Unscented Particle Filter를 이용한 시간영역 비선형 구조계 규명기법

Unscented Particle Filter for Time Domain Identification of Nonlinear Structural Dynamic Systems

구기영* 윤정방**

Koo, Ki Young Yun, Chung Bang

ABSTRACT

본 연구에서는 최근에 개발된 Unscented Particle Filter (UPF)를 사용한 비선형 동적 구조계의 구조계수 규명기법이 연구되었다. 일반적인 비선형 구조계수 추정 문제의 일반 해는 존재하지 않으나, 그에 대한 대안으로써 선형 근사 기법인 extended Kalman filter (EKF)가 비선형 동적 구조계수의 추정에 주로 사용되어왔다. 그러나, EKF는 구간 선형(piecewise linear) 가정으로 인해 biased estimator이고 비선형성이 상대적으로 높을 때 오차가 큰 추정치를 주는 단점을 가진다. 이를 보완하기 위해서 UPF가 개발되었고, 이 기법은 particle filter의 일종으로써 Unscented Kalman filter (UKF)를 사용하여 importance proposal distribution을 생성한다.

수치실험이 SDOF와 MDOF에 대하여 3가지 경우에 대해서 수행되었다. 비선형 SDOF의 수치실험으로부터 잡음이 가해진 상태에서 UKF가 EKF에 비해 초기 공분산 행렬의 가정에 대해 정확하고 강인한 추정결과를 보여줌을 보였다. 최하층의 column에 비선형 거동이 발생하는 5층 전단 빌딩모형의 수치실험으로부터 UKF가 복잡한 구조물의 구조계수 추정능력이 있음을 보여주었다. 여러 가지 수치실험은 UPF가 EKF보다 비선형 동적 구조계수 추정에 있어서 더 나은 방법임을 보여주었다.

1. INTRODUCTION

Detection of structural change or damage is one of the most important and challenging issues in the structural health monitoring system. Statistical inference approaches are more appropriate in the detection of the structural change due to the complex nature of civil infrastructures and noise-polluted measurements. Especially Bayesian approach in statistical inference has useful structure in on-line estimation and has been applied to various fields, i.e., physics, control system and signal processing.

The most well-known Bayesian solution is the Kalman filter which is complete and tractable Bayesian solution for linear dynamic system by making linear and Gaussian assumptions to simplify the optimal recursive Bayesian estimation. Unfortunately, the problem of estimating the structural change often has nonlinear dynamics so the extended Kalman filter (EKF) was applied that is approximation solution for nonlinear dynamic system in a

* 한국과학기술원 건설 및 환경공학과, 박사과정

** 정회원 · 한국과학기술원 건설 및 환경공학과, 교수

sense of piecewise linear approximation and has been reported on its successful applications in numerous state estimation problem. But assumptions on extended Kalman filter may be violated in highly nonlinear systems.

Particle filtering techniques are general approach in estimations of nonlinear dynamic system without assumptions on the form of the probability densities in question; that is, they employ full nonlinear and non-Gaussian estimation. Recently the unscented particle filter has been developed by Wan and Merwe[5] that utilizes the unscented Kalman filter (UKF) to augment and improve the standard particle filter, specifically through generation of the importance proposal distribution. In this study, the performance of the unscented particle filter was evaluated being a more general filtering technique for detection of structural change in civil infra-structures in comparison with the extended Kalman filter.

2. UNSCENTED PARTICLE FILTER

2.1 Bayesian approach

Given measurements of inputs u_k and structural response y_k, the goal is to estimate the parameters of the structural dynamic system. The optimal estimate in a sense of minimum mean-squared error (MMSE) is given by the conditional mean as follows

$$\hat{X} = \mathbf{E}[X_k \mid \mathbf{Y}_{0:k}] \tag{2.1}$$

where $Y_{0:k}$ is the sequence of observations up to time k. Evaluation of this expectation requires information of the a posterior density $p(X_k \mid \mathbf{Y}_{0:k})$. The problems of determine posteriori density given a priori density $p(X_k \mid \mathbf{Y}_{0:k-1})$is referred as the Bayesian approach and can be evaluated recursively according to the following relations:

$$p(X_k \mid \mathbf{Y}_{0:k}) = \frac{p(X_k \mid \mathbf{Y}_{0:k-1})\, p(Y_k \mid X_k)}{p(Y_k \mid \mathbf{Y}_{0:k-1})} \tag{2.2}$$

where

$$p(X_k \mid \mathbf{Y}_{0:k-1}) = \int p(X_k \mid X_{k-1})\, p(X_{k-1} \mid \mathbf{Y}_{0:k-1}) dX_{k-1} \tag{2.3}$$

and the normalizing constant $p(Y_k \mid \mathbf{Y}_{0:k-1})$ is given by

$$p(Y_k \mid \mathbf{Y}_{0:k-1}) = \int p(X_k \mid \mathbf{Y}_{0:k-1})\, p(Y_k \mid X_k) dX_k \tag{2.4}$$

This recursion specifies the current state density as a function of the previous density and the most recent measurement data. The state-space model comes into play by specifying the state transition probability $p(X_k \mid X_{k-1})$ and measurement probability or likelihood, $p(Y_k \mid X_k)$. Unfortunately, the multidimensional integration indicated by Eqn. (2.2). Make a closed-form solution intractable for most systems. The only general approach is to apply Monte Carlo sampling techniques that essentially convert integrals to finite sums, which converge to the true solution in the limit.

2.2 Unscented particle filter

The choice of the importance proposal distribution $q(X_k \mid X_{0:k-1}, Y_{0:k})$ is most important and critical design issue for filter performance. The optimal importance proposal distribution which minimizes the variance on the importance weights is given by

$$q(X_k \mid X_{0:k-1}, Y_{0:k}) = p(X_k \mid X_{0:k-1}, Y_{0:k}) \quad (2.5)$$

That is, the true conditional state density given the previous state hasty and all observations. Sampling from this distribution is impractical for arbitrary densities. Consequently, the transition prior is the most popular choice of importance proposal distribution.

$$q(X_k \mid X_{0:k-1}, Y_{0:k}) \doteq p(X_k \mid X_{k-1}) \quad (2.6)$$

The effectiveness of this approximation depends on how close the importance proposal distribution is to the true posterior distribution. If there is not sufficient overlap, only a few particles will have significant importance weights when their likelihood is evaluated.

An improvement in the choice of importance proposal distribution over the simple transition prior, which also address the problem of sample depletion, can be accomplished by moving the particles toward the regions of high likelihood, based on the most recent observations y_k. An effective approach to accomplish this, is to use an EKF generated Gaussian approximation to the optimal importance proposal, that is,

$$q(X_k \mid X_{0:k-1}, Y_{0:k}) \doteq q_N(X_k \mid Y_{0:k}) \quad (2.7)$$

which is accomplished by using a separate EKF to generate and propagate a Gaussian importance proposal distribution for each particle,

$$q_N(X_k^i \mid Y_{0:k}) = N(\bar{X}_k^i, P_k^i), \qquad i = 1,2,\ldots,N \quad (2.8)$$

That is, at time k one uses the EKF equations, with the new data, to compute the mean and covariance of the importance distribution for each particle from the previous time step $k-1$. Next, we redraw the ith particle (at time k) from this new updated distribution. While still making a Gaussian assumption, the approach provides a better approximation to the optimal conditional importance proposal distribution and has been shown to improve performance on a number of applications.

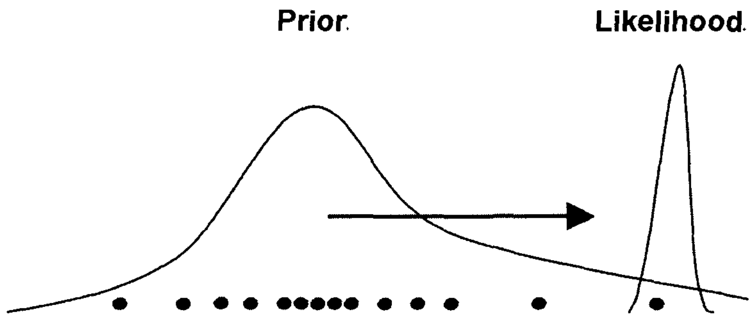


Fig. 2.1 Unscented Particle Filter: Moving importance proposaldistribution to regions of high likelihood using the most current observation

By replacing the EKF with the UKF, we can more accurately propagate the mean and covariance of the Gaussian approximation to the state distribution. Distributions generated by the UKF will have a grater support overlap with the true posterior distribution than the overlap achieved by the EKF estimates. In addition, scaling parameters used for sigma-point selection can be optimized to capture certain characteristic of the prior distribution if known, e.g. the algorithm can be modified to work with distributions that have heavier tail than Gaussian distributions such as Cauchy or Student-t distributions. The new filter that results from using a UKF for importance proposal distribution generation within a particle filter framework is called the unscented particle filter (UPF).

3. Numerical simulations

3.1 Identification of nonlinear SDOF system: the EKF and the UKF

In this numerical simulation, performance of the UKF and the EKF was evaluated on single-degree-of-freedom Bouc-Wen's hysteretic model as following:

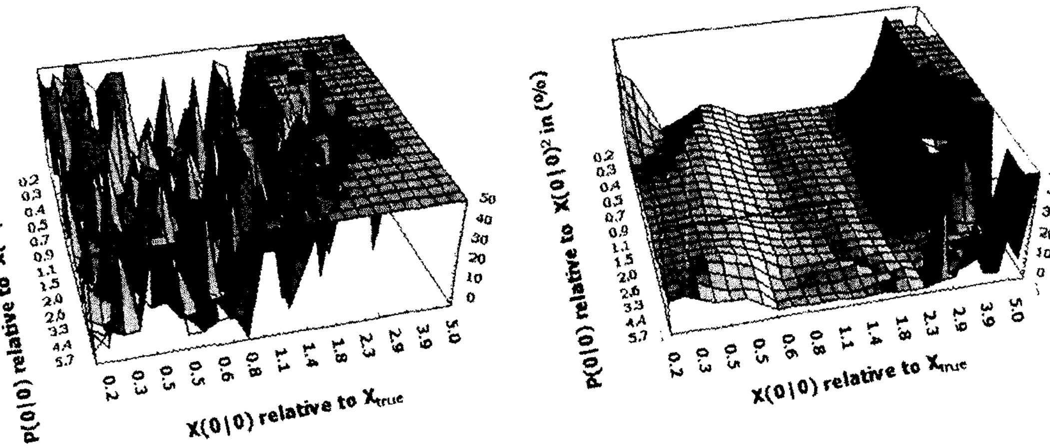
$$\ddot{U}(t) + 2\omega_n \xi \dot{U}(t) + \omega_n^2 \phi(U(t), \dot{U}(t)) = -\ddot{u}_g \tag{3.1}$$

$$\dot{\phi} = \dot{u} - \beta \mid \dot{u} \mid \phi + \gamma \dot{u} \mid \phi \mid \tag{3.2}$$

$$X = \{U \quad \dot{U} \quad \omega_n \quad \xi \quad \phi \quad \beta \quad \gamma\}^T \tag{3.3}$$

where $U(t)$ is displacement, ω_n is natural frequency, ξ is damping ratio, ϕ is Bouc-Wen's nonlinear displacement related to nonlinear restoring force, $\ddot{u}_g$ is ground acceleration and β & γ are shape-control parameters to be identified. X is state vector used in the filtering techniques.

Estimation performance was compared for 13 by 13 cases of initial guesses of $\hat{x}(0 \mid 0)$ and $P(0 \mid 0)$ and estimation error (RMSE in percentile) was shown in Fig. 3.1. Performance between the EKF and the UKF is strongly distinct. The EKF diverges almost region of simulation but the UKF shows robust estimation capability about the initial guesses of $\hat{x}(0 \mid 0)$ and $P(0 \mid 0)$.



a) The extended Kalman filter b) The unscented Kalman filter

Fig. 3.1 Root mean square error (RMSE) of estimation

3.2 Application to 5-story building structure using unscented Kalman filter

Numerical simulations are carried out to show the capability of parameter identification using unscented Kalman filter. Identification performed for the five-degree-of-freedom shear-building model with Bouc-Wen's hysteretic nonlinear spring at the bottom of columns using the artificial responses data with 1% noise in RMS level. In this example, the parameters to be identified are c_i, k_i, β_i and γ (i=1,2,3,4,5). The exact values of system parameters and estimated results are shown in Table 2 and estimationhistories for linear and nonlinear parameters are shown in Fig. 3.3.

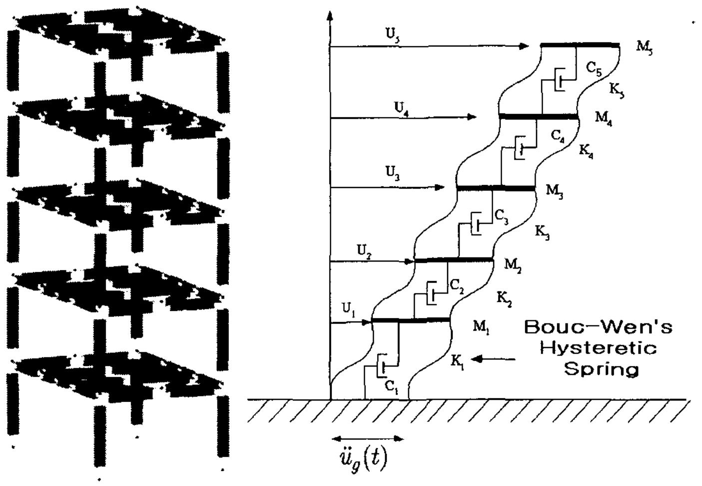


Fig. 3.2 Five story building structure model

Table 2.1 The exact values for simulations and identified values

Story	C			K		
	C_{true}	C_{guess}	$C_{identified}$	K_{true}	K_{guess}	$K_{identified}$
1	0.2	0.1	0.20011	10	20	9.9949
2	0.2	0.1	0.1982	10	30	10.004
3	0.2	0.1	0.20131	10	20	9.9957
4	0.2	0.1	0.20015	10	30	10.005
5	0.2	0.1	0.19839	10	20	9.9959
Story	β			γ		
	β_{true}	β_{guess}	$\beta_{identified}$	γ_{true}	γ_{guess}	$\gamma_{identified}$
1	0	0	0.0999	0	0	0.20

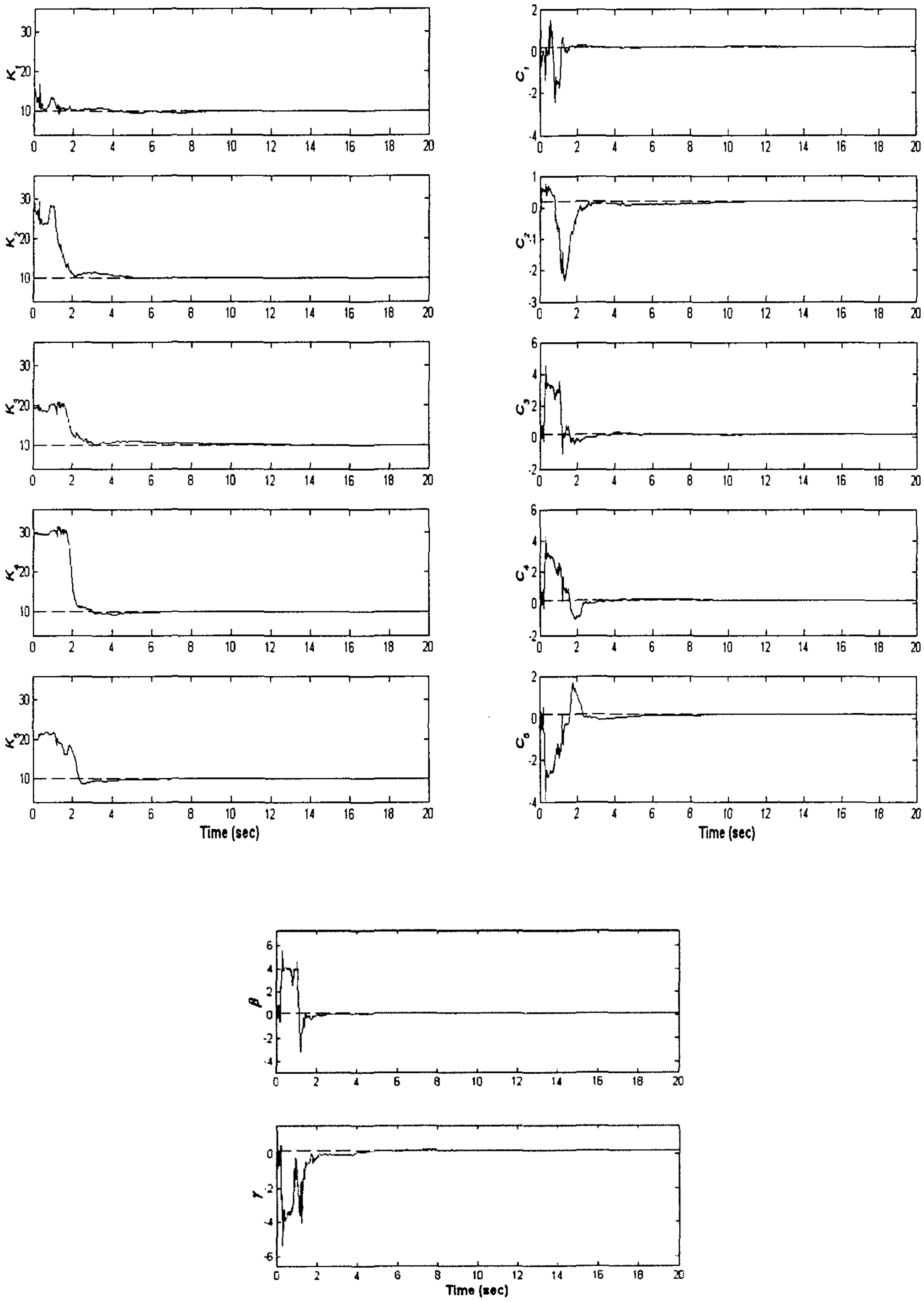
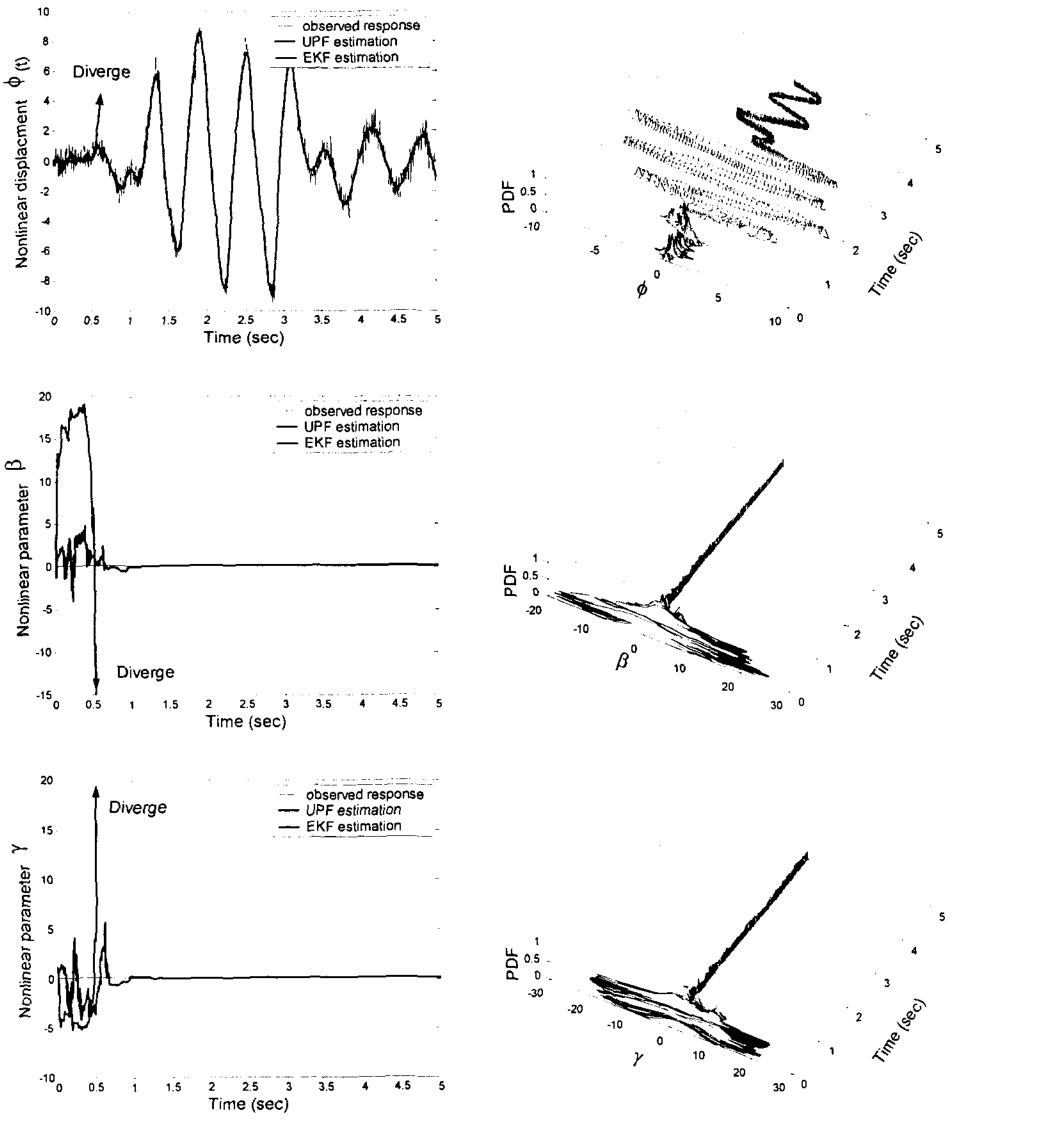


Fig. 3.3 Estimation histories of system parameters (--: true values of system parameters)

3.3 Identification of nonlinear SDOF system

The performance of the UPF and the EKF are evaluated on nonlinear identification problem with Bouc-Wen's hysteretic spring mode mentioned in previous section (see Eqn. (3.2)). Fig. 3.4 compares the estimates generated from a single run of EKF and UPF. The extend Kalman filter shows divergence phenomena whereas unscented particle filter (UPF) shows good estimation capability.



a) states and parameter estimations

b) probability density functions estimated by unscented particle filter

Fig. 3.4 Estimations by the extended Kalman filter and unscented particle filter

4. CONCLUSION

In this study, the recently developed unscented particle filter as an alternative to Extended Kalman filter has been applied to the identification of nonlinear structural dynamic systems. The results from a series of numerical simulation studies are summarized below.

1. Statistical inference for nonlinear dynamic system can be carried out by more generic approach using particle filter than linear approximated schemes such as the extended Kalman filter.

2. Unscented particle filter may show better performance than generic particle filters by using importance proposal distribution estimated from the UKF.

3. For real world applications of statistical filtering technique, the accurate mathematical model on system dynamics including damage phenomena is demanded.

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