Bifurcation of Combinatorial Oscillations in Coupled Duffing's Circuits

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Abstract: This paper studies the bifurcation of combinatorial oscillations in coupled Duffing's circuits when symmetry is broken. The system consists of two periodic forced circuits coupled by a linear resistor. These two periodic external forces are sinusoidal voltage sources with various phase-shift. We investigate the relation between phase-shift and periodic solutions by analyzing many bifurcation diagrams.

1. Introduction

A nonlinear resonance is one of the typical phenomena in nonlinear circuits with a periodic external force. A circuit containing a saturable core [1] described by a Duffing's equation can exhibit a typical nonlinear resonance. If we couple many nonlinear oscillators in one system, it can exhibit a great variety of fundamental dynamical phenomena. So the system of coupled oscillators has attracted a great deal of attention recent years. Here, we consider a basic and typical model coupled by two Duffing's circuits.

According to our previous researches, coupled Duffing's circuits with symmetry in system and periodic solutions have been studied [2]. However, in this paper, two oscillators are chosen as ones without symmetry; as a result, *periodic solutions* also lose their symmetry. Those solutions that had same amplitude and bifurcation structure in symmetrical condition will be separate and independent due to symmetry breakdown.

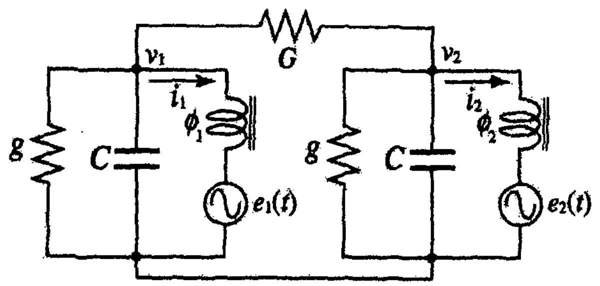

Figure.1 The coupled Duffing's circuit.

We will discuss the variation of periodic oscillations by changing the phase-shift of two external forces We draw many bifurcation diagrams, and emphasize differences between symmetry and asymmetry.

2. Circuit Equations

Let's consider a coupled Duffing's circuit shown in Fig.1 With the notations in the figure, we can describe the circuit equations as follows:

$$\begin{cases} \dfrac{d\phi_1}{dt} = v_1 + e_1(t) \\ \dfrac{d\phi_2}{dt} = v_2 + e_2(t) \\ C\dfrac{dv_1}{dt} + gv_1 + i_1 + G(v_1 - v_2) = 0 \\ C\dfrac{dv_2}{dt} + gv_2 + i_2 + G(v_2 - v_1) = 0 \end{cases} \tag{1}$$

In this circuit, we assume the characteristic of nonlinear inductors as following cubic functions:

$$\begin{aligned} i_1 &= f(\phi_1) = f_1\phi_1 + f_3\phi_1^3 \\ i_2 &= f(\phi_2) = f_1\phi_2 + f_3\phi_2^3 \end{aligned} \tag{2}$$

And normalizing the *state variables and coefficients of* Eqs.(1) as:

$$\begin{aligned} &\phi_1 = x_1, \phi_2 = x_2, v_1 = y_1, v_2 = y_2 \\ &c_1 = \frac{f_1}{C}, c_3 = \frac{f_3}{C}, k = \frac{g}{C}, \delta = \frac{G}{C} \end{aligned} \tag{3}$$

The external forces e_1 and e_2 are sinusoidal voltage forces with phase-shift θ described below:

$$e_1(t) = B\sin(\omega t), \quad e_2(t) = B\sin(\omega t + \theta) \tag{4}$$

Substitution yields

$$\begin{cases} \dfrac{dx_1}{dt} = y_1 + B\sin(\omega t) \\ \dfrac{dx_2}{dt} = y_2 + B\sin(\omega t + \theta) \\ \dfrac{dy_1}{dt} = -c_1x_1 - c_3x_1^3 - ky_1 - \delta(y_1 - y_2) \\ \dfrac{dy_2}{dt} = -c_1x_2 - c_3x_2^3 - ky_2 - \delta(y_2 - y_1) \end{cases} \tag{5}$$

where we fix the system parameters as:

$$k=0.2,\ c_1=0,\ c_3=1,\ \omega=1 \tag{6}$$

3. Combinatorial Oscillations

In our research, we couple two Duffing's oscillators with a linear resistor, so that these two oscillators can interact through voltage difference of this resistor. Because the conjunction is linear element, the global oscillation on weak coupling condition [3] can be considered as the combination of oscillations in each subsystem. Therefore, we call the global dynamic phenomena in a linearly coupled system as combinatorial oscillations.

Let us consider the symmetry of this coupled Duffing's circuit. Observing Eqs.(5), note that elements except for periodic external force are identical, the symmetry of the system is determined by θ which is a phase-shift between two oscillators. Obviously, We can conclude that only $\theta=0$ (in-phase) and $\theta=\pi$ (anti-phase) are two symmetrical conditions in this bi-coupled system. They are complete symmetry and inversion symmetry respectively. If θ equals other value, the system equation will not satisfy symmetrical condition any more (see Fig.2). This type of asymmetrical coupling is a much common case, because in real coupled systems, a delay often exists between the subsystems [4].

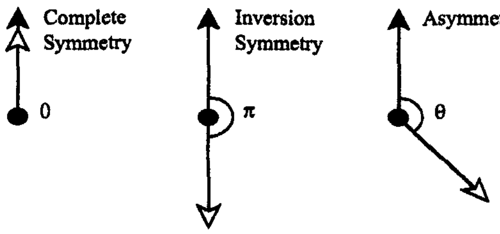

Figure.2 Symmetry and phase-shift.

4. Bifurcation and resonance

To investigate the combinatorial oscillations when symmetry is broken, we draw the bifurcation diagrams by fixing θ as uniform distributed values from 0 to π. Although both 0 and π are discussed in our research before, we also show them here as comparable data. We draw bifurcation diagrams in (B,δ)-plan, which represents the relation between external force and coupling intensity.

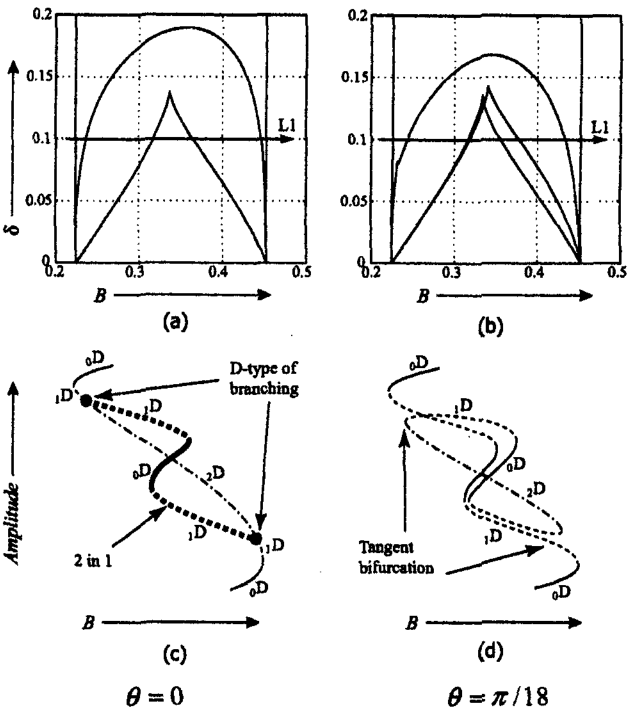

Figure.3 (a) and (b) are bifurcation diagrams of fixed point in (B,δ)-plan when θ equals 0 and $\pi/18$ respectively. (c) and (d) are schematic amplitude characteristic diagrams when B change along L1: δ=0.1.

Firstly, we compare the bifurcation between phase-shift is 0 and $\pi/18$ in Fig.3. We draw a schematic diagram of amplitude characteristic of periodic solutions by changing B along the line L1: δ=0.1, where ${}_0D$, ${}_1D$ and ${}_2D$ represent stable, 1-dimensional unstable and 2-dimensional unstable periodic solution, respectively. If $\theta=0$, a pair of combinatorial solution (bold curve) branches out by D-type of branching. Because system is symmetrical, these two combinatorial solutions are resonant symmetrically, thus, amplitude characteristic and bifurcation structure of them are just the same, so we draw them as one curve. However, if two coupled oscillators have a phase-shift, even if a small value, the system will lose its symmetry. Thus, the solutions can't keep symmetrical, they separate and D-type of branching tune to tangent bifurcation. Moreover, they can occur their tangent bifurcation independently, as shown in the Fig.3(d).

Then we discuss the inversion symmetrical case. Bifurcation diagrams when $\theta=17\pi/18$ and $\theta=\pi$ are shown in Fig.4. Schematic amplitude diagrams when B change along L1: $\delta=0.1$ and L2: $\delta=0.5$ are drawn

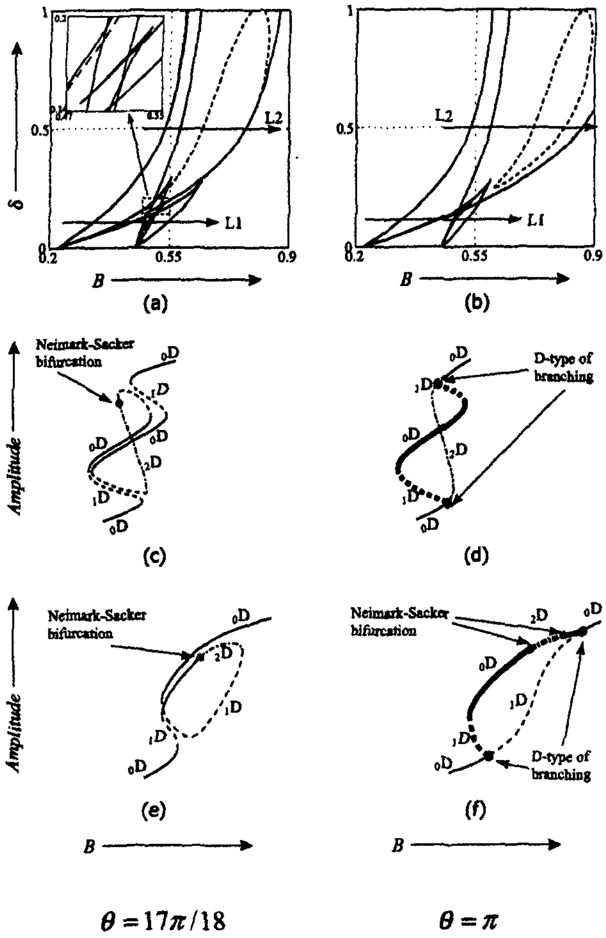

Figure.4 (a) and (b) are bifurcation diagrams of fixed point in (B, δ)-plan when θ equals 0 and $17\pi/18$ respectively. Below ones are schematic amplitude characteristic diagrams representing variations of periodic solutions by changing B along line L1: $\delta = 0.1$ (c, d); and L2: $\delta = 0.5$ (e, f).

respectively. In Fig.4(a, b), dashed curves represent Neimark-Sacker bifurcation. Observing Fig.4, we can conclude similar results to complete symmetrical case. Note that the curve representing variation of periodic solutions is one continuous curve while it is two independent curves in Fig.3(d). Actually, this phenomena is appear when θ is larger than $3\pi/18$, at the same time, Neimark-Sacker bifurcation occurs (Please see Fig.5). Neimark-Sacker bifurcation is a close curve only on inversion symmetrical condition; it is tangent to a tangent bifurcation curve in asymmetrical case, correspondingly.

Above two cases are discussed when phase-shfit is only slightly differ from symmetrical condition. In Fig.5, we show bifurcation diagrams when we change phase-shift step by step. In this way, we can see the change of bifurcation when system is far away from symmetry. We see the most obvious variation occur when phase-shift between $\theta = 5\pi/18$ and $\theta = 8\pi/18$. In addition, variation between $\theta = 13\pi/18$ and $\theta = 14\pi/18$ is remarkable. We found that two tangent bifurcation curves connect together and another occur a peak (see the enlarge region in bifurcation diagram of $\theta = 14\pi/18$). When δ is larger than this peak value, amplitude characteristic curve separate instead of continuous one.

5. Concluding Remarks

We investigated the bifurcation and resonance of a coupled Duffing's circuit losing its symmetry for the phase-shift between two oscillators. Periodic solutions on symmetrical and asymmetrical condition are compared. We found that D-type of branching become two independent tangent bifurcation. The amplitude characteristic is a continuous curve on some condition, see Fig.3(d) and Fig.4(c,d).

The results are worthwhile because it can be referenced when we consider a poly-phase coupled nonlinear oscillators. In a poly-phase coupled system, every oscillator can be regard as one coupled with its neighborhood by a specific phase-shift.

A combinatorial resonance phenomena caused by a finite number of coupled oscillators is an interesting problem in nonlinear circuit open to the future.

References

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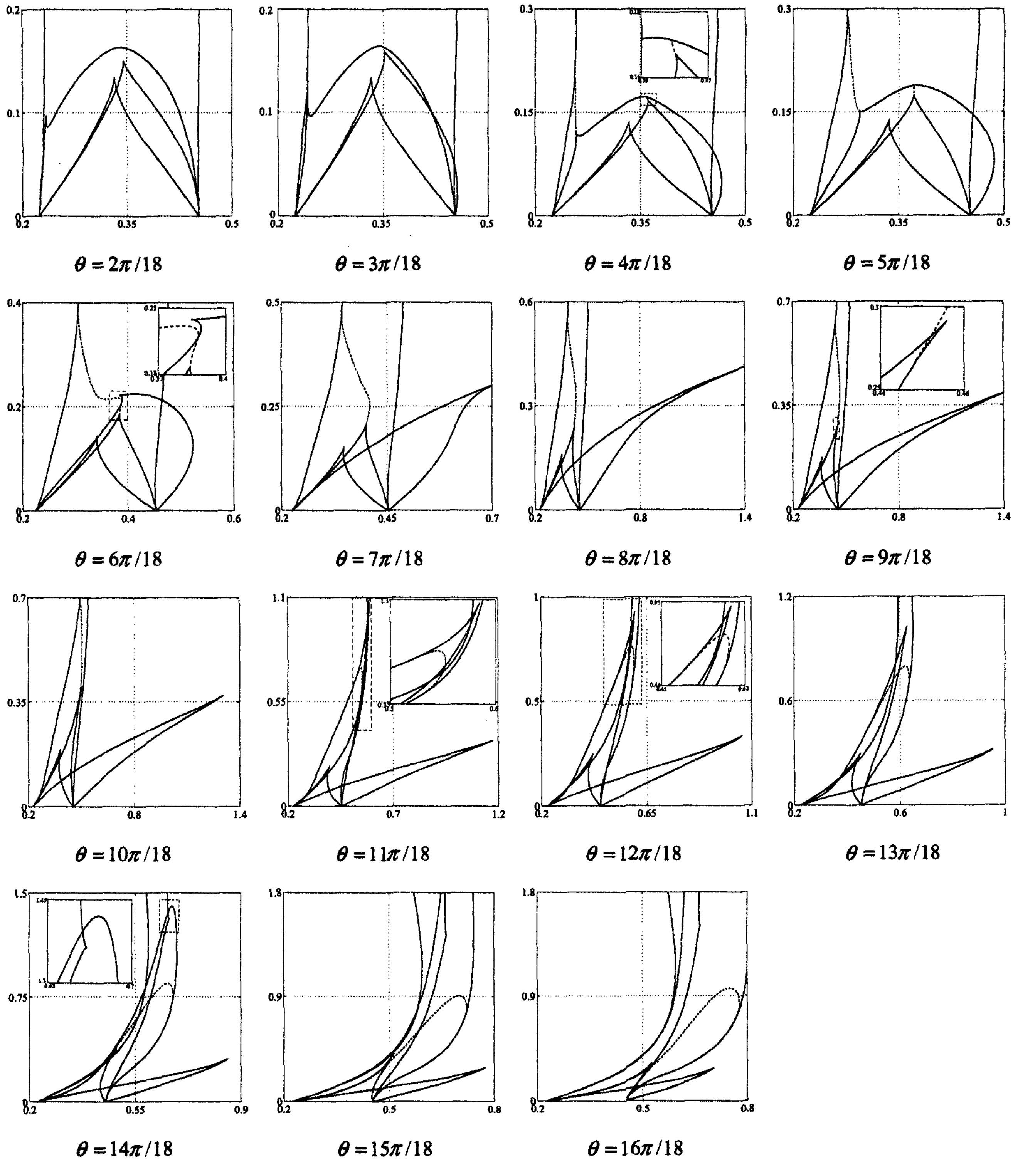

Figure.5 Bifurcation diagrams of fixed point when phase-shift change step by step from $\theta = 2\pi/18$ to $\theta = 16\pi/18$. All of bifurcation diagrams are drawn in (B, δ)-plan. For convenience, we omit denotations of axes in diagrams. Some regions difficult to be recognized are enlarged correspondingly. Dashed curves represent Neimark-Sacker bifurcations. Note that the coordinate of each diagram is adjusted to fit dimension of bifurcation curves.