

# Analysis of the Required Limit on APAA Aperture using Statistical Simulation for HAPS

Bon-Jun Ku, Jong-Min Park, Yang-Su Kim, and Do-Seob Ahn  
Broadband Wireless Communication Research Department,  
ETRI-Radio & Broadcasting Research Laboratory,  
161 Gajeong-Dong, Yuseong-Gu, Daejeon, 305-350, Korea  
Tel. +82-42-860-5719, Fax.: +82-42-860-5740  
e-mail : bjkuo@etri.re.kr

**Abstract:** This paper presents the analysis of the required limit on a multibeam active phased array antenna (APAA) aperture using the statistical simulation for a High Altitude Platform Station (HAPS). The simulation takes into account the random errors caused by the non-identity of the array elements and the inaccuracy of the antenna calibration. The results of our statistical simulation show that the strict requirements on the sidelobe envelope for HAPSs can be met when the amplitude and phase distribution errors are minor, a condition which may be achieved by using digital beam forming.

## 1. Introduction

High Altitude Platform Station (HAPS) is a constituent of future communication system. The system consists of a stratospheric platform HAPS (airship, communication payload, additional mission payload, etc.), a lot of user terminals, several gateways and the ground facilities for TT&C. The system will provide direct user services through several spot beams and links to terrestrial network (PSDN, PSTN, WWW, etc.) through the gateway.

The most important component of HAPS is a multibeam active phased array antenna (APAA) with the high directivity and very low sidelobe level (SLL). To design the antenna satisfying these requirements, it is necessary to realize with adequate accuracy a certain taper amplitude distribution in the antenna aperture. Owing to fabrication errors and dispersion of microwave amplifier parameters, array elements are not absolutely identical. These lead to random errors in amplitude-and-phase distribution over antenna aperture. In this case, it is necessary to process complex gain values of array elements with high accuracy amplitude errors should be less 0.2 dB and phase errors should be less  $1.5^\circ$  in order to suppress the side lobes of the HAPS antenna). Analogue beam former could not provide the accuracy because of imperfection of mentioned microwave devices (due to reflections and insertion loss). The only way is to use digital beamforming (DBF).

In this paper, the influence of amplitude-and-phase aperture distribution errors after calibration is considered by obtaining APAA radiation pattern. The aim is to present the APAA simulation by numerical statistical algorithm for probabilistic evaluation of SLL. Using this algorithm the required overall antenna calibration accuracy is determined in order to estimate the possibility of the system realization.

## 2. Antenna Geometry

The array antenna with 73 elements on the hexagonal grid is considered to fill some circular aperture. The basic radiation element is the cross dipole type as shown in Fig. 1. It is well known that the circular antenna aperture has a minimum of SLL. Element spacing is  $d = 0.7\lambda$ . The antenna geometry is shown in Fig. 2.

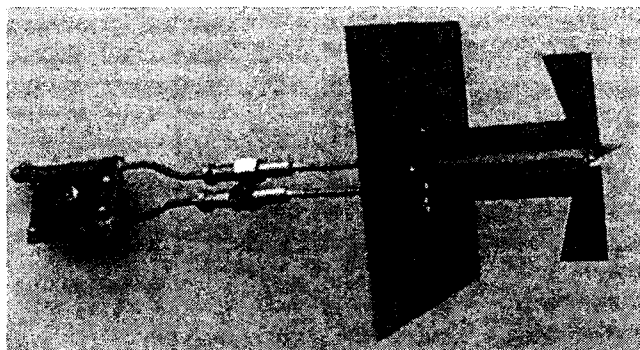


Figure 1. Radiating element

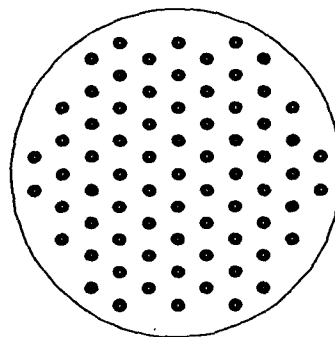


Figure 2. Geometry of APAA

## 3. Main Relations

In this section we define the notations and derive the main statistical relations for array antenna characteristics. On the basis of these relations, the numerical algorithm for statistical analysis of the antenna sidelobe envelope will be developed in section 4.

A field radiated by an APAA is the sum of the element fields; therefore its fluctuation (deviation from the average value) is determined by the error magnitude in each array channel. Usually, the number of elements in the antenna

array is rather big, so errors in different channels fractionally counterveil one another, and the total array field fluctuation may be much lower than the fluctuation of a field in each antenna channel. We will show that the reduction of the total array field dispersion in contrast with the dispersion in each channel is determined by the array gain.

Let's consider the array with radiators placed in points  $\vec{r}_n$ ,  $n=1 \dots N$ . Its radiated field is determined by the equation

$$E(\vec{v}) = \frac{f(\vec{v})}{A} \cdot \sum_{n=1}^N A_n \cdot \exp\{ik \cdot (\vec{r}_n; \vec{v})\}, \quad (1)$$

where  $f(\vec{v})$  is the radiator pattern,  $A_n$  is the amplitude of the  $n$ -th radiator,  $N$  is the total number of radiators in the array,  $\vec{v}$  is the coordinate of the view point,  $k = \frac{2\pi}{\lambda}$  is the wave number, and  $A = \sum_{n=1}^N A_n$  is the normalized factor.

In practice, the greatest interest is on arrays having some center of symmetry, that is, the array structure is invariant to symmetrical transformation at this center. We confine our consideration to only such symmetrical arrays and accept that the coordinate system origin (the origin of vectors  $\vec{r}_n$ ) coincides with the array symmetry center. As (1) reveals, when  $A_n$  is real, the array field is also real.

If there are random errors in antenna channels, the amplitude  $A_n$  is determined by the equation

$$A_n = A_n^0 \cdot \varepsilon_n = A_n^0 \cdot \rho_n \cdot e^{i\varphi_n}, \quad (2)$$

where  $A_n^0$  is the amplitude of the  $n$ -th radiator without errors,  $\rho_n$  and  $\varphi_n$  are amplitude and phase errors, correspondingly. Below, we take the normalized factor in equation (1) as

$$A = \sum_{n=1}^N A_n^0.$$

Assume that amplitude ( $\rho_n$ ) and phase ( $\varphi_n$ ) errors in each antenna channel are independent, and owing to the calibration procedure, the phase errors have zero average values. For simplicity, we assume that a mutual coupling between array elements does not essentially change the element radiation pattern, the array factor, or consequently, the results of the simulation.

In addition, we assume that both  $\rho_n$  and  $\varphi_n$  have an identical distribution (regardless of channel number  $n$ ) and the errors are independent from channel to channel.

To characterize the total channel error  $\varepsilon_n$ , we define its dispersion  $\sigma_\varepsilon$  as follows:

$$\langle \Delta \varepsilon_n \cdot \Delta \varepsilon_n^* \rangle = \delta_{n,n} \cdot \sigma_\varepsilon^2. \quad (3)$$

Here,  $\langle \rangle$  indicates the averaging of an ensemble of realizations, symbol  $*$  denotes a complex conjugation, and

$\Delta$  is a deviation from the average value. The magnitude of  $\sigma_\varepsilon$  is determined by the dispersions of amplitude and phase errors  $\rho_n$  and  $\varphi_n$  and their distributions. When there are normal distributions and minor errors, the magnitude is described as follows:  $\sigma_\varepsilon^2 = \sigma_\rho^2 + \sigma_\varphi^2$ . The Kronecker delta symbol  $\delta_{n,n}$  shows that errors in different channels are uncorrelated.

Using (1)-(3), we can derive the formula for the average radiated power:

$$P_{av}(\vec{v}) = \langle E(\vec{v}) \cdot E(\vec{v})^* \rangle = \langle \varepsilon \rangle^2 \cdot P^0(\vec{v}) + \frac{\sigma_\varepsilon^2}{D} \cdot |f(\vec{v})|^2, \quad (4)$$

where  $P^0(\vec{v})$  is the ideal power pattern for the case of no errors in aperture distribution, and

$$D = A^2 / \sum_{n=1}^N (A_n^0)^2 = \left( \sum_{n=1}^N A_n^0 \right)^2 / \sum_{n=1}^N (A_n^0)^2$$

is the array antenna directivity.

It is necessary to make several comments about (4) as follows:

First of all, (4) is similar to the well known equations of statistical antenna theory, but it is valid for any distribution of random values  $\varepsilon_n$  (not only for normal distribution).

Eq. (4) shows that the average antenna pattern equals the sum of the ideal pattern without errors multiplied by  $\langle \varepsilon \rangle^2$  and the random background  $\frac{\sigma_\varepsilon^2}{D} |f(\vec{v})|^2$  determined by the dispersion of the random error in each channel, decreased by the value of the antenna directivity.

Eq. (4) is very convenient at the initial stage of antenna design because it allows us to estimate the required overall antenna calibration accuracy on the assumption of the desired antenna gain and required sidelobe envelope (and without any assumptions about antenna geometry, number of array elements, etc.).

Under the given assumptions, real and image components of the array field are uncorrelated, that is  $\langle \text{Re}\{\Delta E(\vec{v})\} \cdot \text{Im}\{\Delta E(\vec{v})\} \rangle = 0$ . This relation is helpful for further deriving the numerical algorithm.

#### 4. Numerical Algorithm for Statistical Analysis

Here we present the numerical algorithm we will use for the statistical analysis of the antenna array sidelobes, which takes into account the aperture amplitude and phase distribution errors.

The algorithm includes an analysis of the ensemble of the random array patterns for the given maximum values of amplitude and phase distribution errors. Each pattern realization (1) is calculated by using  $N$  random amplitudes  $A_n$  determined by (2), where

$$\rho_n = 10^{B_0 \mu_n^a / 20}, \quad \varphi_n = \mu_n^\varphi \cdot \Phi_0.$$

Random values  $\mu_n^{a,\varphi}$  are independent and uniformly distributed over the interval  $[-1; +1]$ . Parameters  $B_0$  and  $\Phi_0$

are maximum amplitude (dB) and phase (radian) errors, correspondingly.

We note the following about our choice of error distribution model. In practice, it is very difficult to evaluate the actual amplitude and phase errors distribution in antenna channels, but for our purpose it is not necessary to do this; we are interested in the overall array characteristics. The total array field equals the sum of a large number of independent random array radiator fields with identical properties. According to the central limit theorem of the probability theory, its distribution tends to a normal distribution regardless of the distribution in each antenna channel. Thus, our simple model of randomness is acceptable from an engineering perspective.

Further, we consider the array pattern sections only. Thus, the vector  $\vec{v} = [\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta]^T$  depends on  $\theta \in [-\pi/2; \pi/2]$  only ( $\varphi = \text{const}$ ), and the argument  $\vec{v}$  is replaced by  $\theta$  below. The ideal element radiation pattern  $f(\vec{v}) = \sqrt{\cos \theta}$  is taken into account.

The statistical evaluation of the average radiated power for  $N_l$  independent pattern realizations  $E_l(\theta), l=1 \dots N_l$  is given by the equation

$$P_{av}(\theta) = \frac{1}{N_l} \sum_{l=1}^{N_l} |E_l(\theta)|^2.$$

For the average fluctuation of power, we have the following expression:

$$P_d(\theta) = \sqrt{\frac{1}{N_l} \sum_{l=1}^{N_l} (P_{av}(\theta) - |E_l(\theta)|^2)^2}.$$

The upper sidelobe envelope is determined by the equation

$$P_{up}(\theta) = P_{av}(\theta) + P_d(\theta).$$

Curve  $P_{up}(\theta)$  has the following physical sense. As we mentioned above, the total array field  $E(\theta)$  has a normal distribution. In our analysis, both the real and image part of the array field have normal distribution, and they are uncorrelated (see section 3). Thus, the array field magnitude  $R(\theta) = |E(\theta)|$  has a generalized Rayleigh distribution

$$\omega(R) = \frac{R}{\sigma^2} \exp\left\{-\frac{R^2 + b^2}{\sigma^2}\right\} \cdot I_0\left(\frac{bR}{\sigma^2}\right), \quad (5)$$

where  $b = \sqrt{[\text{Re}(E(\theta))]^2 + [\text{Im}(E(\theta))]^2} = |E(\theta)| = \langle R \rangle$  and  $I_0$  is the modified Bessel function of zero kind.

The level  $\langle R^2 \rangle + \sqrt{\langle (\Delta R)^2 \rangle}$  corresponds to  $P_{up}(\theta)$  for arbitrary direction  $\theta$ .

The probability that antenna power radiated in direction  $\theta$  will not exceed the level  $P_{up}(\theta)$ , following from the law in (5), is equal to approximately 0.85.

The numerical evaluation of this probability for the ensemble of 2000 realizations gives the value 0.83 - 0.87 depending on the level of ideal power pattern  $P^0(\theta)$ .

Thus, the area below the curve  $P_{up}(\theta)$  represents the domain where, at a given range of amplitude and phase errors, a random implementation of  $P_r(\theta)$  is allocated. On a positional relationship of the curve  $P_{up}(\theta)$  and the curve of the required sidelobe envelope, it is possible to decide whether the antenna meets the sidelobe radiation requirements.

The level  $P_{up}(\theta)$  essentially depends on the maximum values of the amplitude and phase errors. Parameters  $B_0$  and  $\Phi_0$ , for which  $P_{up}(\theta)$  is allocated below the required sidelobes envelope in the given beam steering range (or not too much above it), determine the necessary antenna calibration accuracy. The numerical algorithm for statistical analysis of antenna array side lobes with account of aperture amplitude-and-phase errors is presented.

The algorithm includes an analysis of random array patterns ensemble for the given maximum values of amplitude and phase errors.

Each pattern realization is calculated by using  $N$  random amplitudes  $A_n = A_n^0 \cdot \varepsilon_n = A_n^0 \cdot \rho_n \cdot e^{i\varphi_n}$ .

Where,  $A_n^0$  is the amplitude of  $n$ -th radiator without errors,

$\rho_n = 10^{B_0 \mu_n^a / 20}$ ,  $\varphi_n = \mu_n^\varphi \cdot \Phi_0$  and random values  $\mu_n^{a,\varphi}$  are independent and uniformly distributed over the interval  $[-1; +1]$ . Parameters  $B_0$  and  $\Phi_0$  are maximum amplitude (dB) and phase (radian) errors correspondingly.

## 5. Results of Simulation

The array antenna has the 3 dB beamwidth of  $12^\circ$  and the gain of approximately 23 dB. The number of elements, the element gain and aperture distributions are chosen to obtain the same antenna parameters and satisfy the required reference radiation pattern. For the example of simulation, the number of the random generation is 2000.

A tapered amplitude distribution with an axial symmetry is selected and can be calculated according to (6)

$$A_n^0 = 0.58 + 0.42 \cdot \cos\left(\pi \frac{R_n}{R_{\max}}\right), \quad (6)$$

where  $R_n$  is the distance between the circle center and the  $n$ -th element, and  $R_{\max} = \max_n R_n$ . This distribution gives enough high aperture efficiency under low side lobe level. Simulation results are shown in Fig. 3 for one major plane. The results for other planes are nearly similar too.

It is necessary to note that the APAA to be able to meet the requirements of amplitude, phase errors conditions does not exist so far. Really, the parameters of contemporary microwave amplifiers can provide a phase non-identity in limits  $10^\circ$ . Digital beam forming permits to compensate a

dispersion of APAA channel parameters with high precision (about  $0.1^\circ$  on phase and 0.1 dB on amplitude). But in this case, a permanent precise calibration of array channels in order to insert phase errors into processor memory is required. The requirements are near the accessible limit that is reached by using of modern methods of array antenna calibration. The method of satellite antenna calibration with RMS of about  $1^\circ$  was described previously. The feature of the stratospheric platform environment consists in angular fluctuations due to blasts. The special optic-and-radio equipment with high operating speed should be developed to carry out the permanent antenna calibration during APAA service.

## 6. Conclusions

The algorithm of statistic SLL envelope simulation for the HAPS antenna is developed. The tolerances of the antenna calibration are obtained. The implemented calculations show that the quite rigorous requirements on determination of the array channel parameters are needed in order to realize the APAA for the Stratospheric Communication System. According to the results of the statistic simulation, it is shown that the strict requirements on sidelobe envelope for HAPS can be met by using digital beamforming.

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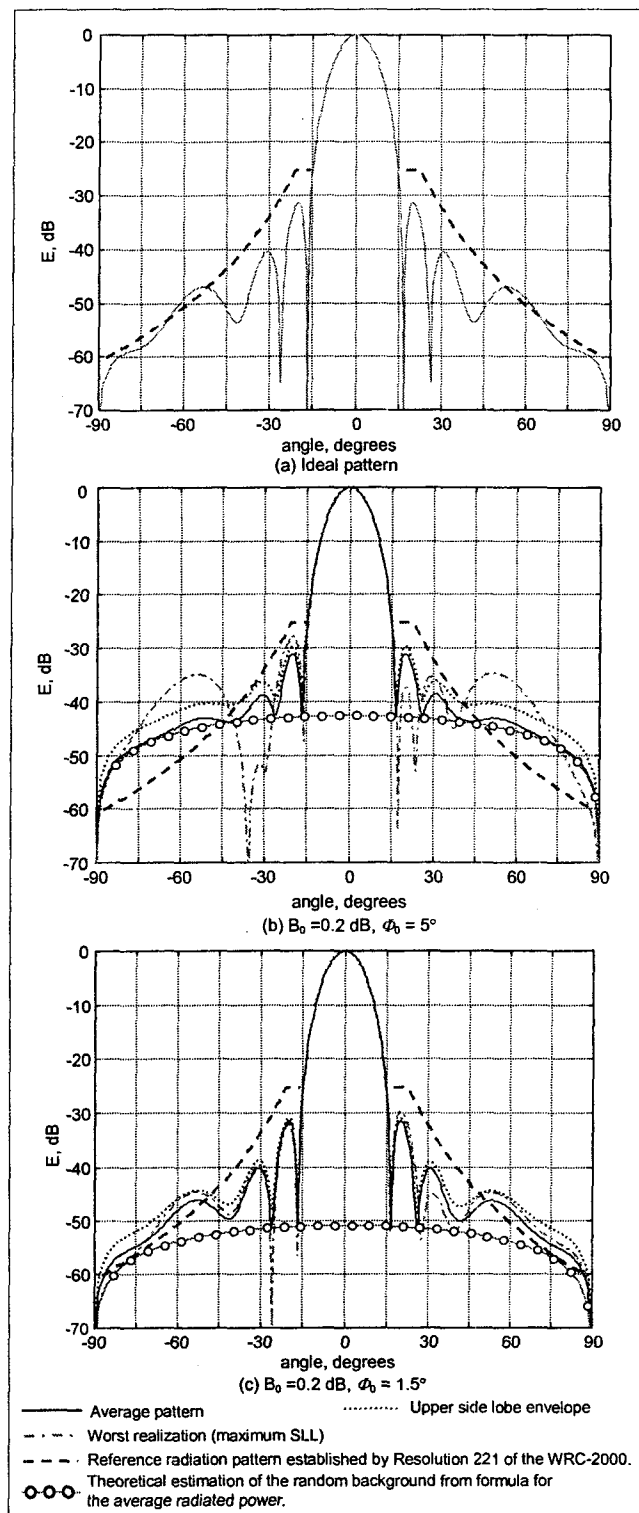


Figure 3. Simulation patterns of 73 element array.