Graph theoretical considerations of a channel assignment problem on multihop wireless networks

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Abstract: Multihop wireless networks consist of mobile terminals with personal communication devices. Each terminal can receive a message from a terminal and send it to the other terminal. In this paper, we discuss edge coloring problems related to multihop wireless networks. We show some relations about the problems.

1. Introduction

The development of economic and social activities is a primary factor of the increasing demand for mobile communication services. The demand stimulates the development of technology in mobile communication including personal communication services. Thus mobile communication has been one of the most active researches in communications in the last several years. There exist various problems to which graph & network theory is applicable in mobile communication services. For example, it is well-known that coloring algorithms of graphs are applicable in channel assignment algorithms in cellular systems[1]. If we formulate some problems in mobile communication systems using graph & network theoretical terms, then it is possible to apply the results on graph & network theory to the problems.

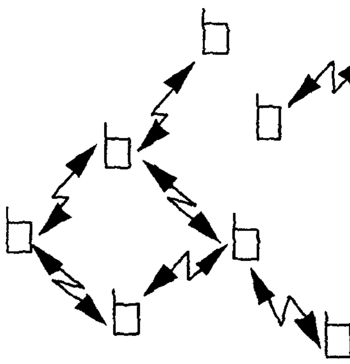


Fig.1 Multihop wireless network.

Multihop wireless networks[2],[3] consist of mobile terminals with personal communication devices. Each terminal can receive a message from a terminal and send it to the other terminal. If a terminal can not communicate the other terminal that sends data directly, some terminals relay the data(Fig.1).

In this network, each terminal communicates to the other terminals using a channel. Since cochannel interference may occur, we can not assign channels randomly. We need to assignchannels efficiently to achieve high spectral efficiency. It is well-known that the channel assignment problem on cellular system is related to the vertex coloring problem in graph theory. So, we formulate channel assignment problems on multihop wireless networks using graph & network theoretical terms. Namely, we define some edge coloring problems related to a channel assignment problem on multihop wireless networks. Then, we show some relations to these coloring problems.

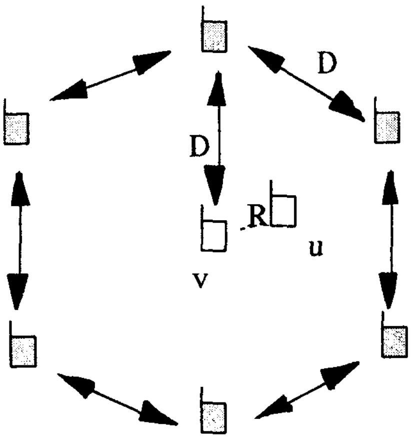
2. An edge coloring problem on wireless multihop networks

First, for simple graphs, we define an edge coloring problem related to a channel assignment problem on multihop wireless networks. We explain this problem using the following model.

A terminal v tries to communicate to the other terminal u using channel A. Here, we assume that a C/I ratio (carrier-to-interference ratio) that is not less than a desired value α is necessary to good communication. In case of FM, 30kHz channel bandwidth and analog telephone system, it is reported α=18dB[4]. And we also assume that the total cochannel interference received by a terminal is determined by the sum of each cochannel interference.

In v, let C be the carrier from u. If terminals using channel A are far enough from v, v can use channel A. For example, we assume the 40 dB/dec mobile radio propagation path loss rule [4] (in propagation path loss, radio signal strength drops proportionate to distance) and α=18dB. Let R be the distance between v and u. And we assume that if the distance between v and terminals using channel A is not less than D, v can use the channel A.

The worst case is the following on this situation(Fig.2).



:terminal using channel A

Fig.2 The worst case on the situation.

In this case,

$$C/I = R^{-4} / 6D^{-4} \geq 18\text{dB}.$$

Then,

$$D/R \geq 4.4.$$

Here, we define a new edge coloring. Let R be the maximum distance between terminals that can directly communicate to each other. If the distance between terminals is larger than R, the terminals can not communicate directly. Therefore the terminals need other terminals to relay data. We construct a graph as follows. Each vertex represents a terminal. If the distance between terminal v and u is less than 4.4R, we join the vertices. In practice, although a terminal v can communicate to the other terminal u, v may not communicate to u. Therefore, the edge set is divided into communication edges and interference edges. We assign channels (=colors) to communication edges only.

For example, in Fig.3, the bold edges represent communication edges and other edges represent interference edges.

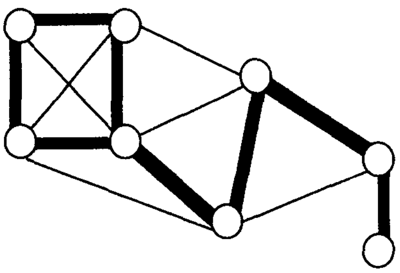


Fig.3. A graph G.

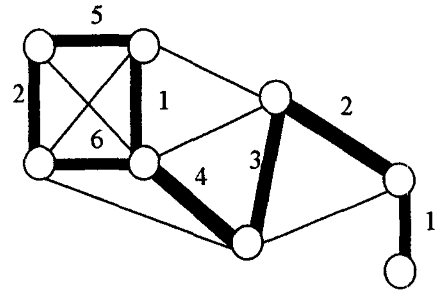


Fig.4 An edge coloring of G.

Clearly, if edge e is adjacent to edge e', we can not assign same color to e and e'. Therefore, this coloring is an edge coloring. Furthermore, the distance between e and e' is two, we also can not assign same color to e and e'. The distance of two edges is two if and only if an endvertex of one is joined to an endvertex of the other. That is to say, if the distance between two vertices is two, there exist two vertices whose distance is less than 4.4R. In Fig.4, we show an edge coloring satisfying the above conditions. Actually, if the edge set is equal to the communication set, this coloring is called strong edge coloring[5]. So, the edge coloring problem in this paper is a generalization of strong edge coloring. It is well-known that the problem of finding a strong edge coloring with the minimum number of colors for general graphs is NP-complete.

We formulate the coloring in this paper using graph theoretical terms.

[Definition 1]

Let G=(V,E) be an undirected graph such that V is the vertex set and E is the edge set. Let E_c and E_i be a partition of E. A strong edge subset coloring of G is an assignment of colors to edges in E_c of G such that every two edges of distance at most two receive different colors.

The minimum number of colors for all strong edge subset colorings of G is called strong edge subset coloring number of G, denoted by χ_S(G).

3. Another edge coloring problem

In the above case, we formulate a channel assignment problem using undirected graphs. Next, we formulate it using directed graphs. This means that the communication from terminal v to terminal u is different from the communication from u to v. So, we use separate channels. We formulate this directed version using graph theoretical terms.

[Definition 2]

Let D=(V,E) be a directed graph such that V is the vertex set and E is the edge set. Let E_c and E_i be a partition of E. A strong edge subset directed coloring of D is an assignment of colors to edges in E_c of D satisfying the following conditions.

1) If two edges share an end vertex, the edges receive different colors.

2) If u is incident to edge e, v is incident from e' and there exists edge (u,v) (Fig.5), e and e' receive different colors.

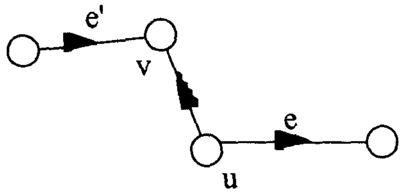


Fig.5 The relation between e and e'.

The minimum number of colors for all strong edge subset directed colorings of D is called strong edge subset directed coloring number of D, denoted by $\chi_D(D)$.

For example, in Fig.6, the bold edges represent communication edges and other edges represent interference edges.

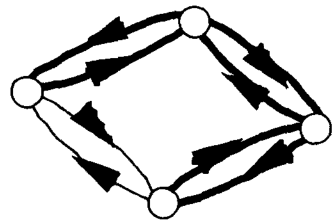


Fig.6 Directed graph D.

Fig.7 shows a strong edge subset directed coloring of D.

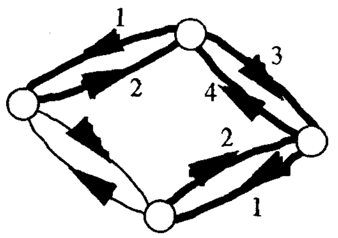


Fig.7 A strong edge subset directed coloring of D.

If the edge set is equal to the communication set, this coloring is discussed in [6]. The paper showed that the problem of finding a strong edge subset directed coloring with the minimum number of colors for general graphs is NP-complete.

4. Relation between the edge coloring problems

To apply the results in this paper to the problems on multihop wireless networks, we assume that directed graph D is symmetric. A directed graph D is symmetric if whenever (u,v) is an edge of D, then so too is (v,u). For example, directed graph G in Fig.6 is symmetric. Let D be a (symmetric) directed graph and G be the underlying graph of D. In case of D in Fig.6, the underlying graph is in Fig.8. In this case, $\chi_D(D)=4$ and $\chi_S(G)=3$.

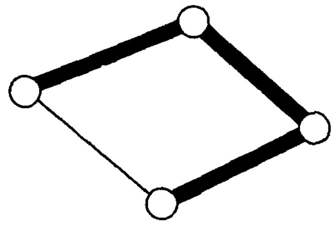


Fig.8 The underlying graph G of D.

Since the number of edges of D is twice the number of edges in G, in almost case,

$$\chi_D(D) \geq \chi_S(G).$$

However, the above inequality does not hold for any graph. Let graph G in Fig.9 be the underlying graph of a directed graph D. We assume that all edges are communication edges. Clearly, the number of edges is equal to the number of colors in G. Therefore,

$$\chi_S(G)=15.$$

Next, we consider the strong edge subset directed coloring of D. In this case, edges on cycle C_3 receive different colors. So, we use 6 colors to assign to edges in the cycle. We assign colors to other edges as follows.

For i=1,2,3,4 and j=1,2,3, we assign

color i to edge (u_j, v_{ji}) and

color i+4 to edge (v_{ji}, u_j).

Obviously, this coloring is a strong edge subset directed coloring. The number of colors is 6+8=14. Therefore, in this case,

$$\chi_D(D) < \chi_S(G).$$

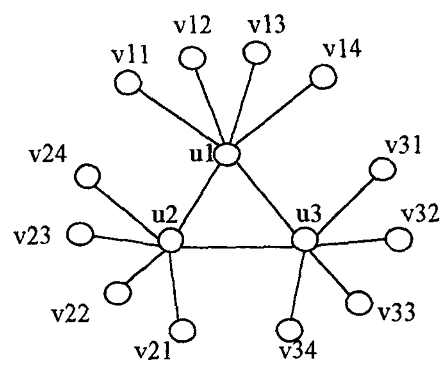


Fig.9 An underlying graph G.

However, in case of trees, we obtain the following result.

[Theorem 1]

Let D be a symmetric directed graph whose edges are communication edges, and G be the underlying graph of D. If G is a tree,

$$\chi_D(D) > \chi_S(G).$$

(Proof)

From [7], the strong edge coloring number $\chi_S(G)$ of tree G is the following.

$$\chi_S(G) = \max\{d_G(u)+d_G(v)-1 \mid (u.v) \in E(G)\},$$

where $d_G(v)$ is the degree of vertex v in G.

Let (v_1, v_2) be an edge of G where

$$d_G(v_1)+d_G(v_2)-1=\chi_S(G). \quad (1)$$

The tree in Fig.10 is the subgraph D' of D, where the vertices are v_1, v_2 and adjacent vertices to v_1 or v_2 and the edges are incident to v_1 or v_2. We can assign a same color to the edge (v_1,u) and (v_2,u') where $u \neq v_2$ and $u' \neq v_1$. We can also assign a same color to the edge (u,v_1) and (u',v_2) where $u \neq v_2$ and $u' \neq v_1$. And we must assign the other colors to (v_1,v_2) and (v_2,v_1). So,

$$\chi_D(D') = \max\{2d_G(v_1), 2d_G(v_2)\}. \quad (2)$$

From (1) and (2),

$$\chi_D(D') > \chi_S(G).$$

Since D is a tree, $\chi_D(D) \geq \chi_D(D')$. Therefore,

$$\chi_D(D) > \chi_S(G).$$

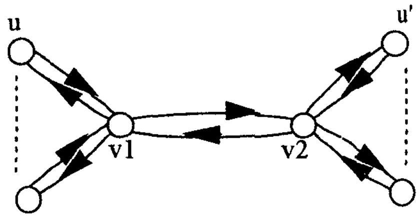


Fig.10 A directed graph D'.

5. Conclusions

In this paper, we formulated channel assignment problems on multihop wireless networks using graph & network theoretical terms. These problems is generalizations of edge coloring problems in graph theory. We showed some relations to these coloring problems. It is hard to obtain optimal solutions on the coloring problems. Because these problems are NP-complete. For practical use, we need to develop heuristic algorithms to solve the problems.

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