Application of Percolation Model for Network Analysis

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Abstract: In order to send the information certainly via the network against the packet lost caused by hardware troubles or limitation of packet transferring, we must construct reliable network infrastructure. However, it is difficult to construct comfortable network early if we construct rely on the prediction or the experience through a lot of troubles.

In this paper, we propose the method to construct reliable network infrastructure based on the computer network simulation. This simulation is based on the percolation model. Percolation model is known as the model that represents connections.

We gave some simulations for the various network topologies: the square lattice network, the cubic lattice network, and the full connection type network.

1. Introduction

Recently, personal computers and mobile telephones are popularized, and anyone can use the communication via networks easily. Therefore, many people who is poor at realistic of communication via networks, also use. They tend to think that the information transmitted by them must be reach objective person. However, the fact is that packets are often lost because of hardware troubles or limitation of packet transferring.

In order to send information to objective person (or point) certainly, we must construct reliable network infrastructure. However, it is difficult to construct comfortable network early if we construct rely on the prediction, or the experience through a lot of troubles.

In this paper, we propose the method to construct the reliable network based on the computer network simulation. The aim of this simulation is examining how much packet permeability is required to send the information to the objective point. This simulation is based on the percolation model. Percolation model is known as the model that represents connections. We considered that percolation model would be useful for computer network simulation.

We gave some simulations for the various network topologies: the square lattice network, the cubic lattice network, and the full connection type network. Then we gave the minimum permeability for each case. This minimum permeability would give us the rating of whether the network is reliable or not.

2. Percolation Model

Percolation model is the model of phenomena of liquid permeation. Additionally, we can treat a blaze of forest or contagious diseases instead of liquid permeation by percolation model.

There are some types of percolation model. For Example, site percolation model, and bond percolation model. These model is represented using lattice like figure 1 and 2.

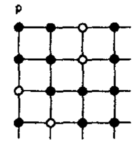

Figure 1. Site Percolation Model

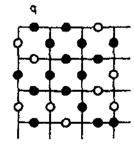

Figure 2. Bond Percolation Model

Site percolation model set its permeability on the node of the lattice. Bond percolation model set its permeation rate on the bond of lattice.

For convenience, we call the cluster which permeate from upper side to lower side on percolation model "cluster α." If the permeability p is enough, cluster α exists certainly. The border value p_c is called "critical rate," and the value is different on each model. On the computer network simulation, this value is the border value that the packet reaches the objective point or not.

However, to simulate computer networks, computers and networks will be treated concurrently, and it is inconvenient to use only one of these models. Thus we used the interlace model of these two models.

3. Bond-Site Percolation Model

In our research, bond-site percolation model (the interlace model of bond percolation and site percolation) is used for network analysis. Bond-site percolation model is the interlace model of bond percolation model and site percolation model. This model is designed for computers and networks which connects each computer. And this model is useful for simulation in case that the computer network has access limitation.

The lattice points indicates the computers, and the bond (each lattice points connects) indicates the connection of computers. Thus each computers can connect neighbor four computers.

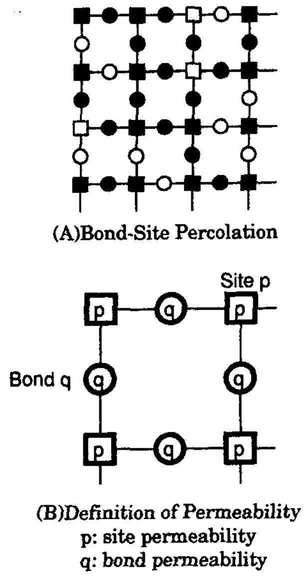

Figure 3. Bond-Site Percolation Model

We also designed the cubic lattice type and full connection (more than 4) type of Bond-Site percolation model (Figure 4, 5). In cubic lattice type model, every node has connection to 6 neighbor nodes. In full connection type model, every node has connection to the all other node.

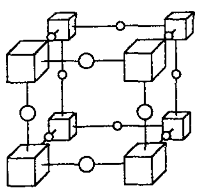

Figure 4. Cubic Type Bond-Site Percolation

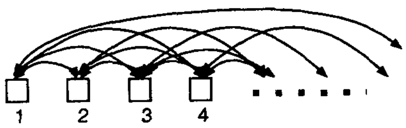

Figure 5. Full Connection Type Bond-Site Percolation

We gave the simulation using these models.

4. Simulation and results

4.1 Square lattice type

We gave the simulation using square type of bond-site percolation model, resizing lattice size 6 ways ; $12 \times 12, 16 \times 16, 24 \times 24, 48 \times 48, 64 \times 64$, and 128×128. In each size of lattice, in each p(or q), we computed the critical rate as the average of the 1000 times simulations. Figure 6,7 is the result. p,q is the permeability shown by (B) of Figure 3, and P_c,Q_c is critical rate in each case. The graph indicates that $P_c(Q_c)$ equals 0 if the cluster α doesn't exist.

For example, in case that lattice size is 12×12, if $p = 0.5$ then $Q_c = 0.868$. In short, if the 72 computers is connected each other with over 86.8%, we can use these computers comfortably.

4.2 Cubic lattice type

We also gave the simulation using cubic type of bond-site percolation model, resizing lattice size 6 ways ; $8 \times 8 \times 8, 16 \times 16 \times 16, 32 \times 32 \times 32, 48 \times 48 \times 48, 64 \times 64 \times 64$, and $96 \times 96 \times 96$. In each size of lattice, in each p(or q), we computed the critical rate as the average of the 1000 times simulations. Figure 8,9 is the result. Naturally, for the cubic type network, it is easier to make the cluster α than the square lattice type network.

4.3 full connection type

We also gave the simulation using cubic type of bond-site percolation model. The number of node is 64,128,256,512, and 1024 . In each number of node, in each p(or q), we computed the critical rate as the average of the 1000 times simulations. Figure 10,11 is the result. Naturally, for the full connection type network, it is easier to make the cluster α than the square and the cubic lattice type network.

4.4 Minimum p (or q) that allows making the cluster α

Minimum p (or q) that allows making the cluster α is important value, because it would be the border value that indicates the network infrastructure is reliable or not.

These values are written in the table 1. The "qPc" and the "pQc" on the the "kind" column means "the variation of P_c along the change of q" and "the variation of Q_c along the change of p". The variance of the P_c (Q_c) written on the "Variance" row. The network scale becomes larger, these values becomes larger.

Naturally, the full connection type model is superior to other two models at this point.

5. Conclusion and future works

We can understand from figures that our model has following feature : the critical rate of bond q (or site p) becomes larger if the size of lattice becomes larger. From this feature, we can expect if the size of lattice becomes infinity, the critical rate of bond q (or site p) will becomes infinity. It's clear if $q(p)$ increase, then the $P_c(Q_c)$ decrease.

We gave the minimum permeability for each case by giving simulation. This minimum permeability would give us the rating of whether the network is reliable or not.

We have treated the logical connection of networks, however, we will also treat the physical connection, that is, activity rate of computers and network cables.

The development of model for the approximation of real network topology (such as star type connection, bus type connection, the token ring type connection, etc.) is also future work.

Moreover, we will think about the case that our model is useful as future work. For example, we expect this model is useful for the simulation of personal handy phone system (PHS) or the wireless. These are stable if we install many antennas all around. However, installing antennas might bring some problems; a baneful influence for human body and environment, cost, and so on. Thus the number of installing should be minimized. Therefore, for simulation to determine the number and location of antennas tuning the stability of radio wave freely, the model in this research might be useful.

References

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Table 1 : Minimum p (or q) that allows making the cluster α

Square lattice type				
Kind	Size	q	Pc	Variance
qPc	12 × 12	0.320	0.995	0.000
	16 × 16	0.340	0.970	-0.000
	24 × 24	0.430	1.000	-0.000
	48 × 48	0.480	0.960	-0.000
	64 × 64	0.510	0.990	0.000
	128 × 128	0.540	1.000	-0.000
Kind	Size	p	Qc	Variance
pQc	12 × 12	0.250	1.000	-0.000
	16 × 16	0.300	1.000	-0.000
	24 × 24	0.360	0.970	-0.000
	48 × 48	0.420	0.990	0.000
	64 × 64	0.440	1.000	-0.000
	128 × 128	0.460	1.000	-0.000

Cubic lattice type				
Kind	Size	q	Pc	Variance
qPc	8 × 8 × 8	0.180	0.250	-0.000
	16 × 16 × 16	0.270	0.990	0.000
	32 × 32 × 32	0.290	0.940	-0.000
	48 × 48 × 48	0.310	0.980	0.000
	64 × 64 × 64	0.310	0.996	0.000
	96 × 96 × 96	0.310	1.000	-0.000
	128 × 128 × 128	0.310	1.000	-0.000
Kind	Size	p	Qc	Variance
pQc	8 × 8 × 8	0.010	0.585	0.022
	16 × 16 × 16	0.010	0.700	0.020
	32 × 32 × 32	0.010	0.824	0.003
	48 × 48 × 48	0.010	0.849	0.002
	64 × 64 × 64	0.010	0.890	0.002
	96 × 96 × 96	0.010	0.924	0.001
	128 × 128 × 128	0.010	0.925	0.000

Full connection type				
Kind	No. of nodes	q	Pc	Variance
qPc	64	0.010	0.010	-0.000
	128	0.010	0.010	-0.000
	256	0.010	0.122	0.004
	512	0.010	0.229	0.004
	1024	0.010	0.192	0.001
Kind	No. of nodes	p	Qc	Variance
pQc	64	0.010	0.010	-0.000
	128	0.010	0.010	-0.000
	256	0.010	0.010	0.000
	512	0.010	0.948	0.002
	1024	0.010	0.554	0.003

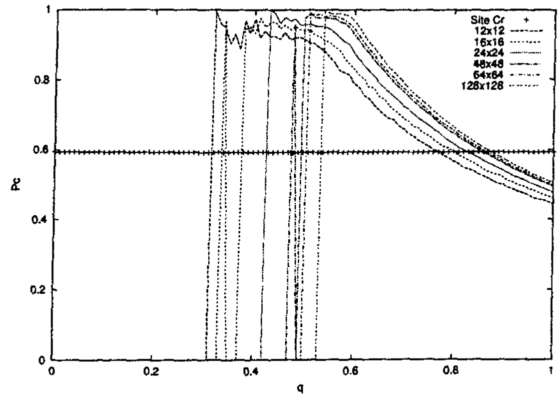

Figure 6. Result of the simulation using
the square lattice bond-site percolation model
The variation of P_c along the change of q

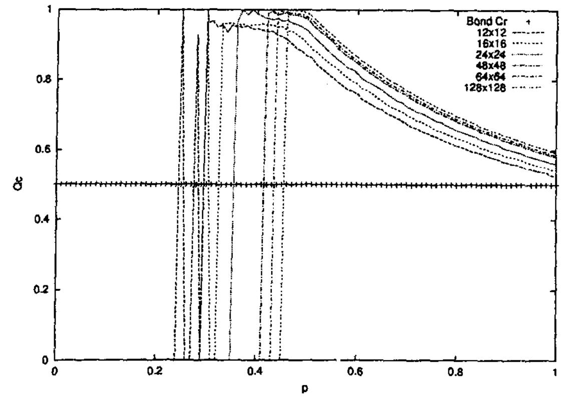

Figure 7. Result of the simulation using
the square lattice bond-site percolation model
The variation of Q_c along the change of p

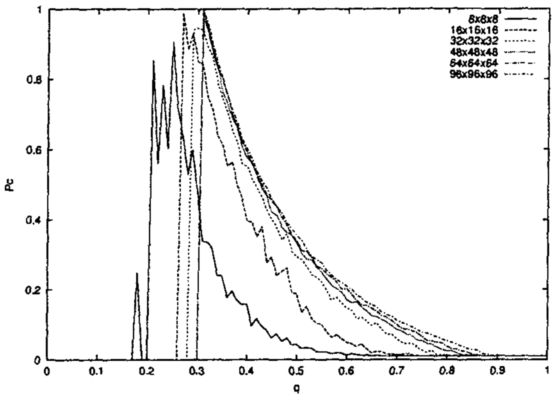

Figure 8. Result of the simulation using
the cubic lattice bond-site percolation model
The variation of P_c along the change of q

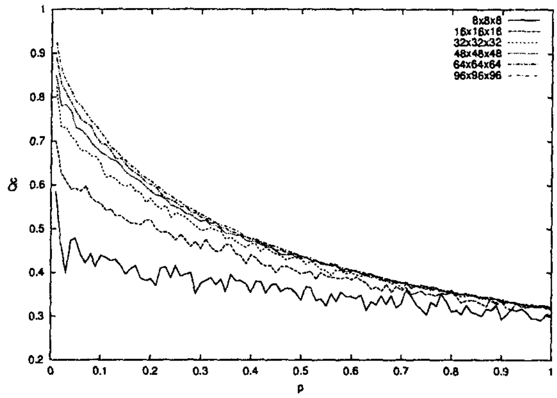

Figure 9. Result of the simulation using
the cubic lattice bond-site percolation model
The variation of Q_c along the change of p

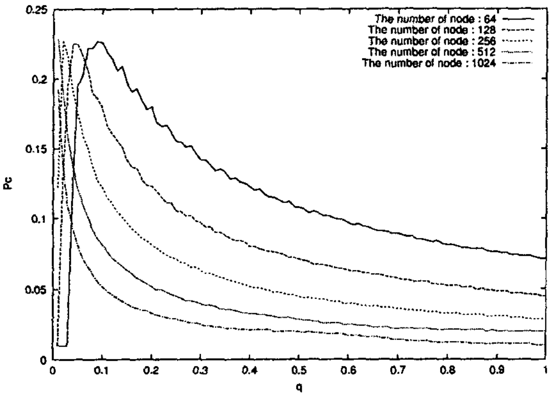

Figure 10. Result of the simulation using
the full connection type bond-site percolation model
The variation of P_c along the change of q

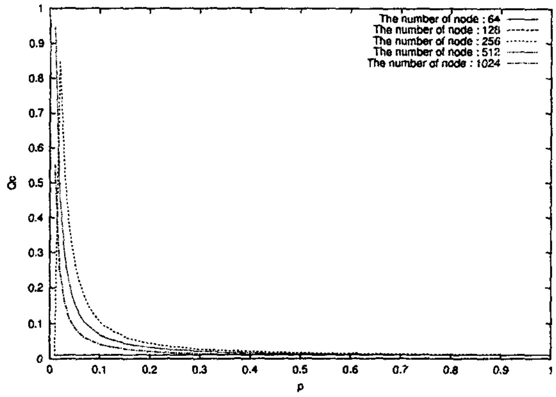

Figure 11. Result of the simulation using
the full connection type bond-site percolation model
The variation of Q_c along the change of p