquad (4)$$

$$C_{ij} \ge tX_{ijt}, \quad \forall i,\ j,\ t \qquad (5)$$

$$X_{ijt} = 0 \text{ or } 1, \quad \forall i,\ j,\ t \qquad (6)$$

$$i=1,2,\cdots,N;\ j=1,2,\cdots,N;\ t=1,2,3,\cdots,T_{\max}$$

The input and output port conflicts are represented by (1) and (2), respectively. And (3) implies that the given traffic between input port i and output port j must be connected(serviced) at all. The result of Property 1 is applied by (4). The MIP problem has $N^2(1+T_{\max})$ decision variables and $N^2+N(N+2)T_{\max}$ constraints, and the number of variable and constraint are depend on the value of $T_{\max}$. However, since the objective function is minimization of the sum of C_{ij} such that $C_{ij} \geq tX_{ijt}$, t=1,2,3, ⋯, $T_{\max}$, the smaller value of $T_{\max}$ rather than $\sum_{i=1}^{N}\sum_{j=1}^{N} p_{ij}$ will be sufficient.

3.2 Special cases

We further characterize the solution properties and develop the optimal solution algorithms for two special cases of 2 × 2 switch and $N \times N$ switch with identical traffic.

Property 2. For 2 × 2 switch, the optimal schedule is S_1-> S_2 if max $\{p_{11}, p_{22}\}$ < max $\{p_{12}, p_{21}\}$, S_2-> S_1 otherwise, where $S_1 = \begin{pmatrix} p_{11} & 0 \\ 0 & p_{22} \end{pmatrix}$, $S_2 = \begin{pmatrix} 0 & p_{12} \\ p_{21} & 0 \end{pmatrix}$.

Proof: By Property 1, there exists two considerable schedules such that S_1-> S_2 and S_2-> S_1. For the sequencing order of S_1-> S_2, $\sum_{i=1}^{2}\sum_{j=1}^{2} C_{ij} = \sum_{i=1}^{2}\sum_{j=1}^{2} p_{ij}$ + 2 max $\{p_{11}, p_{22}\}$. And $\sum_{i=1}^{2}\sum_{j=1}^{2} C_{ij} = \sum_{i=1}^{2}\sum_{j=1}^{2} p_{ij}$ + 2 max $\{p_{12}, p_{21}\}$ for the other sequencing order of S_2-> S_1. Therefore, the order S_1 -> S_2 is better than S_2-> S_1 if max $\{p_{11}, p_{22}\}$ < max $\{p_{12}, p_{21}\}$ and the other order S_2-> S_1 is optimal, otherwise.

For a numerical example, consider a 2 × 2 cross-bar switch with waiting packets $P = \begin{pmatrix} 5 & 10 \\ 9 & 8 \end{pmatrix}$. For the traffic P, there exist two types of switching matrix S_1 and S_2, $S_1 = \begin{pmatrix} 5 & 0 \\ 0 & 8 \end{pmatrix}$, $S_2 = \begin{pmatrix} 0 & 10 \\ 9 & 0 \end{pmatrix}$, where the optimal sequencing is obtained as S_1-> S_2 with total completion time 48, 48 = $\sum_{i=1}^{2}\sum_{j=1}^{2} C_{ij} = \sum_{i=1}^{2}\sum_{j=1}^{2} p_{ij}$ + 2 max $\{p_{11}, p_{22}\}$ by Property 2 since the max $\{p_{11}, p_{22}\}$ =8 is smaller than max $\{p_{12}, p_{21}\}$ = 10.

Property 3. If $p_{ij} = p$, $\forall i, j$, the optimal schedule is obtained by sequencing of minimal set of the switching matrix without intermediate idle time, where the minimal set of switching matrix is a set of minimum number of switching matrix which can compose the given traffic matrix.

The proof is obvious by Property 1 and the optimal schedule results in $\sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij} = \frac{N^2(N+1)p}{2}$ independently on their sequence of the switching matrix. For a numerical example, the traffic $P = \begin{pmatrix} 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \end{pmatrix}$ will be routed according to the four switching matrix S_1, S_2, S_3, and S_4 with total completion time $\frac{N^2(N+1)p}{2}$=400, where $S_1 = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{pmatrix}$, $S_2 = \begin{pmatrix} 0 & 10 & 0 & 0 \\ 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 10 & 0 \end{pmatrix}$, $S_3 = \begin{pmatrix} 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \\ 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \end{pmatrix}$, and $S_4 = \begin{pmatrix} 0 & 0 & 0 & 10 \\ 0 & 0 & 10 & 0 \\ 0 & 10 & 0 & 0 \\ 10 & 0 & 0 & 0 \end{pmatrix}$.

Even though Properties 2 and 3 characterizes the optimal scheduling for special cases, Properties 1 through 3 will be used to develop a scheduling algorithm for a general traffic.

4. Conclusions

This paper considers a scheduling problem on a VOQ router for maximizing the QoS, where the mean tardiness is considered as a measure of QoS under a common due date. The optimal schedule is characterized that the minimization of the sum of completion time is required and no preemptive scheduling is dominant. Based upon the properties, an integer programming is formulated and the optimal solution algorithms are developed for two special cases. The algorithms for the special cases and characterized properties can be utilized to devise solution algorithm for the problem with general due date and switching size which is a subject of further study. And problems with other cell-by-cell based measures for the QoS are also valuable subjects.

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