Numerical Robust Stability Analysis and Design of Fuzzy Feedback Linearization Regulator

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Abstract: In this paper, numerical robust stability analysis method and its design are presented. L_2 robust stability of the fuzzy system is analyzed by casting the systems into the diagonal norm bounded linear differential inclusions (DNLDI) formulation. Based on the linear matix inequality (LMI) optimization programming, a numerical method for finding the maximum stable ranges of the fuzzy feedback linarization control gains is proposed.

1. Introduction

Up to now, Fuzzy feedback linearization has attracted the attention since the nonlinearity can be efficiently modeled and canceled by fuzzy logic system.

Fuzzy feedback linearization is a feedback linearization method which uses a fuzzy model as a nonlinear system model. Since the idea of the fuzzy feedbcak lineariation control based on Takagi-Sugeno (TS) models [1] was presented in [2], various kinds of robust [6-8] and adaptive techniques [3-5] have been applied to the fuzzy feedback linearization control. While the adaptive fuzzy feedback linearization guarantees Lyapunov stability in the presence of uncertainty, it has some practical limitations due to its complex structures. From a practical point of view, robust approach is more suitable for the fuzzy feedback linearization to overcome the uncertainty. The stability analysis was made in the frequency domain in [6] and the robust stability condition and design method using multivariable circle criterion have been presented in [7]. However, they based on graphical stability analysis, there exist some difficulties in being applied to the control problems directly. In order to obtain the numerical solutions for the fuzzy feedback linearization control systems, Linear Matrix Inequality (LMI) based robust stability condition which can be solved numerically for the fuzzy feedback linearization regulator has been presented in [8]. in which, however, the only stability analysis was done.

In this paper, we study a controller design as well as numerical stability analysis for the robust fuzzy feedback linearization control systems using TS fuzzy model. For the structured uncertainty, the L_2 robust stability of the closed system are analyzed by applying the LMI based convex optimization method. The stability problems are cast into diagonal norm bounded linear differential inclusions (DNLDI) and a generalized eigenvalue problem (GEVP) [9]. We present a systematic numerical method for finding the maximum stable ranges of the fuzzy feedback linearization control gains.

2. Problem formulation

The fuzzy model represents a nonlinear system with the following form of fuzzy rules.

i-th plant rule:

$IF\ x\ is\ M_{i1}$ and $\dot{x}\ is\ M_{i2}$ and $\cdots$ and $x^{(n-1)}\ is\ M_{in}$

$$THEN\ x^{(n)} = (a_i + \Delta a_i(t))^T \cdot x + (b_i + \Delta b_i(t))u + d, \qquad i = 1, 2, \cdots, r \qquad (1)$$

where $x = [x, \dot{x}, \cdots, x^{(n-1)}]^T$ is the state vector which is assumed to be available and $a_i, \Delta a_i(t) \in R^{1 \times n}$, $b_i, \Delta b_i(t) \in R$ and $d \in R$ denotes unknown external disturbance which belongs to L_2 space such that $\int_0^\infty d(t)^2 dt < \infty$ (2)

Also, M_{ij} is the fuzzy set and r is the number of fuzzy rules and Also, $\Delta a_i(t)$ and $\Delta b_i(t)$ denote the norm-bounded time-varying modeling uncertainties for system and input matrices, respectively. The TS fuzzy model can be inferred as

$$x^{(n)} = \frac{\sum_{i=1}^{r} w_i(x)\{(a_i + \Delta a_i(t))^T \cdot x + (b_i + \Delta b_i(t))u\}}{\sum_{i=1}^{r} w_i(x)} + d$$

$$= \sum_{i=1}^{r} h_i(x)\{(a_i + \Delta a_i(t))^T \cdot x + (b_i + \Delta b_i(t))u\} + d \qquad (3)$$

where $w_i(x) = \prod_{j=1}^{n} M_{ij}(x^{(j-1)})$, $h_i(x) = \frac{w_i(x)}{\sum_{i=1}^{r} w_i(x)}$

For the robust stability, consider the following control law,

$$u = \frac{(a_R^T + \sum_{i=1}^{r} h_i(x)(a_d^T - a_i^T)) \cdot x}{\sum_{i=1}^{r} h_i(x) b_i} \qquad (4)$$

where, $a_R \in R^n$ is the appended input vector in order to reduce the disturbance, which comes from the uncertainties.

By substituting (4) into (3), the closed loop system can be written as (5).

$$x^{(n)} = a_d^T \cdot x + a_N(t)^T \cdot x + d \qquad (5)$$

where, $a_N(t) = a_R + \sum_{i=1}^{r} h_i(x)\ \Delta a_i(t)$

$$+ \frac{\sum_{i=1}^{r} h_i(x) \Delta b_i(t)}{\sum_{i=1}^{r} h_i(x) b_i}\{\sum_{i=1}^{r} h_i(x)(a_d + a_R - a_i)\} \qquad (6)$$

In the following, the robust stability analysis and the design of a_R for (6) is presented.

3. L_2 robust stability analysis of fuzzy feedback linearization control systems

In order to give the numerical L_2 stability condition, the closed system (6) is cast into Diagonal Norm-bound Linear Differential Inclusions (DNLDI). DNLDI is a linear system with scalar, uncertain and time-varying feedback gains, each of which is bounded by one. The DNLDI formulation of the closed system (6) is given by

$$\dot{x} = Ax + Bp + w, \quad p = \Delta(t)Cx, \quad z = Dx \qquad (7)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 \\ & & \vdots & & \\ a_{d1} & a_{d2} & a_{d3} & \cdots & a_{dn} \end{bmatrix} \in R^{n \times n}, \quad B = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ & & \vdots & & \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix} \in R^{n \times n},$$

$$C = \begin{bmatrix} c_1 & 0 & 0 & \cdots & 0 \\ 0 & c_2 & 0 & \cdots & 0 \\ 0 & 0 & c_3 & \cdots & 0 \\ & & \vdots & & \\ 0 & 0 & 0 & \cdots & c_n \end{bmatrix} \in R^{n \times n}, \quad \Delta(t) = \begin{bmatrix} \delta_1(t) & 0 & 0 & \cdots & 0 \\ 0 & \delta_2(t) & 0 & \cdots & 0 \\ 0 & 0 & \delta_3(t) & \cdots & 0 \\ & & \vdots & & \\ 0 & 0 & 0 & \cdots & \delta_n(t) \end{bmatrix}$$

$$\delta_j(t) = \begin{cases} \dfrac{a_{Nj}(t)}{c_j} & \text{if } c_j \neq 0 \\ 0 & \text{if } c_j = 0 \end{cases} \tag{8}$$

constraint : $| \Delta a_{Nj}(t) | \le c_j \; (i = 1, 2, \cdots, n)$ (9)

or equivalently, $p^T p \le x^T C^T C x$ (10)

$$D = I \in R^{n \times n}, \quad p \in R^n, \quad z \in R^n, \quad w = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ d \end{bmatrix} \in R^n \tag{11}$$

Remark 1: In (8), $c_j \; (j = 1, 2, \cdots, n)$ can be any non-negative real scalar satisfying the constraint (9) or C can be any diagonal positive semidefinite matrix satisfying the constraint (10). Note that c_j can be set to 0, only if there is no uncertainty in the corresponding a_j, i.e. $\Delta a_{Nj}(t) = 0$. In Appendix A, the selecting method of $c_j \; (j = 1, 2, \cdots, n)$ for the stability analysis is proposed.

In (11), w is the unknown external disturbance input which belongs to L_2 space such that

$$\int_0^\infty w^T w \; dt < \infty \tag{12}$$

and z is the output which is the same as the state x.

Theorem 1: The system (7) is L_2 stable and its L_2 gain is less than γ if there exist $P > 0$ and $\tau \ge 0$ such that

$$\begin{bmatrix} A^T P + PA + D^T D + \tau C^T C & PB & P \\ B^T P & -\tau I & 0 \\ P & 0 & -\gamma^2 I \end{bmatrix} \le 0 \tag{13}$$

Proof: Now, suppose there exist a quadratic function $V(x) = x^T P x$, $P > 0$, and $\gamma \ge 0$ such that for all t,

$$\frac{d}{dt} V(x) + z^T z - \gamma^2 w^T w$$

$$= x^T (A^T P + PA + D^T D) x + 2 x^T PBp + 2Pw - \gamma^2 w^T w \le 0 \tag{14}$$

for all x and p satisfying $p^T p \le x^T C^T C x$

Using the S-procedure of LMI techniques [9], (14) is equivalent to the existence of P and τ satisfying

$$\begin{bmatrix} A^T P + PA + D^T D + \tau C^T C & PB & P \\ B^T P & -\tau I & 0 \\ P & 0 & -\gamma^2 I \end{bmatrix} \le 0$$

To show the L_2 gain (10) is less than γ, we integrate (14) from 0 to T, with the initial condition $x(0) = 0$, to get

$$V(x(T)) + \int_0^T (z^T z - \gamma^2 w^T w) \, dt \le 0$$

Since $V(x(T)) \ge 0$, this implies

$$\frac{\|z\|_2}{\|w\|_2} < \gamma .$$

Therefore, from the Theorem 1, we can obtain the upper bound on the L_2 gain by solving the following EigenValue Problem (EVP).

minimize γ

$$P > 0, \; \tau \ge 0,$$

$$\begin{bmatrix} A^T P + PA + D^T D + \tau C^T C & PB & P \\ B^T P & -\tau I & 0 \\ P & 0 & -\gamma^2 I \end{bmatrix} \le 0$$

Based on the Theorem 1, the analysis procedure can be summarized as follows.

STEP 1. Cast the closed system (6) into DNLDI (7).

STEP 2. Select $c_j \; (j = 1, 2, \cdots, n)$ as in Appendix A.

STEP 3. Check the stability condition of Theorem 1. This can be easily done by solving the feasibility problem.

STEP 4. If there exists a feasible EVP solution $\gamma_{min} > 0$, then the closed system is robust stable in L_2 sense and L_2 gain is less than γ_{min}.

Therefore, from the Theorem 1, we can obtain the upper bound on the L_2 gain by solving the following EigenValue Problem (EVP).

minimize γ,

$$P > 0, \; \tau \ge 0, \; \begin{bmatrix} A^T P + PA + D^T D + \tau C^T C & PB & P \\ B^T P & -\tau I & 0 \\ P & 0 & -\gamma^2 I \end{bmatrix} \le 0 \tag{15}$$

4. L_2 robust stable design of fuzzy feedback linearization control systems

Our problem is that of determining the L_2 robust stability range of $a_{Nj} \; (j = 1, 2, \cdots n)$ which can maintain the L_2 gain of the closed system (5) within the specified upper bound γ_{max}. From the constraint, c_j (9) can be regarded as the upper bound on $| a_{Nj}(t) | \; (j = 1, 2, \cdots, n)$.

Therefore, in order to determine a robust stable range on a_{Ri}, we need to find the largest possible c_i for which Theorem 1 holds with $\gamma = \gamma_{max}$ should be obtained. This can be obtained by solving the following optimization problem (17).

maximize $c_1, c_2, \cdots, c_n$ subject to

$$P > 0, \; \tau \ge 0, \; \begin{bmatrix} A^T P + PA + D^T D + \tau C^T C & PB & P \\ B^T P & -\tau I & 0 \\ P & 0 & -\gamma_{max}^2 I \end{bmatrix} \le 0 \tag{17}$$

However, it is difficult to solve the multiple parameter optimization problem (17) straightforward. Instead, by splitting (17) into the single parameter optimization problems (18) for each i, it is possible to derive the feasible solution of (17) from the solutions of (18).

maximize c_i subject to

$$P_i > 0, \; \tau_i \ge 0,$$

$$\begin{bmatrix} A^T P_i + P_i A + D^T D + \tau_i C_i^T C_i & P_i B & P_i \\ B^T P_i & -\tau_i I & 0 \\ P_i & 0 & -\gamma_{max}^2 I \end{bmatrix} \le 0 \tag{18}$$

where $C_i = \mathbf{diag}(\underbrace{0, \cdots, 0}_{i-1}, c_i, \underbrace{0, \cdots, 0}_{n-i})$

If we define $\lambda_i = c_i^2$, the optimization problem (18) can be viewed as the Generalized Eigen-Value Problem (GEVP) (19).

maximize λ_i subject to

$$P_i > 0, \; \begin{matrix} \tau_i \ge 0 \\ \lambda_i \ge 0 \end{matrix},$$

$$\begin{bmatrix} A^T P_i + P_i A + D^T D + \lambda_i \tau_i E_i^T E_i & P_i B & P_i \\ B^T P_i & -\tau_i I & 0 \\ P_i & 0 & -\gamma_{max}^2 I \end{bmatrix} \le 0 \tag{19}$$

where $E_i = \dfrac{C_i}{c_i}$

Thus, the above GEVP can be easily solved by well-established LMI optimization techniques [9].

Denote the solutions of GEVP (19) as $\overline{\lambda_i} \; (i = 1, 2, \cdots, n)$.

Then, the solutions of the optimization problem (18) can be written as $\overline{c_i} = \sqrt{\overline{\lambda_i}} \; (i = 1, 2, \cdots, n)$. Now, it should be noted that $\overline{c_1}, \overline{c_2}, \cdots, \overline{c_n}$ can not be a feasible solution of the optimization problem (17).

$$\text{For } \; \overline{C} = \begin{bmatrix} \overline{c_1} & 0 & 0 & \cdots & 0 \\ 0 & \overline{c_2} & 0 & \cdots & 0 \\ 0 & 0 & \overline{c_3} & \cdots & 0 \\ & & \vdots & & \\ 0 & 0 & 0 & \cdots & \overline{c_n} \end{bmatrix} = \sum_{i=1}^{n} \overline{C_i} , \tag{20}$$

where $\overline{C_i} = \mathbf{diag}(\underbrace{0, \cdots, 0}_{i-1}, \overline{c_i}, \underbrace{0, \cdots, 0}_{n-i})$, it can not be guaranteed (14) holds. Thus, some modifications are needed to obtain a feasible solution. The modified $\overline{C^m}$ can be written as

$$\overline{C^m} = \begin{bmatrix} \overline{c_1^m} & 0 & 0 & \cdots & 0 \\ 0 & \overline{c_2^m} & 0 & \cdots & 0 \\ 0 & 0 & \overline{c_3^m} & \cdots & 0 \\ & & \vdots & & \\ 0 & 0 & 0 & \cdots & \overline{c_n^m} \end{bmatrix} = \frac{1}{\sqrt{\sum_{i=1}^{n} \overline{\tau_i}}} \begin{bmatrix} \sqrt{\overline{\tau_1}}\,\overline{c_1} & 0 & 0 & \cdots & 0 \\ 0 & \sqrt{\overline{\tau_2}}\,\overline{c_2} & 0 & \cdots & 0 \\ 0 & 0 & \sqrt{\overline{\tau_3}}\,\overline{c_3} & \cdots & 0 \\ & & \vdots & & \\ 0 & 0 & 0 & \cdots & \sqrt{\overline{\tau_n}}\,\overline{c_n} \end{bmatrix}$$

$$= \frac{\sum_{i=1}^{n} (\sqrt{\overline{\tau_i}}\, \overline{C_i})}{\sqrt{\sum_{i=1}^{n} \overline{\tau_i}}} \tag{21}$$

where $\overline{\tau_i}$ denotes τ_i corresponding to $\overline{\lambda_i}$ or $\overline{c_i}$ $(i = 1, 2, \cdots, n)$.
In Theorem 2, it is shown that Theorem 1 holds for $\overline{C^m}$ in (21).

Theorem 2: For C^m in (21), there exists $P > 0$ and $\tau \geq 0$ which satisfy LMI L_2 stability condition (22).

$$\begin{bmatrix} A^TP+PA+D^TD+\tau\,\overline{C^m}^T\overline{C^m} & PB & P \\ B^TP & -\tau I & 0 \\ P & 0 & -\gamma_{\max}^2 I \end{bmatrix} \leq 0 \tag{22}$$

Proof: Since $\overline{c_i}$ $(i = 1, 2, \cdots, n)$ is the solution of the optimization problem (18), the following holds for all i

$$\begin{bmatrix} A^T\overline{P_i}+\overline{P_i}A+D^TD+\overline{\tau_i}\,\overline{C_i}^T\overline{C_i} & \overline{P_i}B & \overline{P_i} \\ B^T\overline{P_i} & -\overline{\tau_i}I & 0 \\ \overline{P_i} & 0 & -\gamma_{\max}^2 I \end{bmatrix} \leq 0 \tag{23}$$

where $\overline{P_i}$ denotes P_i corresponding to $\overline{\lambda_i}$ or $\overline{c_i}$ $(i = 1, 2, \cdots, n)$.
Hence, from the property of the negative semi-definite matrix (24) also holds.

$$\frac{1}{n}\sum_{i=1}^{n}\begin{bmatrix} A^T\overline{P_i}+\overline{P_i}A+D^TD+\overline{\tau_i}\,\overline{C_i}^T\overline{C_i} & \overline{P_i}B & \overline{P_i} \\ B^T\overline{P_i} & -\overline{\tau_i}I & 0 \\ \overline{P_i} & 0 & -\gamma_{\max}^2 I \end{bmatrix} \leq 0 \tag{24}$$

By rearranging the summations, (24) becomes

$$\begin{bmatrix} A^T(\sum_{i=1}^{n}\frac{\overline{P_i}}{n})+(\sum_{i=1}^{n}\frac{\overline{P_i}}{n})A+D^TD+\sum_{i=1}^{n}\frac{\overline{\tau_i}}{n}\overline{C_i}^T\overline{C_i} & (\sum_{i=1}^{n}\frac{\overline{P_i}}{n})B & (\sum_{i=1}^{n}\frac{\overline{P_i}}{n}) \\ B^T(\sum_{i=1}^{n}\frac{\overline{P_i}}{n}) & -(\sum_{i=1}^{n}\frac{\overline{\tau_i}}{n})I & 0 \\ (\sum_{i=1}^{n}\frac{\overline{P_i}}{n}) & 0 & -\gamma_{\max}^2 I \end{bmatrix} \leq 0 \tag{25}$$

Using the property of $\overline{C_i}$, it can be easily shown that

$$\sum_{i=1}^{n}\frac{\overline{\tau_i}}{n}\overline{C_i}^T\overline{C_i} = (\sum_{i=1}^{n}\sqrt{\frac{\overline{\tau_i}}{n}}\,\overline{C_i}^T)(\sum_{i=1}^{n}\sqrt{\frac{\overline{\tau_i}}{n}}\,\overline{C_i}) \tag{26}$$

holds.
Employing (25), (26) can be written as

$$\begin{bmatrix} A^T(\sum_{i=1}^{n}\frac{\overline{P_i}}{n})+(\sum_{i=1}^{n}\frac{\overline{P_i}}{n})A+D^TD+(\sum_{i=1}^{n}\sqrt{\frac{\overline{\tau_i}}{n}}\,\overline{C_i}^T)(\sum_{i=1}^{n}\sqrt{\frac{\overline{\tau_i}}{n}}\,\overline{C_i}) & (\sum_{i=1}^{n}\frac{\overline{P_i}}{n})B & (\sum_{i=1}^{n}\frac{\overline{P_i}}{n}) \\ B^T(\sum_{i=1}^{n}\frac{\overline{P_i}}{n}) & -(\sum_{i=1}^{n}\frac{\overline{\tau_i}}{n})I & 0 \\ (\sum_{i=1}^{n}\frac{\overline{P_i}}{n}) & 0 & -\gamma_{\max}^2 I \end{bmatrix} \leq 0 \tag{27}$$

Let us choose

$$P = \sum_{i=1}^{n}\frac{\overline{P_i}}{n} \quad \text{and} \quad \tau = \sum_{i=1}^{n}\frac{\overline{\tau_i}}{n} \tag{28}$$

Using (21), (28) and (27) can be expressed as

$$\begin{bmatrix} A^TP+PA+D^TD+\tau\,\overline{C^m}^T\overline{C^m} & PB & P \\ B^TP & -\tau I & 0 \\ P & 0 & -\gamma_{\max}^2 I \end{bmatrix} \leq 0 \tag{29}$$

Therefore, for $\overline{C^m}$ in (21), there exists $P > 0$ and $\tau \geq 0$ which satisfies LMI L_2 stability condition (22). Since Theorem 2 holds for $\overline{C^m}$ in (27), $\overline{c_1^m}, \overline{c_2^m}, \cdots, \overline{c_n^m}$ can be a feasible solution of the optimization problem (17). Therefore $\overline{c_j^m}$ $(j = 1, 2, \cdots, n)$ can be used as the largest possible c_j $(i = 1, 2, \cdots, n)$ for which Theorem 1 holds.

Thus, using the admissible bounds of $|a_{Nj}(t)|$ with respect to a_{Rj}, the robust stable range of a_{Rj} can be expressed by the following set representation (30).

$$\left\{ a_{Rj} \;\middle|\; |a_{Rj}| + \max_i |\Delta a_{ij}(t)| + \frac{\max_i |\Delta b_i(t)|}{\min_i |b_i|}\left(\max_i |a_{dj} + a_{Rj} - a_{ij}|\right) \leq c_j^m \right\},\quad j = 1, 2, \cdots, n \tag{30}$$

The control design procedure is summarized as follows.

STEP 1. Cast the closed loop system (5) into DNLDI (7).
STEP 2. Solve the GEVP (19).
STEP 3. Find the stable range (30) of a_{Rj} from $\overline{C^m}$ in (21)
STEP 4. Select proper a_{Rj} in the set (30)

5. Numerical examples

Consider the problem of balancing and swing-up of an inverted pendulum on a cart. The dynamic equations can be approximated by the following two fuzzy rules [8] and the membership functions used in this fuzzy model are shown in [8].

Rule 1 : *IF x is about* 0
THEN $\dot{x} = (\mathbf{a}_1 + \Delta\mathbf{a}_1(t))^T \cdot \mathbf{x} + (b_1 + \Delta b_1(t))u + d$
Rule 2 : *IF x is about* $\pm\frac{\pi}{2}$ $(|x| < \frac{\pi}{2})$
THEN $\dot{x} = (\mathbf{a}_2 + \Delta\mathbf{a}_2(t))^T \cdot \mathbf{x} + (b_2 + \Delta b_2(t))u + d$ (31)

(31) can be inferred as

$$\dot{x} = \frac{\sum_{i=1}^{2} w_i(\mathbf{x})\{(\mathbf{a}_i + \Delta\mathbf{a}_i(t))^T \cdot \mathbf{x} + (b_i + \Delta b_i(t))u\}}{\sum_{i=1}^{2} w_i(\mathbf{x})} + d$$
$$= \sum_{i=1}^{2} h_i(\mathbf{x})\{(\mathbf{a}_i + \Delta\mathbf{a}_i(t))^T \cdot \mathbf{x} + (b_i + \Delta b_i(t))u\} + d \tag{32}$$

where $w_i(\mathbf{x}) = \prod_{j=1}^{2} M_{ij}(x^{(j-1)})$, $h_i(\mathbf{x}) = \frac{w_i(\mathbf{x})}{\sum_{i=1}^{2} w_i(\mathbf{x})}$ and,

$\mathbf{a}_1 = \left[\frac{g}{4l/3 - aml} \quad 0\right] = [17.29 \quad 0]$, $\mathbf{a}_2 = \left[\frac{2g}{\pi(4l/3 - aml\beta^2)} \quad 0\right] = [9.35 \quad 0]$,
$b_1 = -\frac{a}{4l/3 - aml} = -0.1765$, $b_2 = -\frac{a\beta}{4l/3 - aml\beta^2} = -0.0052$

we assume that $\Delta\mathbf{a}_1(t)$, $\Delta\mathbf{a}_2(t)$, $\Delta b_1(t)$, $\Delta b_2(t)$ are unknown but bounded as follows.
$-1 \leq \Delta a_{11}(t) \leq 1$, $-0.5 \leq \Delta a_{12}(t) \leq 0.5$, $-1 \leq \Delta a_{21}(t) \leq 1$,
$-0.5 \leq \Delta a_{22}(t) \leq 0.5$, $-0.001 \leq \Delta b_1(t) \leq 0.001$,
$-0.001 \leq \Delta b_2(t) \leq 0.001$
we use the feedback linearizartion control law as (33).

$$u = \frac{\mathbf{a}_R^T + \sum_{i=1}^{r} h_i(\mathbf{x})(\mathbf{a}_d^T - \mathbf{a}_i^T) \cdot \mathbf{x}}{\sum_{i=1}^{r} h_i(\mathbf{x}) b_i} \tag{33}$$

and then, the closed loop system by substituting (33) into (39) yields $\dot{x} = \mathbf{a}_d^T \cdot \mathbf{x} + \mathbf{a}_N(t)^T \cdot \mathbf{x} + d$
where, $\mathbf{a}_N(t) = \mathbf{a}_R \cdot \mathbf{x} + \sum_{i=1}^{r} h_i(\mathbf{x})\, \Delta\mathbf{a}_i(t) \cdot \mathbf{x}$

$$+ \frac{\sum_{i=1}^{r} h_i(\mathbf{x})\Delta b_i(t)}{\sum_{i=1}^{r} h_i(\mathbf{x}) b_i}\left\{\sum_{i=1}^{r} h_i(\mathbf{x})(\mathbf{a}_d + \mathbf{a}_R - \mathbf{a}_i) \cdot \mathbf{x}\right\} \tag{34}$$

Consider the design problem for a_{Rj}, $j=1,2$, for the feedback linearization control system (34) with $\mathbf{a}_d$ = [-1 -1]. Following the *STEPs* in Section 4, a_{Rj} can be obtained. In the design procedure, $\gamma = \gamma_{\max} = 0.01$ was specified.
Figure 1 shows the region of a_{R1} and a_{R2}, where we choose the parameters as a_{R1} = -2.5 and a_{R2} = -8.
In the computer simulation, as a disturbance $d(t)$ which belongs to L_2 space, the signal shown in Fig. 2 is used. Figs 3 and 4 illustrate the simulation results in which the initia condition is zero. In Figs. 5 and 6, the energy of the disturbance and the output are plotted with respect to time respectively.

L_2 norm of the input and output can be computed as

$$\|w\|_2 = \int_0^\infty w^T w\, dt = \int_0^\infty d(t)^2\, dt = 10 \tag{35}$$
$$\|z\|_2 = \int_0^\infty z^T z\, dt = \int_0^\infty x^T x\, dt = 0.0858 \tag{36}$$

Thus L_2 gain is

$$\sup_{\|w\|_2 \neq 0} \frac{\|z\|_2}{\|w\|_2} = 0.00858 \tag{37}$$

The simulation results illustrate that the closed system (34) i robust stable in L_2 sense and L_2 gain is less than 0.01 whic is specified in the design procedure, *STEP2*.
Also, in order to analyze the Lyapunov stability, th simulation results for the unforced system, i.e. $d(t) = 0$ and th

initial condition x_0 = [1 0] are presented in Fig. 7 and 8.

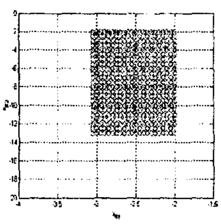

Fig. 1 a_{R1} and a_{R2}

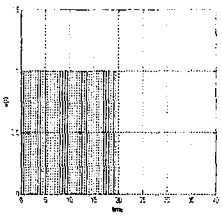

Fig. 2 $d(t)$

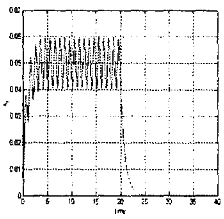

Fig. 3 state x_1

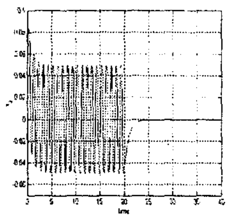

FIg. 4 state x_2

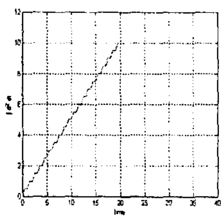

Fig. 5 disturbance energy

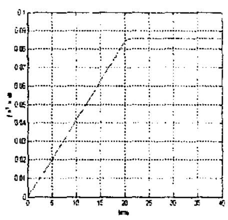

Fig. 6 output energy

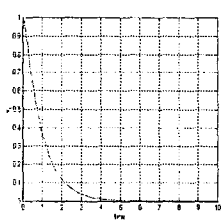

Fig. 7 state x_1

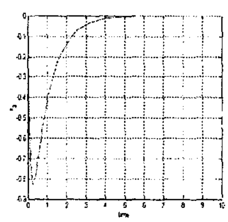

Fig. 8 state x_2

5. Conclusion

In this work, we have presented the LMI-based L_2 robust stability analysis and design method for the fuzzy feedback linearization control systems. The plant was represented by well-known TS fuzzy model and the analysis and design problems was numerically solved by casting the closed loop system into DNLDI and GEVP form. In the examples, the fuzzy feedback linearization controller was developed efficiently and the validity of the proposed analysis and design scheme was shown.

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Appendix A

Although c_j $(j = 1, 2, \cdots, n)$ can be any positive real scalar satisfying the constraint (9), c_j $(j = 1, 2, \cdots, n)$ should be chosen as the minimum upper bound for $|a_{Nj}(t)|$ to avoid the conservative analysis.

In order to obtain the minimum upper bound for $|a_{Nj}(t)|$, (6) is written in the component form as in (A.1)

$$a_{Nj}(t) = a_{Rj} + \sum_{i=1}^{r} h_i(x(t))\Delta a_{ij}(t) + \frac{\sum_{i=1}^{r} h_i(x(t))\Delta b_i(t)}{\sum_{i=1}^{r} h_i(x(t))b_i}\sum_{i=1}^{r} h_i(x(t))(a_{dj}+a_{Rj}-a_{ij}) \quad (j = 1, 2, \cdots, n) \tag{A.1}$$

Then, the following inequality (A.2) holds for all j in which we used basic assumption,

$$\sum_{i=1}^{r} h_i(x(t)) = 1 \quad \text{and} \quad \max_{x} h_i(x(t)) = 1 .$$

$$|a_{Nj}(t)| \le |a_{Rj}| + |\sum_{i=1}^{r} h_i(x(t))\Delta a_{ij}(t)| + \left|\frac{\sum_{i=1}^{r} h_i(x(t))\Delta b_i(t)}{\sum_{i=1}^{r} h_i(x(t))b_i}\sum_{i=1}^{r} h_i(x(t))(a_{dj}+a_{Rj}-a_{ij})\right| \tag{A.2}$$

The second and third terms in the right side of (A.2) satisfy (A.3) and (A.4).

$$|\sum_{i=1}^{r} h_i(x(t))\Delta a_{ij}(t)| \le \max_{i} |\Delta a_{ij}(t)| \tag{A.3}$$

$$\left|\frac{\sum_{i=1}^{r} h_i(x(t))\Delta b_i(t)}{\sum_{i=1}^{r} h_i(x(t))b_i}\sum_{i=1}^{r} h_i(x(t))(a_{dj}+a_{Rj}-a_{ij})\right| \le \frac{\max_i |\Delta b_i(t)|}{\min_i |b_i|}\left(\max_i |a_{dj}+a_{Rj}-a_{ij}|\right) \tag{A.4}$$

Then, the following inequality holds for all j.

$$|a_{Nj}(t)| \le |a_{Rj}| + \max_i |\Delta a_{ij}(t)| + \frac{\max_i |\Delta b_i(t)|}{\min_i |b_i|}\left(\max_i |a_{dj}+a_{Rj}-a_{ij}|\right)$$

Therefore, we choose

$$c_j = |a_{Rj}| + \max_i |\Delta a_{ij}(t)| + \frac{\max_i |\Delta b_i(t)|}{\min_i |b_i|}\left(\max_i |a_{dj}+a_{Rj}-a_{ij}|\right) \quad j = 1, 2, \cdots, n \tag{A.5}$$

for less conservative stability analysis.