Online SNR Estimation for Turbo Coded Multicode DS/SS Systems

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Abstract: In this paper, an online SNR estimator is derived for turbo coded multicode DS/SS systems in Nakagami fading channels. The multicode DS/SS approach is one of promising solutions to obtain higher-rate data transmission in DS/SS technologies. Turbo coding has paid much attention because of the significant improvements on error rate performances in various communication systems including multicode DS/SS systems. However, in the turbo decoding, channel state information, especially signal-to-noise ratio (SNR) at the correlator outputs, is desired in order to obtain such improvements. We evaluate the accuracy of the derived SNR estimation. It is shown that the bit error rate performance using our SNR estimation is close to the performance with perfect knowledge of channel state information.

1. Introduction

Direct sequence spread spectrum (DS/SS) technique has been successfully implemented in mobile cellular radio systems as code division multiple access (CDMA) [1] and in wireless LAN such as IEEE 802.11 [2] standard. Demand on high-rate data transmission is increasing in various wireless communication systems. To increase the data rate in DS/SS technique, an attractive solution is multi-code DS/SS technique [3]. In this multi-code DS/SS technique, the data rate is directly proportional to the number of transmitting PN codes.

Since transmitted signal encounters severe channel environment such as multipath fading, some appropriate error correction coding (ECC) techniques are usually applied in recent wireless data communication systems. Since the development of turbo codes [4], serially and parallel concatenated coding with a random interleaver and iterative decoding has been recognized as a powerful ECC technique to bring a significant improvement in error rate performance. To obtain such significant improvement, knowledge of signal-to-noise ratio (SNR) is crucial at the decoder so that the turbo decoder based on the MAP or log-MAP algorithm [5, 6] can work successfully. So, study on an online SNR estimation without any additional insertion is an attractive topic.

In this paper, we discuss online SNR estimation using based on a statistical ratio of received signals in the turbo coded multicode DS/SS systems. Our SNR estimation method is a modified version of the SNR estimation method proposed by Summers et al. [7] and Ramesh et al.[8] for multi-code DS/SS systems. This SNR estimator uses statistical ratio obtaining from correlator outputs. We evaluate the accuracy of the SNR estimation and the error rate performance on the turbo coded multicode DS/SS systems over Nakagami fading channels.

The organization of this paper is as follows. In the next section, the baseband system model for turbo coded multicode DS/SS system is shown. Online SNR estimation in Nakagami fading channels is derived in section 3. Simulation results and discussions are settled in section 4. Finally, we draw some conclusions in section 5.

2. System Model

Multicode DS/SS systems convey information data in parallel by transmitting pre-assigned M orthogonal PN codes. In Fig. 1, the baseband system model of turbo coded multicode DS/SS systems is shown.

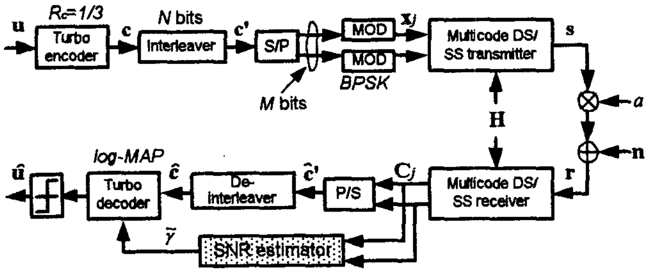

Figure 1. Baseband system model for turbo coded multicode DS/SS systems through an interleaver

Turbo encoder with a quasi-random interleaver converts the information data **u** into coded sequence **c**. In this paper, turbo encoder with coding rate R_c=1/3 is used without puncturing. The constraint length is three and the generation polynomial is [7,5] in an octal form. The size of the interleaver in turbo encoder is denoted as N_T. The code **c** is entering by M bits the multicode DS/SS transmitter. BPSK modulation is employed, the modulated vector per symbol is denoted as

$$\mathbf{x}_j = \{x_{j,1},\dots,x_{j,i},\dots,x_{j,M}\} \quad x_{j,i} = \{+1,-1\}, 1 \le j \le N_T/(M \cdot R_C) \quad (1)$$

Note that the number of mapping symbols N/M has to be an integer in this system. Then the transmitting signal **s** is represented as

$$\mathbf{s} = \sqrt{E_c}\ \mathbf{x}_j \cdot \mathbf{H} \quad (2)$$

where **H** is an $M \times l$ matrix whose individual rows correspond to the pre-assigned orthogonal PN codes with length l. In (2), E_c denotes the energy per code which is related to the energy per bit E_b by $E_c = E_b \cdot R_c$.

The transmitting signal **s** is corrupted by a fading channel. In this paper, we assume that the fading amplitude a is constant during a PN period. We also assume that the probability density function (pdf) of the fading attenuation a follows Nakagami-m distribution, which is given by

$$p(a) = \frac{2m^m a^{2m-1}}{\Gamma(m)} e^{-ma^2} \quad (3)$$

where $\Gamma(.)$ is the Gamma function. Note that the second moment of the fading amplitude, $E(a^2)$, is normalized to unity. The Nakagami m-distribution spans, via the parameter m, the widest range of multipath fading distributions. For instance, it includes the Rayleigh distribution (m=1) as a special case. In the limit as $m \to +\infty$, the Nakagami fading channel converges to a non-fading AWGN channel. When $m \geq 1$, a one-to-one mapping between the parameter m and the Ricean factor allows the Nakagami m-distribution to closely approximate the Rice distribution.

At the receiver, the received signal **r** corrupted by the fading is represented by

$$\mathbf{r} = a \cdot \mathbf{s} + \mathbf{n} \qquad (4)$$

where **n** is an AWGN sample vector with mean zero and variance

$$\sigma_c^2 = \left(2R_c E_b / N_0 \cdot l \right)^{-1} \qquad (5)$$

A set of correlator outputs from the bank of M correlators is given by

$$\mathbf{C}_j = \mathbf{r} \cdot \mathbf{H}'/l = \{ C_{j,1}, \dots, C_{j,i}, \dots, C_{j,M} \} \qquad (6)$$

Assuming coherent detection, each correlator output in $\mathbf{C}_j$ contains signal and noise such as

$$C_{j,i} = \pm a \sqrt{E_c} + n \qquad (7)$$

where n is a Gaussian random variable having zero mean and variance

$$\sigma^2 = \sigma_c^2 / l \qquad (8)$$

The detector provides the received code sequence $\hat{\mathbf{c}}$ in soft value for turbo decoder via the deinterleaver. Turbo decoder is based on symbol log-MAP algorithm [5][6]. After a fixed number of iterations, hard decision for information data is made.

III. Online SNR Estimation for Multicode DS/SS Systems

We want to estimate the average received SNR at the correlator outputs, $\gamma = (E_c/2\sigma^2) \cdot E(a^2)$, in a blind algorithm. Accordingly, we formulate an SNR estimator based on a block observation of $\mathbf{C}_j$'s. We define the ratio of two statistical computations as a parameter Z that is given by

$$Z \equiv \frac{E\left[\sum_{i=1}^{M} \left(C_{j,i}^2 \right) \right]}{E\left[\sum_{i=1}^{M} \left(|C_{j,i}| \right) \right]^2} \qquad (9)$$

In the following, Z is derived as a function of the received SNR which is required in the turbo decoder. The expectation, $E[C_{j,i}^2]$, is given in

$$E[C_{i,j}] = E_c + \sigma^2 \qquad (10)$$

On the other hand, to derive $E[|C_{j,i}|]$, we first derived the conditional expectation of $|C_{j,i}|$ conditioned by the fading amplitude a, and then took its expectation over a to get $E[|C_{j,i}|]$ [8]. As a result, the expectation can be written as

$$E[|C_{j,i}|] = \sqrt{\frac{2}{\pi}} \sigma \left(\frac{m}{m + E_c/2\sigma^2} \right)^m + \frac{\sqrt{E_c}\,\Gamma(m+1/2)}{\sqrt{m}\,\Gamma(m)} \left(1 - \frac{2}{\pi} I(m) \right) \qquad (11)$$

where $I(m)$ is given by

$$I(m) = \cos^{-1}\left(\sqrt{\frac{\beta}{1+\beta}} \right) + \sum_{k=0}^{m-1} \sum_{l=0}^{2(m-k)-1} \binom{m}{k} \binom{2(m-k)-1}{l} \cdot \left(\frac{-\beta}{1+\beta} \right)^{m-k} \cdot \frac{1}{2^{2(m-k)-1}} \frac{e^{\theta_1 (2l - 2(m-k)+1)} - 1}{(2l - 2(m-k)+1)} \qquad (12)$$

where

$$\beta = \frac{E_c}{2m\sigma^2} \quad \text{and} \quad \theta_1 = \cosh^{-1} \sqrt{\frac{1+\beta}{\beta}}$$

Because the correlator outputs $C_{j,i}$ is independent each other, the statistical ratio Z is expressed as

$$Z = \frac{2\gamma + 1}{M \cdot \left\{ \sqrt{\frac{2}{\pi}} \left(\frac{m}{m+\gamma} \right)^m + \sqrt{\frac{2\gamma}{m}} \frac{\Gamma(m+1/2)}{\Gamma(m)} \left(1 - \frac{2I(m)}{\pi} \right) \right\}^2} = f(\gamma, m, M) \qquad (13)$$

where $\gamma = E_c/2\sigma^2$. Note that (13) assumes the knowledge of the Nakagami parameter m, but the accurate estimation method of m has given in [9]. We assume that the Nakagami parameter m is estimated with high accuracy in this paper. The number of pre-assigned orthogonal codes M is known at the receiver surely, consequently, Z is a function of the desired parameter γ.

Next, we show the SNR estimation method using the statistic ratio given in (13). The relation between Z and γ can be expressed as

$$\Gamma \equiv Z \cdot M = \frac{2\gamma + 1}{\left\{ \sqrt{\frac{2}{\pi}} \left(\frac{m}{m+\gamma} \right)^m + \sqrt{\frac{2\gamma}{m}} \frac{\Gamma(m+1/2)}{\Gamma(m)} \left(1 - \frac{2I(m)}{\pi} \right) \right\}^2} \qquad (14)$$

However, it is difficult to determine a closed-form solution for γ from the statistics. This difficulty is reduced by utilizing a simple polynomial approximation. In this paper, the fourth-order polynomial is employed such as

$$\gamma \approx a_4\Gamma^4 + a_3\Gamma^3 + a_2\Gamma^2 + a_1\Gamma + a_0 \qquad (15)$$

where the coefficients are determined by the minimum squared error method. For example, when the Nakagami parameter m is 1, the statistical ratio is expressed as

$$\Gamma_{m=1} = \frac{\pi}{2} \cdot \frac{2\gamma + 1}{\left[1 + \sqrt{\gamma} \left(\frac{\pi}{2} - \cos^{-1} \sqrt{\frac{\gamma}{1+\gamma}} \right) \right]^2} \qquad (16)$$

and the coefficients in (15) are determined as

$$\begin{cases} a_4 = 5.6930\text{e}+3 \\ a_3 = -3.3705\text{e}+4 \\ a_2 = 7.4887\text{e}+4 \\ a_1 = -7.4017\text{e}+4 \\ a_0 = 2.7464\text{e}+4 \end{cases} \qquad (17)$$

The estimation of the SNR is obtained by substituting $\Gamma = Z \cdot M$ into the approximation-polynomial. However, the expectations Z in (9) could not be obtained from the finite number of observations of $C_{j,i}$. Thus, the approximation of γ in (14), $\hat{\Gamma}$, is introduced which is the block averages given as

$$\hat{\Gamma} = M \cdot \frac{\frac{1}{N_B}\sum_{j=1}^{N_B}\sum_{i=1}^{M} C_{j,i}^2}{\left[\frac{1}{N_B}\sum_{j=1}^{N_B}\sum_{i=1}^{M}\left|C_{j,i}\right|\right]^2} \quad (18)$$

where N_B is the number of symbols in this block. Substituting $\hat{\Gamma}$ for the Γ in (15), the SNR estimate $\tilde{\gamma}$ is obtained.

4. Simulation Results and Discussions

4.1 Accuracy of the derived SNR estimator

We tested the accuracy of the polynomial approximation given in (15) by evaluating the mean and standard derivation of the SNR estimates $\tilde{\gamma}$, determined by over 2000 trials. In these simulations, we evaluate the estimation accuracy in uncoded multicode DS/SS systems.

Table 1 shows the accuracy of estimation with different block size N_B. The true SNR value ranges from 0.0dB to 5.0dB. The block size is set to 50, 100 and 500 symbols. It is found that the accuracy of the estimation is superior as the block size increases and 500 symbol-block size brings accurate estimation. From these results, the block size is set to about 500 symbols to estimate SNR with accuracy. Table 2 illustrates the accuracy of estimation when the number of pre-assigned orthogonal PN codes M is changed ranging from 16 to 64. The true SNR value ranges from 0.0 dB to 5.0 dB. The block size N_B is 500 symbols. We observed that the estimation-accuracy is almost same with different value of M.

Table 1. Accuracy of the SNR estimator. Block size N_B is 100, 500 and 1000 symbols and M =8.

True γ [dB]	N_B=50		N_B=100		N_B=500	
	$E[\tilde{\gamma}]$ dB	$SD[\tilde{\gamma}]$ dB	$E[\tilde{\gamma}]$ dB	$SD[\tilde{\gamma}]$ dB	$E[\tilde{\gamma}]$ dB	$SD[\tilde{\gamma}]$ dB
0.0	0.71	2.06	0.29	1.59	0.01	0.77
1.0	1.77	2.19	1.37	1.67	1.04	0.73
2.0	2.89	2.40	2.39	1.72	2.07	0.76
3.0	4.08	2.61	3.60	1.85	3.08	0.79
4.0	5.18	2.64	4.59	1.95	4.16	0.86
5.0	6.32	2.86	5.67	2.06	5.16	0.91

Table 2. Accuracy of the SNR estimator. Block size N_B is 500 symbols and Nakagami parameter m=1.

True γ [dB]	M=16		M=32		M=64	
	$E[\tilde{\gamma}]$ dB	$SD[\tilde{\gamma}]$ dB	$E[\tilde{\gamma}]$ dB	$SD[\tilde{\gamma}]$ dB	$E[\tilde{\gamma}]$ dB	$SD[\tilde{\gamma}]$ dB
0.0	0.03	0.64	0.01	0.55	0.03	0.51
1.0	1.04	0.64	1.04	0.57	1.04	0.56
2.0	2.05	0.69	2.03	0.61	2.05	0.60
3.0	3.08	0.77	3.04	0.65	3.05	0.67
4.0	4.10	0.79	4.08	0.76	4.07	0.72
5.0	5.11	0.86	5.14	0.83	5.11	0.85

Table 3 demonstrates the accuracy of estimation over the Nakagami fading channels. The Nakagami parameter m is set to 2, 4 and 6. When $m \geq 1$, Nakagami fading channels closely approximates the Ricean fading channels with Rice factor K_r through the following relation:

$$K_r = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}} \qquad m \geq 1 \quad (19)$$

The Nakagami parameters m=2, 4 and 6 correspond to the Rice factor K_r=3.8dB, 8.1dB and 10.2dB, respectively. The block size is set to 100 symbols. The coefficients of the approximation-polynomial are listed in Table 4. It is found that the estimator can provide accurate SNR estimates in each channel. Especially, the derived SNR estimates become more accurate when the Nakagami parameter m is larger.

Table 3. Accuracy of the SNR estimator. Block size N_B is 100 symbols and M = 8

True γ [dB]	m=2		m=4		m=6	
	$E[\tilde{\gamma}]$ dB	$SD[\tilde{\gamma}]$ dB	$E[\tilde{\gamma}]$ dB	$SD[\tilde{\gamma}]$ dB	$E[\tilde{\gamma}]$ dB	$SD[\tilde{\gamma}]$ dB
0.0	0.07	0.89	0.03	0.63	0.03	0.58
1.0	1.03	0.81	1.03	0.57	1.04	0.49
2.0	2.03	0.79	2.03	0.53	2.01	0.44
3.0	3.01	0.80	3.02	0.51	3.01	0.44
4.0	3.93	0.85	3.98	0.53	4.03	0.44
5.0	4.91	0.91	4.99	0.55	5.03	0.43

Table 4. The coefficients of the approximation polynomial

	a_4	a_3	a_2	a_1	a_0
m=2	1.1897e+3	-6.7624e+3	1.4434e+4	-1.3723e+4	4.9064e+4
m=4	6.8517e+2	-3.7984e+3	7.9111e+3	-7.3431e+3	2.5663e+3
m=6	5.8755e+2	-3.2273e+3	6.660e+3	-6.1271e+3	2.1231e+3

4.2 BER performance on turbo coded multicode DS/SS systems

The derived online SNR estimator is applied to the turbo decoder at the receiver of turbo coded multicode DS/SS systems over both i.i.d Nakagami fading channels and correlated Rayleigh fading channels. The number of pre-assigned orthogonal PN codes M is set to 8. In turbo decoding, the number of decoding iterations is set to five. The size of the interleaver of the turbo encoder N_T is set to 800 bits. The average SNR over one codeword is provided for the turbo decoder. In this case, the coding rate R_c is 1/3 and M is 8, thus the block size N_B is 300 symbols. In each simulation result, the bit error rate performance is evaluated for the two cases: assuming the perfect channel state information (perfect CSI) on the fade amplitude and the SNR at the receiver, and using the SNR estimates from the SNR estimator (SNR estimation).

Figure 2 shows the bit error rate performance on i.i.d Nakagami fading channels. We evaluate the error rate performance in the channels with the Nakagami parameter m=1 and 2. At the 10^{-4} BER, the difference between the systems with perfect CSI and with the SNR estimates is within 1.5dB. We observed that the difference decreases as the Nakagami parameter m increases.

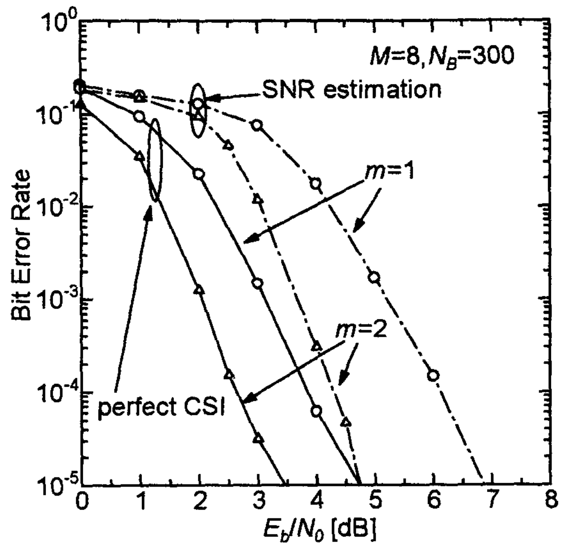

Figure 2. Comparison of the bit error rate performance between using perfect CSI and SNR estimates on i.i.d Nakagami fading channels.

5. Conclusions

In this paper, an online SNR estimator is derived for multicode DS/SS systems, when the signal undergoes Nakagami fading channels. This SNR estimator computes the estimation based on a statistical ratio of the correlator outputs. A polynomial approximation of relation between the SNR and the statistical ratio was introduced in order to easy implementation. The accuracy of the estimation was investigated by computer simulations. It was found that the derived estimator works well when the block size is more than 500 symbols. The derived estimator is applied to the channel state estimator in turbo coded multicode DS/SS systems in both i.i.d Nakagami fading channels. These simulation results shown that the bit error rate performance using the SNR estimates were close to that with perfect CSI, within 1.5dB in each channel. The theoretical study on the estimation accuracy is the future problem.

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