Topological Consideration on Switched Mode DC-DC Converters

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Abstract: In this paper we analyze a class of switched mode DC-DC converters and derive some topological conditions for the circuit to work as DC-DC converter, in particular, buck, booster, and buch/booster cponverters.

1. Introduction

A DC-DC converter is a key device for personal computers and many electronic appliances and therefore various kinds of converters have long been discussed. However there are few discussions on the structure of the circuits from topological points of view.

This paper first shows the analysis of a class of switched mode DC-DC converters based on the graph theory and derives the explicit formulae for the output voltage. Based on the above results, several topological conditions are for the circuits to work as the converter, in particular, buck, booster, and buck/booster converters.

2. Derivation of a dynamic equation by the averaging method

We consider a circuit consisting of m inductors, $L_i(i = 1,\cdots,m)$, n capacitors, $C_i(i = 1,\cdots,n)$, one dc voltage source, E, one output resistor, R_0, and k ideal switches, $S_i(i = 1,\cdots,k)$. Though transformers are usually used in floating-type DC-DC converters, we exclude them in this paper for simplicity. In this paper we also excluded diodes, which are basic elements of DC-DC converters, since in our analysis we regarded diodes as ideal switches.

2.1 Graph Representation of the circuit and Assumptions

Let the graph of the circuit be denoted by G, which consist of m inductor edges, $L \equiv \{L_i\}$, n capacitor edges, $C \equiv \{C_i\}$, a dc voltage source edge, E, an output resistor edge, R_0, and k switch edges, $\{S_i\}$. That is, each circuit element is identified with the corresponding edge.

We assume the following in our analysis.

Assumption 1: The circuit operates in 2 phases synchronously. We partition all switches into two classes; $S^I \equiv \{S_1,\cdots,S_{k_1}\}$ and $S^{II} \equiv \{S_{k_1+1},\cdots,S_k\}$. When $S^I = \{S_1,\cdots,S_{k_1}\}$ is ON, then $S^{II} = \{S_{k_1+1},\cdots,S_k\}$ is OFF, and in the next phase the converse holds.

We call the state where switches S^I is ON and S^{II} is OFF "Phase I" and the state where S^I is OFF and S^{II} is ON "Phase II".

Assumption 2: There exists no loop consisting of E and $\{C_i\}$ only.

Assumption 3: There exists no cutset consisting of R_o and $\{L_i\}$ only.

Assumption 4: There exists no loop consisting of E, $\{C_i\}$, and some (≥ 1) switches in S^I. Also there exists no loop consisting of E, $\{C_i\}$, and some (≥ 1) switches in S^{II}.

Similarly,

Assumption 5: There exists no cutset consisting of R_o, $\{L_i\}$ and some(≥ 1) switches in S^I. Also there exists no cutset consisting of R_o, $\{L_i\}$ and some(≥ 1) switches in S^{II}.

If we do not assume Assumption 4, then the switching loss as well as impulsive surge current and surge noise necessarily occurs due to the KVL and charge preservation rule. This is undesirable for DC-DCconverters. Similarly Assumption 5 is reasonable.

Let Q_1 and Q_2 be disjoint subset of edges in the graph G. Then the graph obtained from G by short-circuiting the edges Q_1 and open-circuiting the edges Q_2 be denoted by $G(Q_1;Q_2)$. In particular let $G_1 = G(S^I;S^{II})$ and $G_2 = G(S^{II};S^I)$.

Assumption 4 shows that edges S^I, E and $\{C_i\}$ constitute (a part of) a tree of G and also Assumption 6 shows that all edges of S^{II}, R_o and $\{L_i\}$ constitute (a part of) a cotree of G. We therefore have:

Lemma 1: S^I, E and $C(\equiv \{C_i\})$ form a tree of G_1 and S^{II}, E and C also form a tree of G_2. Similarly, S^{II}, L and R_o form a cotree of G_1, and S^I, $L(\equiv \{L_i\}$ and R_o form a cotree of G_2.

From Lemma 1 we have:

Lemma 2:

$$\|S^I\| = \|S^{II}\|, \text{ i.e., } k_1 = \frac{k}{2} \tag{1}$$

Let the fundamental loop matrix of G with respect to the tree T consisting of S^I, C and E as follows:

$$B_f = B_f(T) = \begin{array}{c} \\ S^{II} \\ L \\ R_o \end{array} \begin{array}{c} \begin{array}{cccccc} S^I & C & E & S^{II} & L & R \end{array} \\ \begin{bmatrix} B_{js} & B_{jc} & B_{je} & 1 & 0 & 0 \\ B_{ls} & B_{lc} & B_{le} & 0 & 1 & 0 \\ B_{rs} & B_{rc} & B_{re} & 0 & 0 & 1 \end{bmatrix} \end{array} \tag{2}$$

where the subscripts in the above equation have meanings as follows:

$$\left.\begin{array}{ccc} j & \leftrightarrow & S^{II} \\ l & \leftrightarrow & L \\ r & \leftrightarrow & R_o \end{array}\right\}, \qquad \left.\begin{array}{ccc} c & \leftrightarrow & C \\ s & \leftrightarrow & S^I \\ e & \leftrightarrow & E \end{array}\right\} \tag{3}$$

The fundamental loop matrix, $B_f(T')$, with respect to the tree T' consisting of S^{II}, C and E can be derived from $B_f(T)$ in (2) by the well-known pivot-transformation as follows:

Note that:

Lemma 3: $|B_{js}| = \pm 1 \neq 0$ (4)

Multiplying the first row of B_f by B_{js}^{-1}, we have:

$$\tilde{B}_f = \begin{bmatrix} 1 & B_{js}^{-1}B_{jc} & B_{js}^{-1}B_{je} & B_{jc}^{-1} & 0 & 0 \\ B_{ls} & B_{lc} & B_{le} & 0 & 1 & 0 \\ B_{rs} & B_{rc} & B_{re} & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Then adding the first row multiplied by $-B_{ls}$ to the second row and adding the first row multiplied by $-B_{rs}$ to the third row, we have $B_f(T')$ as follows:

$$B_f(T') = \begin{array}{cc} & \begin{array}{cccccc} S^I & C & E & S^{II} & L & R \end{array} \\ \begin{array}{c} S^I \\ L \\ R_o \end{array} & \left[\begin{array}{cccccc} 1 & B'_{sc} & B'_{se} & B'_{sj} & 0 & 0 \\ 0 & B'_{lc} & B'_{le} & B'_{lj} & 1 & 0 \\ 0 & B'_{rc} & B'_{re} & B'_{rj} & 0 & 1 \end{array}\right] \end{array} \qquad (6)$$

where

$$\left.\begin{array}{lcl} B'_{sc} & \equiv & B_{js}^{-1}B_{jc} \\ B'_{se} & \equiv & B_{js}^{-1}B_{je} \\ B'_{sj} & \equiv & B_{js}^{-1} \\ B'_{lc} & \equiv & B_{lc} - B_{ls}B_{js}^{-1}B_{jc} \\ B'_{le} & \equiv & B_{le} - B_{ls}B_{js}^{-1}B_{je} \\ B'_{rc} & \equiv & B_{rc} - B_{rs}B_{js}^{-1}B_{jc} \\ B'_{re} & \equiv & B_{re} - B_{rs}B_{js}^{-1}B_{je} \\ B'_{lj} & \equiv & -B_{ls}B_{js}^{-1} \\ B'_{rj} & \equiv & -B_{rs}B_{js}^{-1} \end{array}\right\} \qquad (7)$$

Assumption 6: The output resistor R_o is connected in parallel with C_n as shown in Fig. 1:

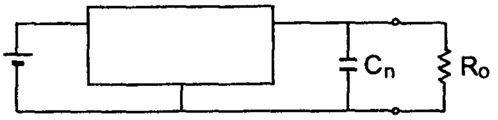

Figure 1.

Since by Assumption 6 R_o forms a fundamental loop with the tree branch C_n in both phases, we have in Eqs. (2) and (6):

$$B_{rc} = B'_{rc} = [\ \overbrace{0,\ 0,\ \cdots,\ 0}^{n-1}\ \ \overbrace{1}^{1}\ ] \qquad (8)$$

$$B_{rs} = B'_{rj} = 0, \quad B_{re} = B'_{re} = 0 \qquad (9)$$

Let the branch voltages and branch currens of $G(N)$ in Phase I as follows:

$$v = \left[\begin{array}{c} v_s \\ v_c \\ v_e \\ v_j \\ v_l \\ v_r \end{array}\right], \quad i = \left[\begin{array}{c} i_s \\ i_c \\ i_e \\ i_j \\ i_l \\ i_r \end{array}\right] \begin{array}{l} \leftarrow S^I \\ \leftarrow C \\ \leftarrow E \\ \leftarrow S^{II} \\ \leftarrow L \\ \leftarrow R \end{array} \qquad (10)$$

We use the same notation except for the prime $'$ as in Eq. (10) for the variables in Phase II.

Considering (9), we rewrite $B_f(T)$, $C_f(T)$, $B_f(T')$, and $C_f(T')$ as follows:

$$B_f \equiv B_f(T) = \begin{array}{cc} & \begin{array}{cccccc} S^I & C & E & S^{II} & L & R \end{array} \\ \begin{array}{c} S^{II} \\ L \\ R_o \end{array} & \left[\begin{array}{cccccc} B_{js} & B_{jc} & B_{je} & 1 & 0 & 0 \\ B_{ls} & B_{lc} & B_{le} & 0 & 1 & 0 \\ 0 & B_{rc} & 0 & 0 & 0 & 1 \end{array}\right] \end{array} \qquad (11)$$

$$C_f \equiv C_f(T) = \begin{array}{cc} & \begin{array}{cccccc} S^I & C & E & S^{II} & L & R \end{array} \\ \begin{array}{c} S^I \\ C \\ E \end{array} & \left[\begin{array}{cccccc} 1 & 0 & 0 & -B_{js}^T & -B_{ls}^T & 0 \\ 0 & 1 & 0 & -B_{jc}^T & -B_{lc}^T & -B_{rc}^T \\ 0 & 0 & 1 & -B_{je}^T & -B_{le}^T & 0 \end{array}\right] \end{array} \qquad (12)$$

$$B'_f \equiv B_f(T') = \begin{array}{cc} & \begin{array}{cccccc} S^I & C & E & S^{II} & L & R \end{array} \\ \begin{array}{c} S^I \\ L \\ R_o \end{array} & \left[\begin{array}{cccccc} 1 & B'_{sc} & B'_{se} & B'_{sj} & 0 & 0 \\ 0 & B'_{lc} & B'_{le} & B'_{lj} & 1 & 0 \\ 0 & B'_{rc} & 0 & 0 & 0 & 1 \end{array}\right] \end{array} \qquad (13)$$

$$C'_f \equiv C_f(T') = \begin{array}{cc} & \begin{array}{cccccc} S^I & C & E & S^{II} & L & R \end{array} \\ \begin{array}{c} S^{II} \\ C \\ E \end{array} & \left[\begin{array}{cccccc} -B'^T_{sj} & 0 & 0 & 1 & -B'^T_{lj} & 0 \\ -B'^T_{sc} & 1 & 0 & 0 & -B'^T_{lc} & -B'^T_{rc} \\ -B'^T_{se} & 0 & 1 & 0 & -B'^T_{le} & 0 \end{array}\right] \end{array} \qquad (14)$$

2.2 Averaging State Equation

We will derive the circuit equations for two hases.

(I) Phase I (i.e., S^I is ON and S^{II} is OFF)

$$B_f v = 0 \qquad (15)$$

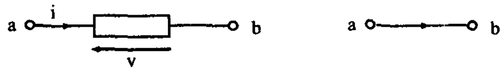

Figure 2.

$$C_f i = 0 \qquad (16)$$

$$\left.\begin{array}{lcl} i_c & = & C\frac{dv_c}{dt} \\ v_s & = & 0 \\ v_e & = & E \end{array}\right\}, \quad \left.\begin{array}{lcl} i_j & = & 0 \\ v_l & = & L\frac{di_l}{dt} \\ v_r & = & R_o i_r \end{array}\right\} \qquad (17)$$

From Eqs. (10), (11), (12), (15), and (16), we have:

$$\left.\begin{array}{l} B_{js}v_s + B_{jc}v_c + B_{je}v_e + v_j = 0 \\ B_{ls}v_s + B_{lc}v_c + B_{le}v_e + v_l = 0 \\ B_{rc}v_c + v_r = 0 \end{array}\right\} \qquad (18)$$

$$\left.\begin{array}{l} i_s - B_{js}^T i_j - B_{ls}^T i_l = 0 \\ i_c - B_{jc}^T i_j - B_{lc}^T i_l - B_{rc}^T i_r = 0 \\ i_e - B_{je}^T i_j - B_{lc}^T i_l = 0 \end{array}\right\} \qquad (19)$$

Note that the first equation of Eq. (18) is the equation for determining v_j and the first and the third equations of Eq. (19) are for determining i_s and i_e.

Since from Eq. (18) $R_o i_r + B_{rc}v_c = 0$ holds, we have:

$$i_r = -\frac{1}{R_o}B_{rc}v_c \qquad (20)$$

Substituting (20) into the first equation of (19) and referring to the second equation of (18) and the first equation of (19), we get:

$$\left[\begin{array}{c} L\frac{di_l}{dt} \\ C\frac{dv_c}{dt} \end{array}\right] = \left[\begin{array}{cc} 0 & -B_{lc} \\ B_{lc}^T & -\frac{1}{R_o}B_{rc}^T B_{rc} \end{array}\right] \left[\begin{array}{c} i_l \\ v_c \end{array}\right] - \left[\begin{array}{c} B_{le} \\ 0 \end{array}\right] E \qquad (21)$$

where the relation between the directions of currents and voltages and the direction of each edge is shown as in Fig. 2.

(II) Phase II (i.e., S^I is OFF and S^{II} is ON)

In this case we have the same equation as Eqs. (15)-(17) with exception B_f, C_f, i etc. having the prime($'$). The we have the equation corresponding to (21) as follows:

$$\left[\begin{array}{c} L\frac{di_l}{dt} \\ C\frac{dv_c}{dt} \end{array}\right] = \left[\begin{array}{cc} 0 & -B'_{lc} \\ B'^T_{lc} & -\frac{1}{R_o}B'^T_{rc} B'_{rc} \end{array}\right] \left[\begin{array}{c} i_l \\ v_c \end{array}\right] - \left[\begin{array}{c} B'_{le} \\ 0 \end{array}\right] E \qquad (22)$$

where the prime $'$ of the variables i'_l and v'_c are omitted.

We rewrite (21) and (22) as follows:

$$K\dot{x} \;\; = Ax + b \qquad \text{(in Phase I)} \qquad (23)$$

$$K\dot{x} \;\; = A'x + b' \qquad \text{(in Phase II)} \qquad (24)$$

where

$$K = L\dot{+}C = \mathrm{diag}[L_1, \cdots, L_m, C_1, \cdots, C_n] \qquad (25)$$

$$A = \left[\begin{array}{cc} 0 & -B_{lc} \\ B_{lc}^T & -\frac{1}{R_o}B_{rc}^T B_{rc} \end{array}\right] \qquad (26)$$

$$A' = \left[\begin{array}{cc} 0 & -B'_{lc} \\ B'^T_{lc} & -\frac{1}{R_o}B'^T_{rc} B'_{rc} \end{array}\right] \qquad (27)$$

$$b = -\left[\begin{array}{c} B_{le} \\ 0 \end{array}\right] E, \quad b' = -\left[\begin{array}{c} B'_{le} \\ 0 \end{array}\right] E \qquad (28)$$

Let the time durations of Phases I and II be D and D' respectively and let

$$D_{total} = D + D', \quad d \equiv \frac{D}{D_{toal}}, \quad d' \equiv \frac{D'}{D_{total}} \qquad (29)$$

Then

$$1 \geq d \geq 0, \quad 1 \geq d' \geq 0, \quad d + d' = 1 \qquad (30)$$

Assumption 7: The variation of the voltages and currents of all elements during D_{total} is slow.

On Assumption 7 we can utilize the state averaging method and obtain the final averaging equation as follows:

$$K\frac{dx}{dt} = \bar{A}x + \bar{b} \qquad (31)$$

$$\bar{A} = dA + d'A', \quad \bar{b} = db + d'b' \qquad (32)$$

that is,

$$\bar{A}=\left[\begin{array}{cc} 0 & -dB_{lc}-d'B'_{lc} \\ dB_{lc}^{T}+d'B_{lc}^{\prime T} & -\frac{1}{R_o}\left(dB_{rc}^{T}B_{rc}+d'B_{rc}^{\prime T}B'_{rc}\right) \end{array}\right] \quad (33)$$

$$\bar{b}=-\left[\begin{array}{c} dB_{le}+d'B'_{le} \\ 0 \end{array}\right]E\left(\equiv-\left[\begin{array}{c} b_e \\ 0 \end{array}\right]E\right) \quad (34)$$

$$b_e=dB_{le}+d'B'_{le} \quad (35)$$

Assumption 8 : $|\bar{A}|\neq 0$ (36)

In Eq. (33) as well as (21) we have by using (8)

$$B_{rc}^{T}B_{rc}=B_{rc}^{\prime T}B'_{rc}=\left[\begin{array}{cccc} 0 & \cdots & 0 & 0 \\ \vdots & \ddots & & \vdots \\ 0 & & 0 & 0 \\ 0 & \cdots & 0 & 1 \end{array}\right] \quad (37)$$

Then setting $a=-\frac{1}{R_o}$, we have

$$\bar{A}=\left[\begin{array}{c|cccc} 0 & & -H & & \\ \hline & 0 & \cdots & 0 & 0 \\ H^T & \vdots & \ddots & & \vdots \\ & 0 & & 0 & 0 \\ & 0 & \cdots & 0 & a \end{array}\right] \quad (38)$$

where $H=dB_{lc}+d'B'_{lc}$. We will investigate the condition for $\bar{A}$ to satisfy (36).

2.3 Steady State Solution

Lemma 4: $|\bar{A}|\neq 0$ holds only if

$$n=m,\ \text{or}\ n=m+1 \quad (39)$$

Under the condition $|\bar{A}|\neq 0$ the steady state solution of (31) can be derived by solving $dx/dt=0$ as:

$$\bar{A}x+\bar{b}=0 \quad (40)$$

Using the Cramer formula we have

$$x_{m+n}=\left[\begin{array}{c|c|c} 0 & -H_n & b_e \\ \hline H^T & 0 & 0 \end{array}\right]/|\bar{A}| \quad (41)$$

where H_n is the matrix obtained by deleting the n-th column of H.

$$x_{m+n}\neq 0 \quad (42)$$

should be satisfied. Suppose that $n=m+1$ holds in (41). Then since the number of the rows of H^T is larger than that of columns, we see that the numerator of (41) vanishes identically, which contradicts (42). に Thus we have **Lemma5:** (36) and (42) holds if and only if

$$m=n, \quad (43)$$

that is, the number of inductors is equal to that of capacitors.

Under the condition (43) we have from (38):

$$|\bar{A}|=(-1)^n|H^T||-H|=|H|^2\geq 0 \quad (44)$$

We also have

$$x_{m+k}=\frac{\left|\begin{array}{cc} 0 & -H_k \\ H^T & 0 \end{array}\right|}{|\bar{A}|}=\frac{|H_k||H^T|}{|\bar{A}|} \quad (1\leq k\leq n)(45)$$

where H_k is the matrix obtained by replacing the k-th column of H with $-\bar{b}_e$. We can get similar formulae corresponding to (45) for x_k $(k=1,2,\cdots,m)$.

We can describe H and b_e in Eqs. (33) and (35) by using the elements C_{ij} of the fundamental cutset matrix as follows:

$$H=-(dC_{cl}^{T}+d'C_{cl}^{\prime T}),\quad b_e=dC_{el}^{T}+d'C_{el}^{\prime T} \quad (46)$$

3. Topological Conditions

We will investigate $|H|$ and $|H_n|$ in Eqs. (44) and (45) in detail and show some topological conditions for them.

3.1 The case where switches are one pair

We first consider the simplest case:

$$m=n=1 \quad (47)$$

Then

$$C_f\equiv\begin{array}{c} \\ S^I \\ C \\ \\ \\ \\ E \end{array}\begin{array}{c} \begin{array}{cccccc} S^I & C & E & S^{II} & L & R \end{array} \\ \left[\begin{array}{c|c|c|c|c|c} 1 & & & \alpha_0 & p_0 & \\ \hline & & & \alpha_1 & p_1 & \\ & 1 & & \alpha_2 & p_2 & \\ & & & \vdots & \vdots & \\ & & & \alpha_n & p_n & \\ \hline & & 1 & \alpha_{n+1} & p_{n+1} & \end{array}\right] \end{array} \quad (48)$$

where α_i are ± 1 or 0, in particular, $\alpha_0=\pm 1$.

3.1.1 Graphical interpretation of $|H|$ Let one of the CL block(submatrix) of the matrix C_f be

$$C_{cl}=\left[\begin{array}{c} p_1 \\ p_2 \\ \vdots \\ p_n \end{array}\right](\equiv P\text{とおく}) \quad (49)$$

Since the $(S^I,\ S^{II})$ element of C_f is α_0, we can derive (C,L)-block C'_{cl} of C'_f (which corresponds to the C_f with S^I and S^{II} interchanged) in a similar way as in Eqs. (2) and (6) as follows:

$$C'_{cl}=\left[\begin{array}{c} p_1-\alpha_0^{-1}\alpha_1p_0 \\ p_2-\alpha_0^{-1}\alpha_2p_0 \\ \vdots \\ p_n-\alpha_0^{-1}\alpha_np_0 \end{array}\right]=C_{cl}-\alpha_0^{-1}\left[\begin{array}{c} \alpha_1 \\ \vdots \\ \alpha_n \end{array}\right]p_0 \quad (50)$$

We therefore have by (46):

$$\begin{aligned} -H^T &= d\left[\begin{array}{c} p_1 \\ p_2 \\ \vdots \\ p_n \end{array}\right]+d'\left[\begin{array}{c} p_1-\alpha_0^{-1}\alpha_1p_0 \\ p_2-\alpha_0^{-1}\alpha_2p_0 \\ \vdots \\ p_n-\alpha_0^{-1}\alpha_np_0 \end{array}\right] \\ &= \left[\begin{array}{c} (d+d')p_1-d'\alpha_0^{-1}\alpha_1p_0 \\ (d+d')p_2-d'\alpha_0^{-1}\alpha_2p_0 \\ \vdots \\ (d+d')p_n-d'\alpha_0^{-1}\alpha_np_0 \end{array}\right] \end{aligned} \quad (51)$$

Since each row of $-H^T$ is the sum of the vector $(d+d')p_i$ and p_0 multiplied by some constant, $|-H^T|$ can be calculated as follows:

$$\begin{aligned} |-H^T| &= (d+d')^n|P|-(d+d')^{n-1}\sum_{i=1}^{n}|P_i| \\ &+ (d+d')^{n-2}\sum_{i,j=1}^{n}|P_{ij}| \\ &- (d+d')^{n-3}\sum_{i,j,k=1}^{n}|P_{ijk}|+\cdots \end{aligned} \quad (52)$$

where P_i is the matrix obtained from P by replacing the i-th row of P by p_0, P_{ij} is the matrix obtained from P by replacing the i, and$j$-th rows of P by p_0, P_{ijk} is the matrix obtained from P by replacing the $i,j$and$k$-th rows of P by p_0, and so on. We therefore see that The third, forth, ... of the right-hand side of (52) vanish.

Now the first and second terms of (52) can be rewritten as follows:

$$|-H^T| = (d+d')^n|P|-(d+d')^{n-1}\sum_{i=1}^{n}|P_i|$$

$$= \frac{1}{d+d'} \begin{vmatrix} (d+d') & (d+d')p_0 \\ \alpha_1\alpha_0^{-1}d' & (d+d')p_1 \\ \alpha_2\alpha_0^{-1}d' & (d+d')p_2 \\ \vdots & \vdots \\ \alpha_n\alpha_0^{-1}d' & (d+d')p_n \end{vmatrix} \tag{53}$$

We define $|n \times n$ matrix C_{scjl} as follows:

$$C_{scjl} = \begin{bmatrix} \alpha_0 & p_0 \\ \alpha_1 & p_1 \\ \vdots & \vdots \\ \alpha_n & p_n \end{bmatrix} \tag{54}$$

Lemma 6: If $m = n = 1$, then

$$\begin{aligned} |-H^T| &= d(d+d')^{n-1}|C_{cl}| + \alpha_0^{-1}d'(d+d')^{n-1}|C_{scjl}| \\ &= (d+d')^{n-1}\left\{d\,|C_{cl}| + \alpha_0^{-1}d'\,|C_{scjl}|\right\}. \end{aligned} \tag{55}$$

Lemma7: $|A| \neq 0$ only if $|C_{cl}| \neq 0$ or $|C_{scjl}| \neq 0$.

Lemma 8 : $|C_{cl}| \neq 0$ holds if and only if L and C form the complementary tree structure of $G(S^I, E; S^{II}, R_o)$.

Lemma 9: $|C_{scjl}| \neq 0$ holds if and only if $\{C, S^I\}$ and $\{L, S^{II}\}$ form the complementary tree structure of $G(E; R_o)$.

If $|C_{cl}|$ and $\alpha_0^{-1}|C_{scjl}|$ have opposite sign, the denominator $|H|^2$ of (55) vanishes for some values of d, d'. This is not inconvenient for DC-DC converters.

The followin equality holds:

$$|C_{scjl}| = |C_{cl}|\beta \tag{56}$$

where

$$\beta = \alpha - p_0 P^{-1}[\alpha_1, \cdots, \alpha_n]^T \tag{57}$$

β represents the direction of the cutset consisting of S^I and S^{II} in the graph $G(E, L; R_0, C)$, while α_0 represents that in the graph $G(E, C; R_0, L)$. We therefore see that:

Lemma 10: $|C_{cl}|$ and $\alpha_0^{-1}|C_{scjl}|$ has opposite sign if and only if $G(E, L; R_0, C)$ and $G(E, C; R_0, L)$ has opposite direction cutset.

Lemma 11: If switches are only one pair, then $\alpha_0 = \beta$. This means that the denominator does not vanish.

3.1.2 Numerator of x_{m+n} In the above we consider the denominator of x_{m+n}. In this section we consider the numerator of (41).

We examine it by using H, b_e in (46) instead of (41). Then we see that α_n and p_n in Eqs. (49), (50), (51), (53), and (54) should be replaced by α_{n+1} and p_{n+1} in (48). For example, (49) and (54) should be relaced as:

$$C_{cl}^* = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_{n-1} \\ p_{n+1} \end{bmatrix}, \quad C_{scjl}^* = \begin{bmatrix} \alpha_0 & p_0 \\ \alpha_1 & p_1 \\ \vdots & \vdots \\ \alpha_n & p_n \end{bmatrix} \tag{58}$$

This means that $G(E; R_0)$ should be replaced by the graph $G(C_n; R_0)$, which is obtained from it by short-circuiting C_n and leaving E. We therefore get the following lemmas corresponding to Lemmas 8-11.

Lemma 12: If $m = n = 1$, then

$$|-H_n b_e| = (d+d')^{n-1}\left\{d\,|C_{cl}^*| + \alpha_0^{-1}d'\,|C_{scjl}^*|\right\} \tag{59}$$

Lemma 13: The numerator of x_{m+n} does not vanish only if $|C_{cl}^*| \neq 0$ or $|C_{scjl}^*| \neq 0$ holds.

Lemma 14: $|C_{cl}^*| \neq 0$ holds if and only if L and $\{C_i(i = 1, \cdots, n-1), E\}$ form the complementary tree structure of the graph $G(S^I, C_n; S^{II}, R_o)$.

Lemma 15 : $|C_{scjl}^*| \neq 0$ holds if and only if $\{C_i(i = 1, \cdots, n-1), E, S^I\}$ and $\{L, S^{II}\}$ form the complementary tree structure of the graph $G(C_n; R_o)$.

3.1.3 Booster, Buck, Buck/Booster Converters The above circuit works as a booster converter, a buck converter, a buck/booster converter depending on the values of the coefficients of d and d' in Eqs. (55) and (59) That is, it depends on the values of $|C_{cl}|$, $|C_{scjl}|$, $|C_{cl}^*|$, and $|C_{scjl}^*|$ being 0 or not.

$$\begin{array}{lcl} \text{Booster type} & : & \frac{d+d'}{d} \text{ or } \frac{d+d'}{d'} \\ \text{Buck type} & : & \frac{d}{d+d'} \text{ or } \frac{d'}{d+d'} \\ \text{Buck/booster type} & : & \frac{d'}{d} \text{ or } \frac{d}{d'} \end{array}$$

Booster	one of $\|C_{cl}\| = 0$, $\|C_{scjl}\| = 0$ holds and $\|C_{cl}^*\| \neq 0$, and $\|C_{scjl}^*\| \neq 0$
Buck	$\|C_{cl}\| \neq 0$ $\|C_{scjl}\| \neq 0$, and one of $\|C_{cl}^*\| = 0$ $\|C_{scjl}^*\| = 0$ holds
Buck/booster	$\left.\begin{array}{l} \|C_{cl}\| = 0,\ \|C_{scjl}\| \neq 0, \\ \|C_{cl}^*\| \neq 0\ \|C_{scjl}^*\| = 0 \end{array}\right\}$ or $\left.\begin{array}{l} \|C_{cl}\| \neq 0,\ \|C_{scjl}\| = 0, \\ \|C_{cl}^*\| = 0,\ \|C_{scjl}^*\| \neq 0 \end{array}\right\}$

From this table and the previous lemmas we can easily derive the conditions for Buck, Booster, Buck/booster types.

3.2 More than one pair of switches

In this section we extend the result for $m = n = 1$ to general $m = n > 1$ case. For simplicity we describe only the $m = n = 2$ case.

We write only the equations corresponding to Eq. (48).

$$C_f = \begin{array}{c} \\ S^I \\ \\ C \\ \\ E \end{array} \begin{array}{c} \begin{array}{cccccc} S^I & C & E & S^{II} & L & R \end{array} \\ \left[\begin{array}{cc|c|c|cc|c|c} 1 & 0 & p_{01} & & \alpha_{011} & \alpha_{012} & 0 & 0 \\ 0 & 1 & p_{02} & & \alpha_{021} & \alpha_{022} & & \\ \hline & & p_1 & & & \alpha_1 & & \\ & 0 & \vdots & & & \vdots & 1 & 0 \\ & & p_n & & & \alpha_n & & \\ \hline & 0 & p_{n+1} & & & \alpha_{n+1} & 0 & 1 \end{array}\right] \end{array} \tag{60}$$

where $\alpha_0 = [\alpha_{0ij}]$ is a 2×2 matrix and α_i and $p_i(i = 1, \cdots, n+1)$ are 1×2 matrix.

We omit the details, but almost the same lemmas and equations as $m = n = 1$ are obtained for the $m = n = 2$ case, except for the calculation of 2×2 matrix in the case of topological criterion. We can further extend the result to $m = n \geq 2$ cases.

4. Conclusion

In this paper we consider the circuit without transformer, but we can generalize the result to those containing transformers.

5. Acknowlegment

This research was partially supported by Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research (C), 13650414, 2001.

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