A Design Method of Lossy Linear Tapered Transmission Line with Quasi Non-distortion Characteristic in the Time Domain

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Abstract: An exact solution of the lossy linear tapered transmission line is derived. As its application, a simple design method of the quasi non-distortion lossy linear tapered transmission line in the time domain is described. A design example is presented to show the validity and usefulness of the method.

1. Introduction

With the rapid increase of signal frequency and decrease of feature sizes of high-speed electronic circuits, interconnect has become a dominating factor in determining circuit performance and reliability in VLSI and microwave IC designs. Transmission line effects, such as delay, reflection, distortion, dispersion, and crosstalk, have severe impact on circuit performance. In recent years, interconnect modeling and simulation have become hot topics in the research of advanced CAD techniques[1]. At the next stage, we think that the design method of interconnect will become important. As interconnect is modeled by uniform and nonuniform transmission line, then, to specify the transmission line characteristics in the time domain is needed[2].

In this paper, we show an exact solution of the lossy linear tapered transmission. And using these equations, we present a simple design method to obtain the quasi non-distortion lossy linear tapered transmission line in the time domain, and design examples are shown demonstrate the validity of the proposed method.

2. Exact Solution

Lossy nonuniform transmission line equations in the time domain are expressed as follows:

$$-\frac{\partial V(x,t)}{\partial x} = sL(x)\frac{\partial I(x,t)}{\partial t} + R(x)I(x,t) \quad (1)$$

$$-\frac{\partial I(x,t)}{\partial x} = C(x)\frac{\partial V(x,t)}{\partial t} + G(x)V(x,t) \quad (2)$$

where $V(x,t)$ and $I(x,t)$ are the voltage and current at point x and time t on the line. $L(x)$, $C(x)$, $R(x)$ and $G(x)$ are inductance, capacitance, resistance and conductance parameters per unit length at point x, respectively. Figure 1 shows a terminated lossy linear tapered transmission line with length ℓ. Here, parameters are

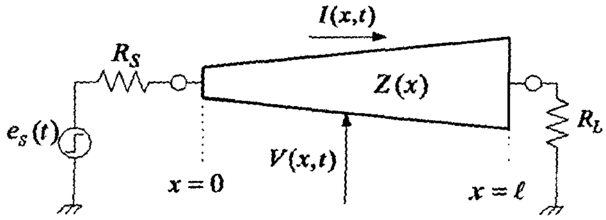

Figure 1. A terminated lossy linear tapered transmission line.

$$L(x) = L_0(1+kx) \quad (3)$$
$$C(x) = C_0(1+kx)^{-1} \quad (4)$$
$$R(x) = R_0(1+kx) \quad (5)$$
$$G(x) = G_0(1+kx)^{-1}. \quad (6)$$

We obtained the frequency domain solution of these as follows:

$$\begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V(\ell,s) \\ rI(\ell,s) \end{bmatrix} \quad (7)$$

$$A = u_0[I_1(u_0)K_0(u_\ell) + I_0(u_\ell)K_1(u_0)] \quad (8)$$
$$B = Z_{c0}u_\ell[I_1(u_0)K_1(u_\ell) - I_1(u_\ell)K_1(u_0)] \quad (9)$$
$$C = \frac{u_0}{Z_{c0}}[I_0(u_0)K_0(u_\ell) - I_0(u_\ell)K_0(u_0)] \quad (10)$$
$$D = u_\ell[I_1(u_\ell)K_0(u_0) + I_0(u_0)K_1(u_\ell)] \quad (11)$$

where

$$u_x = -(1+kx)\frac{\gamma_0}{k} \quad (x=0,\ell) \quad (12)$$
$$Z_{c0} = \sqrt{\frac{sL_0+R_0}{sC_0+G_0}} \quad (13)$$
$$\gamma_0 = \sqrt{(sL_0+R_0)(sC_0+G_0)} \quad (14)$$

and s is complex frequency. $I_n(z)$ and $K_n(z)$ are the first and the second kind modified Bessel functions respectively. Another representation of the solution exists and it is as follows:

$$A = \frac{\pi u_0}{2}[J_1(u_0)Y_0(u_\ell) - J_0(u_\ell)Y_1(u_0)] \quad (15)$$

$$B = jZ_{c0}\frac{\pi u_\ell}{2}[J_1(u_0)Y_1(u_\ell) - J_1(u_\ell)Y_1(u_0)] \quad (16)$$

$$C = j\frac{1}{Z_{c0}}\frac{\pi u_0}{2}[J_0(u_0)Y_0(u_\ell) - J_0(u_\ell)Y_0(u_0)] \quad (17)$$

$$D = \frac{\pi u_\ell}{2}[J_1(u_\ell)Y_0(u_0) - J_0(u_0)Y_1(u_\ell)] \quad (18)$$

where

$$u_x = -j(1+kx)\frac{\gamma_0}{k} \qquad (x = 0, \ell) \quad (19)$$

and $J_n(z)$ and $Y_n(z)$ are the first and the second kind Bessel functions respectively. If it is a lossless here, Equation(19) becomes

$$u_0 = \frac{\beta_0}{k} \qquad u_\ell = (1+k\ell)u_0 \quad (20)$$

where $\beta_0 = \omega\sqrt{L_0C_0}$, $Z_{c0} = \sqrt{L_0/C_0}$. This is corresponding to the previous work[3]–[5].

3. Initial Values of Step Response

By using the initial and the final value theorem of the Laplace transformation, we derived the intial and final values of step response from equation (7).

$$V(\ell,\tau) = \frac{2U_0 r_L\sqrt{S_r}}{(1+r_S)(S_r+r_L)}e^{-\frac{1}{2}(r_0+g_0)} \quad (21)$$

$$V(\ell,\infty) = \frac{U_0}{p_0P_1 + \sqrt{\frac{r_0}{g_0}\frac{p_\ell}{r_L}}P_2 + \sqrt{\frac{g_0}{r_0}}p_0r_SP_3 + \frac{p_\ell r_S}{r_L}P_4} \quad (22)$$

where

$$P_1 = I_1(p_0)K_0(p_\ell) + I_0(p_\ell)K_1(p_0) \quad (23)$$

$$P_2 = I_1(p_0)K_1(p_\ell) - I_1(p_\ell)K_1(p_0) \quad (24)$$

$$P_3 = I_0(p_0)K_0(p_\ell) - I_0(p_\ell)K_0(p_0) \quad (25)$$

$$P_4 = I_1(p_\ell)K_0(p_0) + I_0(p_0)K_1(p_\ell) \quad (26)$$

$$\tau = \frac{\ell}{c} \qquad Z(\ell) = Z_\ell = (1+k\ell)Z_0 \quad (27)$$

$$p_0 = \frac{\sqrt{r_0g_0}}{1-S_r} \qquad p_\ell = S_rp_0 \qquad S_r = \frac{Z_\ell}{Z_0} \quad (28)$$

$$r_0 = \frac{R_0}{Z_0}\ell \quad g_0 = Z_0G_0\ell \quad r_S = \frac{R_S}{Z_0} \quad r_L = \frac{R_L}{Z_0} \quad (29)$$

and τ is the delay time and c is the velocity of the nonuniform transmission line. Z_x is the time domain characteristic impedance of the nonuniform transmission line defined by

$$Z_x = Z(x) = \sqrt{\frac{L(x)}{C(x)}} \quad (30)$$

4. Quasi Non-distortion Design

It is equivalent with the step response to the step wave to make a transmission line a non-distortion one. We propose the simple conditions of the quasi non-distortion as follows:

$$\lim_{\Delta\tau\to 0} V(\ell, n\tau - \Delta\tau) = \lim_{\Delta\tau\to 0} V(\ell, n\tau + \Delta\tau) \qquad (n = 3,5,7,\cdots) \quad (31)$$

$$V(\ell,\tau) = V(\ell,\infty) \quad (32)$$

Equation (31) is a condition because discontinuity is not caused during the response waveform, and (32) is a condition to equate an initial arriving value and the steady state value of the response waveform. From (31), we obtained

$$R_S = Z_0 \quad \text{or} \quad R_L = Z_\ell \quad (33)$$

These equations show that it is necessary to make matching at least by either of the near or the far end. The relation of other tansmission line constants are obtained solving (32) numerically.

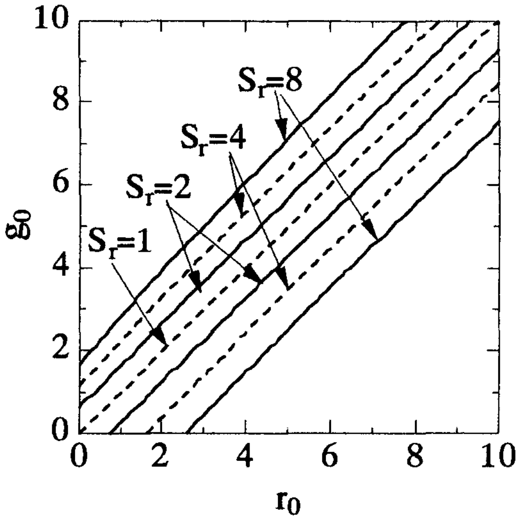

Figure 2. Quasi non-distortion conditions and initial voltages at the far end of lossy linear tapered transmission line.

5. Design Example

5.1 Example 1

Figure 2 shows the quasi non-distortion conditions of lossy linear tapered transmission line obtained by solving equation(31) and (32) where the characteristic impedance ratio at the near and the far end S_r is defined by equation (28) and the terminated resistances are $R_S/Z_0 = 1$ and $R_L/Z_\ell = 1$. Here line loss parameters are normalized as shown equation (29). It is understood that there are two solution curves and that the linear tapered transmission line can have non-distortion characteristic even if g_0 is zero. This is convenient in the design of IC. For example, the solution is $g_0 = 0.374$ or $g_0 = 3.29$ for $r_0 = 2$. Figure 3 shows the step response in this case and others. It is understood that the step response is almost becomes a step wave when the quasi non-distortion conditions are satisfied.

5.2 Example 2

A transmission line parameters obtained from [6] is considered in this example. The parameters are

$$L_0 = 3138[\text{nH/m}] \qquad C_0 = 123[\text{pF/m}]$$
$$R_0 = 358[\text{k}\Omega/\text{m}] \qquad G_0 = 0$$

and line length $\ell = 1.0 \times 10^{-3}$[m]. When source impedance and load impedance are

$$R_S = Z_0 = \sqrt{\frac{L_0}{C_0}} = 159.7[\Omega]$$
$$R_L = Z_\ell = 4Z_0 = 638.8[\Omega]$$

the step response does not become a non-distortion characteristic as shown in Figure 3 by broken line. Then, when R_0 which almost becomes a non-distortion characteristic in the state of $G_0 = 0$ is obtained from equation (32), we obtain with $R_0 = 263\text{k}\Omega$ The characteristic at this time is a quasi non-distortion characteristic to showing in Figure 3 by solid line. In a word, it is understood to become a quasi non-distortion characteristic by decreasing some the losses to 2/3.

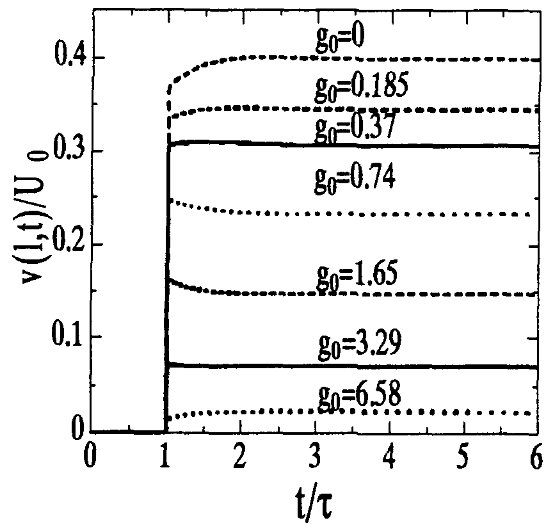

Figure 3. Step responses of lossy linear tapered transmission line($r_0 = 2$).

6. Conclusion

We obtained an exact solution of lossy linear tapered transmission line. Next, using this solution, we showed a design method to obtain the quasi non-distortion lossy linear taper transmission line.

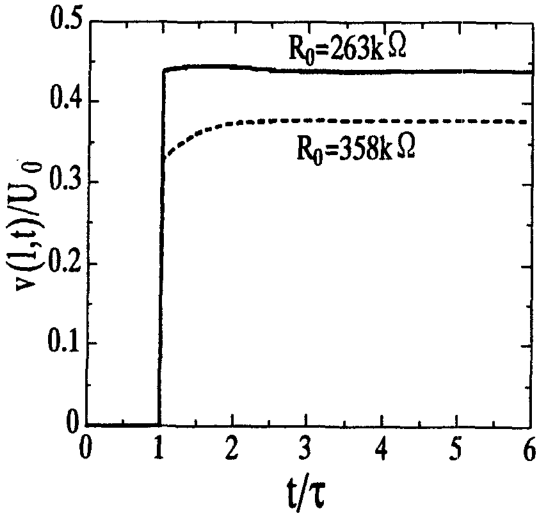

Figure 4. Step responses of lossy linear tapered transmission line($g_0 = 0$).

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