Performance Analysis of an Improved NLMS Algorithm

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Abstract: This paper presents a performance analysis of an improved adaptive algorithm proposed by the authors recently. It is based on the normalized least mean square (NLMS) algorithm, which is one of the major techniques to adapt the coefficients of a transversal filter. Generally, the performance of an adaptive algorithm is often discussed by investigating the misadjustment. In this paper, unlike these approaches, a novel analytical method is considered. Setting the parameters so that the residual mean square error (MSE) after the convergence of the algorithm is equal to that of the NLMS algorithm, the MSE level is compared. It is shown that the theoretical analysis is agreed with the simulation results.

1. Introduction

Many modefied LMS algorithms have been proposed for the use of FIR adaptive filters [4–10], because the standard LMS algorithm [1] is simple and has robustness to numerical calculations.

It is well known that, for any algorithm with a fixed value of step size in adaptation processes, a trade-off between the convergence speed and the residual error does exist. If a larger value of the step size is used, a faster convergence is attained as long as the updating of the coefficient vector is not insufficient. On the other hand, if a smaller value is used for the step size, a more accurate performance is obtained.

The least mean fourth (LMF) algorithm [7] is one of the modified LMS algorithms and had been introduced for realizing both fast convergence and low residual error. Some adaptive algorithms that combine the LMS and LMF [8–10] also have been derived and studied along this line.

Recently, we introduced a novel adaptive algorithm [11], which is based on the NLMS algorithm and provides better performance than the LMS. An advantage of this algorithm is to have both characteristics of the NLMS and LMF. The step size is given by the following equation:

$$\mu = \frac{2}{X(n)^T X(n) + \frac{\gamma}{|e(n)|+\delta}} \tag{1}$$

where $X(n)$ is the input vector, $e(n)$ is the error sequence at time n, T is transpose, δ is a small value and γ is a constant which corresponds to the step size in the LMS and NLMS algorithms. It has been shown by experiments that the algorithm provides an excellent equalization performance.

Adaptive algorithms are usually compared with respect to the misadjustment and convergence properties. We, however, consider the residual MSE of each algorithm in this paper to show the performance of the algorithm with an analytical approach. Subtracting the residual MSE of the algorithm from that of the NLMS algorithm, we investigate whether if the algorithm provides better performance than the NLMS algorithm.

This paper is organized as follows. Section 2 develops the analysis, where it is indicated why the algorithm accomplishes faster start-up than the NLMS algorithm. Section 3 gives simulation results in the scenario of channel equalization. Finally, conclusions are drawn in Section 4.

2. Performance Comparison Between NLMS and Improved NLMS Algorithms

2.1 General Formula of excess MSE

The error coefficient vector of FIR adaptive filters is given by the following equation:

$$C(n) = W_{opt} - W(n) \tag{2}$$

where W_{opt} is defined as the optimal Wiener solution and $W(n)$ is the coefficient vector at nth iteration. In [1], the excess MSE has been obtained as

$$\epsilon_{ex} = E[C(n)^T R(n) C(n)] \tag{3}$$

where $R(n)$ is the autocorrelation matrix of $X(n)$.

2.2 Difference of excess MSEs

Equation (3) may be reexpressed as

$$\epsilon_{ex} = E[C(0)^T (I - \mu\Lambda)^{2n} \Lambda C(0)] \tag{4}$$

$$= E\left[\sum_{k=1}^{M} w_{opt}^2(k)\lambda_k (I - \mu\lambda_k)^{2n}\right] \tag{5}$$

where Λ is the diagonal matrix of $R(n)$, λ_k is the kth element of Λ, and M is the filter order. Comparing the NLMS with the improved NLMS, we consider a different value defined by the following equation:

$$\Xi = (1 - \mu\lambda_k)^2 - (1 - \mu'\lambda_k)^2 \tag{6}$$

where μ is the step size of the NLMS and μ' is that of the improved NLMS. The improved NLMS algorithm gives

Table 1. Conditions in simulation experiments

Filter Order		$M = 11$
Delay	MP Channel	$D = 0$
	NMP Channel	$D = 7$
Transmitted Signal		± 1 Random Sequences
Additive Noise		White Gaussian Noise
Signal Noise Ratio[dB]		$SNR = 40[dB]$
Independent Trial		$T = 100$

Table 2. Parameters of adaptive algorithms

	Channel Model	
	NMP	MP
LMS	$\mu = 0.02$	$\mu = 0.01$
NLMS	$\alpha = 0.3$ $\beta = 0.0001$	$\alpha = 0.1$ $\beta = 0.0001$
Combined LMS/F	$\mu = 0.012$ $V_{th} = 0.0001$	$\mu = 0.008$ $V_{th} = 0.0001$
Improved NLMS	$\gamma = 3.0$ $\delta = 0.0001$	$\gamma = 5.0$ $\delta = 0.0001$

Fig.2 shows the convergence comparison in the case of NMP channel. The improved algorithm provides better performance than any other algorithm, setting the parameters so that the residual MSE levels are common. From the above analysis, it is observed that $\theta\epsilon_{re}$ in (19) is close to 0.316 which means $-5[dB]$ of MSE. Fig.3 also shows that the improved algorithm provides better performance. In this case, we can consider that $\theta\epsilon_{re}$ is close to 1. From these results, it is suggested that the algorithm is not affected by the channel condition through the analysis and the experiments.

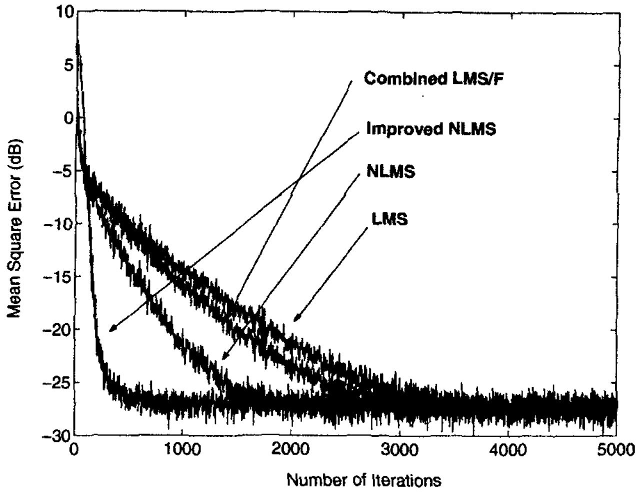

Figure 2. Convergence comparison in the case of NMP channel

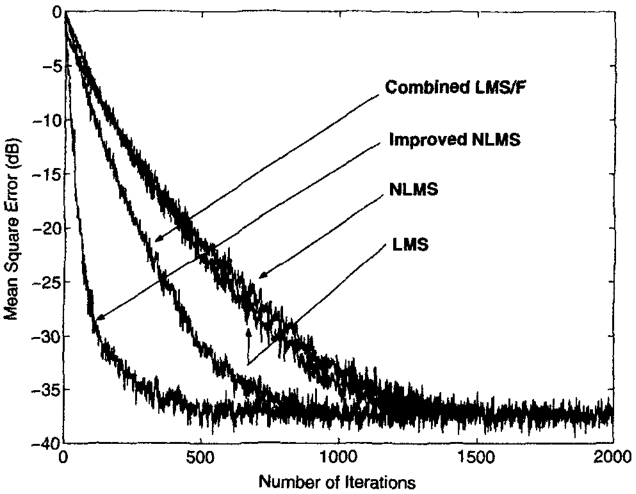

Figure 3. Convergence comparison in the case of MP channel

4. Concluding Remarks

In this paper, we have analyzed the performance of the improved NLMS algorithm we devised recently. In the analysis, it has been shown that the improved algorithm gives an advantage over the NLMS by setting the parameters so that the residual errors after convergence are equivalent. Simulation experiments have also demonstrated that the analysis results are validated.

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