Synchronization of Chaos in a Dual-structured System Consisting of Two Identical Piecewise-linear Systems

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Abstract: Synchronization phenomena of chaos observed in a dual-structured system is presented. The system is consisting of two identical piecewise-linear systems and the simple coupling between the two systems enables the synchronization of the chaotic behavior. An application of the proposed dual-structure to a real power system for the parameter value identification is also presented.

1. Introduction

Two systems of the same structure and of the same parameter values can be synchronized by supplying the signal from one to the other even when they behave chaotically. This phenomenon is called synchronization of chaos [1].

In this paper a dual-structured system is proposed and is presented as a simple example that enables the chaos synchronization without complicated system configurations. The system is consisting of two identical piecewise-linear systems that the author has derived and investigated [2,3] to have found that they exhibit chaos for a wide range of the parameter value. And using a real system including laboratory generator the similar dual-structured system is experimentally studied to indicate that the validity of the proposed structure in the real situations.

2. Configuration of the two identical systems

The system to be investigated in this paper consists of two identical subsystems as shown later in Figure 4. In this section, one of the two identical subsystems as in Figure 1 is described. It consists of a second-order lag element, two first-order lag elements, a gain element and a feedback-loop with a piecewise-linear limiter [2,3]. This model has been derived from a typical power system model, in which chaotic behavior were observed both numerically [4] and experimentally. Though it has been derived from a power system model, the model rather represents generic characteristics of 4-dimensional systems not limited to power systems.

This simple system consists of serial connection of typical basic linear elements, the nonlinearity lies only in the feedback loop which is piecewise-linear saturation element, and only one equilibrium point exists. But it exhibits chaotic behavior for a fairly wide range of the parameter values.

If the state variables x_2, x_3, x_4 are chosen as in Figure 1, the differential equation of the model becomes

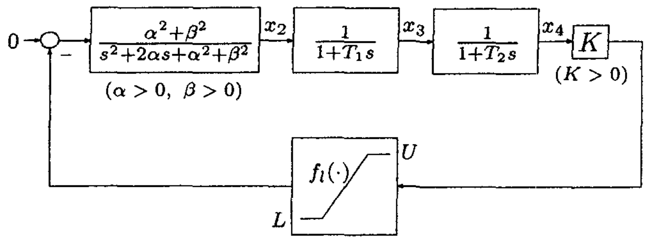


Figure 1. The model

Table 1. parameter values

α	0.4
β	5
T_1	1
T_2	3
U	1
K	34

$$\dot{x_1} = -2\alpha x_1 - (\alpha^2+\beta^2)\{x_2 + f_l(Kx_4)\} \quad (1)$$
$$\dot{x_2} = x_1 \quad (2)$$
$$T_1 \dot{x_3} = x_2 - x_3 \quad (3)$$
$$T_2 \dot{x_4} = x_3 - x_4 \quad (4)$$

where

$$f_l(Kx_4) = \begin{cases} U \ (Kx_4 > U) \\ x \ (L \le Kx_4 \le U) \\ L \ (Kx_4 < L) \end{cases} \quad (5)$$

Parameter values shown in Table 1 are used in the simulation. These values were chosen based on the dynamical characteristic of the original power system model. The lower bound L of the limiter is canceled for simplicity in the analysis of this paper.

Equations (1)-(5) can be expressed with matrices as

$$\dot{\boldsymbol{x}} = \begin{cases} A\boldsymbol{x} & (L \le Kx_4 \le U) \\ B(\boldsymbol{x}-\boldsymbol{p}) & (U < Kx_4) \\ B(\boldsymbol{x}-\boldsymbol{q}) & (Kx_4 < L) \end{cases} \quad (6)$$

where

$$\boldsymbol{x} = {}^t(x_1 \quad x_2 \quad x_3 \quad x_4) \quad (7)$$

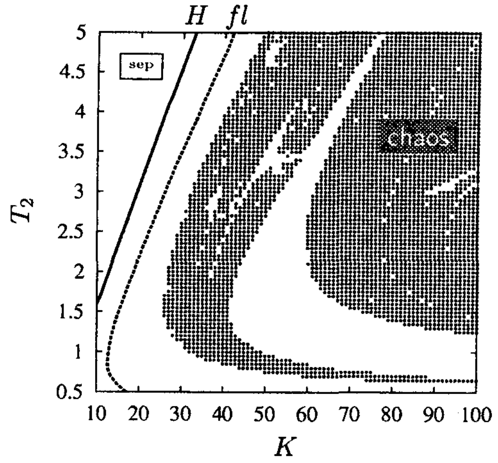


Figure 2. Bifurcation set on the parameter plane of K-T_2

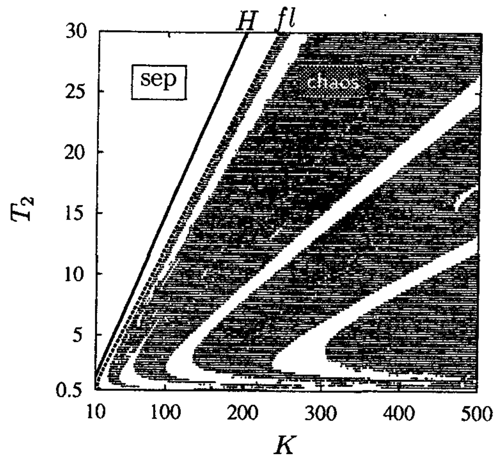


Figure 3. Bifurcation set on the parameter plane of K-T_2 (in a wider range)

$$A = \begin{pmatrix} -2\alpha & -(\alpha^2+\beta^2) & 0 & -(\alpha^2+\beta^2)K \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{T_1} & -\frac{1}{T_1} & 0 \\ 0 & 0 & \frac{1}{T_2} & -\frac{1}{T_2} \end{pmatrix} \tag{8}$$

$$B = \begin{pmatrix} -2\alpha & -(\alpha^2+\beta^2) & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{T_1} & -\frac{1}{T_1} & 0 \\ 0 & 0 & \frac{1}{T_2} & -\frac{1}{T_2} \end{pmatrix} \tag{9}$$

$$\boldsymbol{p} = B^{-1} \begin{pmatrix} (\alpha^2+\beta^2)U \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{10}$$

$$\boldsymbol{q} = B^{-1} \begin{pmatrix} (\alpha^2+\beta^2)L \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{11}$$

Figure 2 shows the various bifurcation points and the region where chaotic motion was observed for the values of the feedback gain K and the time constant of a first-order lag element T_2. At the curve denoted by H, we saw the jumping from the equilibrium point to a limit cycle and at the curve fl, flip bifurcation (period-doubling bifurcation). In the left region of the curve H (labeled as [sep]) the equilibrium point is stable. The region where chaotic behavior arises is indicated by chaos.

If the values of K and T_2 are varied in a wider range, Figure 3 is obtained. It indicates that the chaotic region spreads over the large region and the periodic windows are located in a regular manner.

3. Dual-structured system

Combining the two identical systems of the configuration shown in Figure 1 by adding the coupling from one to the other, a dual-structured system as in Figure 4 has been derived.

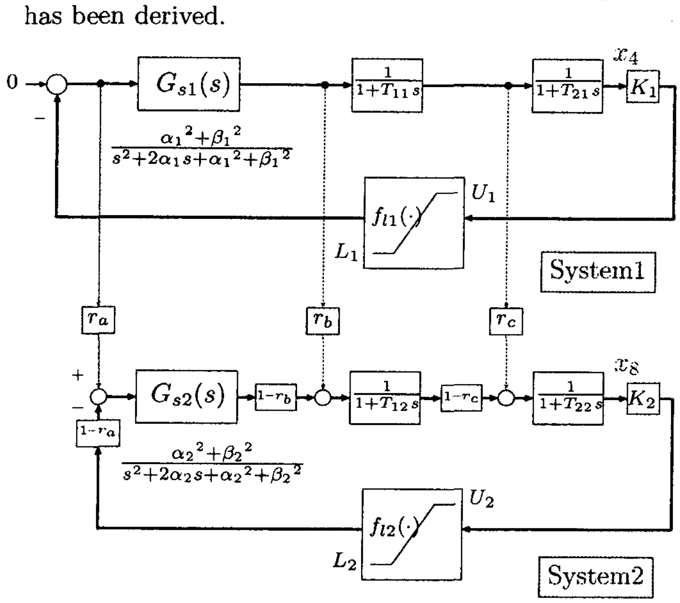
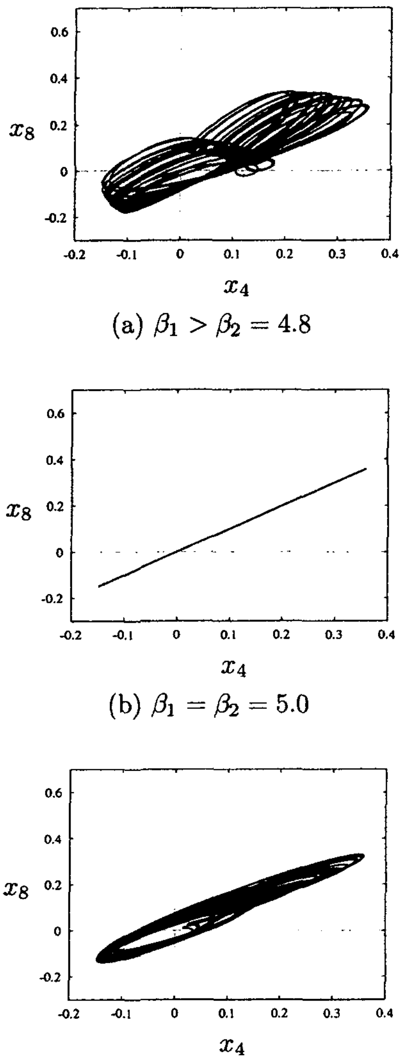


Figure 4. Dual-structured system

As seen in Figure 4, System1 is not disturbed by System2, and System2 is disturbed by the signals from System1. The extent of the disturbance into System2 depends on the coupling coefficients r_a, r_b, r_c which range from 0 to 1. When the coupling coefficient is 0, no signal from System1 is fed into System2, i.e., there is no coupling. When it is 1, the feedback loop is cut out at the coupling point, and the signal at the same point of System1 is directly fed to System2. So, if the coupling coefficient is 0, System1 and System2 behave just separately, while if it is 1, System1 and System2 are naturally supposed to make the identical behavior under the condition that the open-loop characteristic is stable, i.e., non-oscillatory. According to this characteristic, you can expect these two system's chaotic behavior will synchronize when the coupling coefficients r_a, r_b and r_c are set to appropriate values from 0 till 1.



(a) $\beta_1 > \beta_2 = 4.8$

(b) $\beta_1 = \beta_2 = 5.0$

(c) $\beta_1 < \beta_2 = 5.2$

Figure 5. Synchronization of the chaotic behavior of System1 and System2

4. Synchronization of chaos

Simulation results of this dual-structured system are shown in Figure 5. Figure 5(a), (b) and (c) indicate how correlated System1's behavior and System2's behavior are when β_2 is varied as depicted below each figure. These results are obtained with $r_a = 0.5$, $r_b = r_c = 0$ and other parameter values are same as in Table 1. Figure 5 means that the chaotic behaviors of both systems are in synchronization when the parameter values of System1 and System2 are identical.

5. Case study of a real power system

To see the practical validity of the proposed dual-structured system and the observed phenomena of chaos synchronization in it, experimental studies of the chaos synchronization were done using the real laboratory power system. Figure 6 shows the dual-structured power

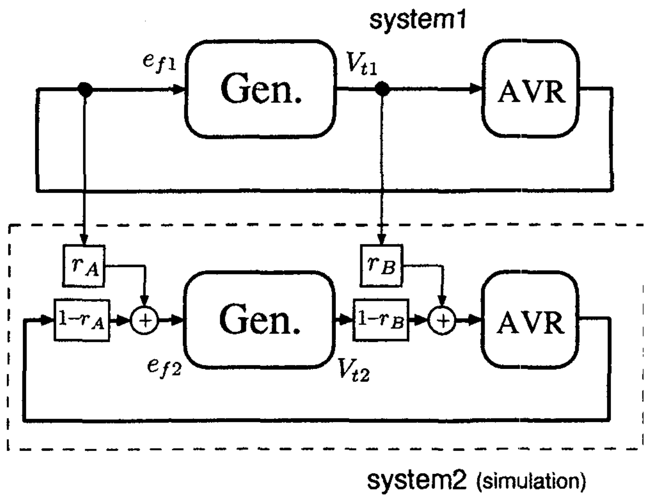


Figure 6. Dual-structured power system

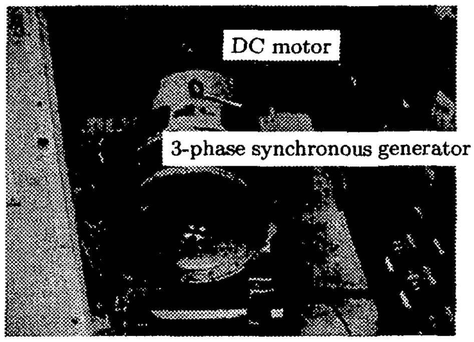


Figure 7. Laboratory motor-generator set

system to be investigated experimentally. System1, the upper half of Figure 6, is the real power system in a laboratory equipment. And System2, the lower half, is the virtual system computationally realized in a computer.

In Figure 6, **Gen.** indicates a generator, **AVR** indicates Automatic Voltage Regulator that controls the generator voltage V_t by adjusting the excitation voltage e_f. Note that the subscripts of 1 and 2 indicate that the variables or parameters are involved with System1 and System2, respectively. For example, e_{f1} indicates the excitation voltage of System1.

The laboratory power system (System1) consists of the motor-generator set, the combination of a direct current motor (the substitution for the steam turbine of the real power plant) and a 3-phase synchronous generator coupled through a rotation shaft as shown in Fig. 7. The generator is electrically connected to the external system of a power company via equivalent transmission lines.

Each figure in Figure 8 shows the correlation of the two subsystems in the dual-structured power system. The generator inertia time constant M_2 (s), one of the parameters of the virtual power system (System2) is varied as indicated below the figures. The coupling coefficient r_A and r_B are set to be 0.7 and 0.6, respectively.

The correlation diagrams like these for many other values of M_2 yield Figure 9, where V_{t2P} indicates cross-sectional points of V_{t2}. The cross section is the vertical line of $V_{t1} = 0.9$ p.u.

Though the true value of the corresponding parameter of M_2 in System1, i.e. M_1, is not known because System1 is not a mathematical model but a real one, the expected value of M_1 by the designer of the machine is 10 s. Unlike Figure 5, even when the parameter values of the two systems are supposed to be identical, System1 and System2 do not completely synchronize, but higher correlation can still be observed.

6. Conclusion

The application aimed with the proposed dual-structured system is the parameter value identification. In order to identify the parameter values in System1 (which is supposed to correspond to a real system), the value of the corresponding parameter is varied in System2 (realized with computer simulation) and check if the chaotic behavior of the both systems synchronize or not, thereby the parameter value can be identified.

How easily the both system are synchronized depends on the values of coupling coefficients. That means we can make an adaptive way to let the estimation value converge to the true value. For example, first we set the coupling coefficients to 1 or a value very close to 1, so that we can easily estimate the parameter value roughly. And gradually decreasing the value of the coupling coefficients, better estimations of the parameter value can be made by utilizing the increasing sensitivity of the chaos synchronization to the parameter value inconsistency.

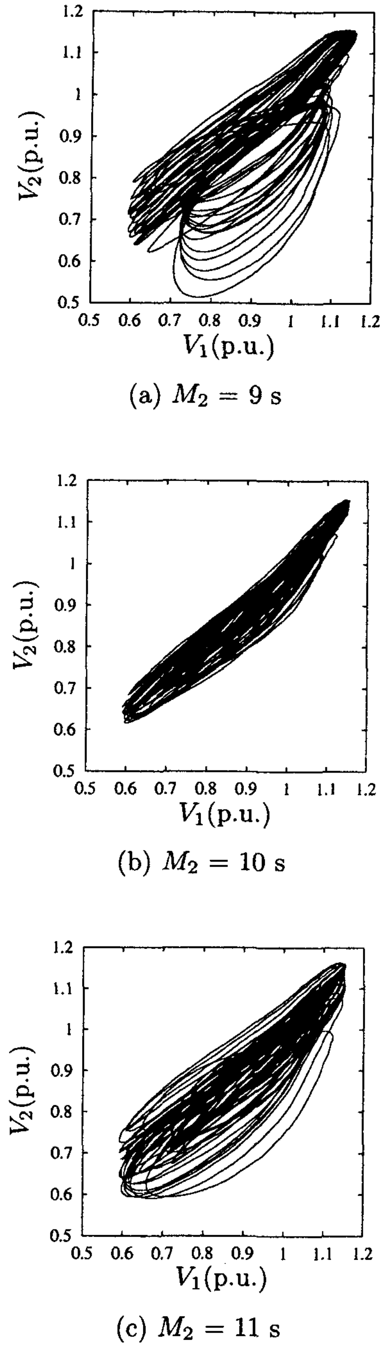


Figure 8. Correlation of the chaotic behavior of the dual-structured power system

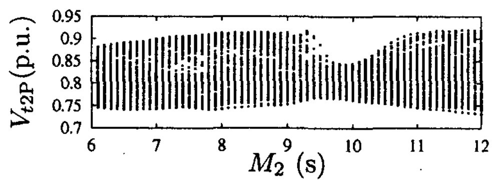


Figure 9. Variation of cross-sectional points according to M_2

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