Edge preserving method using mean curvature diffusion in aerial imagery

Chul-Soo Ye*, Kyoung-Ok Kim*, Young-Kyu Yang*, Kwae-Hi Lee**

*Spatial Imagery Information Research Team

Spatial Information Technology Center, ETRI- Computer & Software Research Laboratory

161 Gajeong-dong, Yuseong-gu, Daejeon, 305-600, Korea

E-mail : {csye, kokim}@etri.re.kr

**Dept. of Electronic Engineering, Sogang University

1 Shinsu-Dong, Mapo-Gu, Seoul, Korea, 121-742

Email : khlee@sognag.ac.kr

Abstract

Mean curvature diffusion (MCD) is a selective smoothing technique that promotes smoothing within a region instead of smoothing across boundaries. By using mean curvature diffusion, noise is eliminated and edges are preserved. In this paper, we propose methods of automatic parameter selection and implementation for the MCD model coupled to min/max flow. The algorithm has been applied to high resolution aerial images and the results show that noise is eliminated and edges are preserved after removal of noise.

Key words: mean curvature diffusion, edge preserving filtering.

1. Introduction

El-Fallah and Gary E. Ford represented the image as a surface and proved that setting the inhomogeneous diffusion coefficient equal to the inverse of the magnitude of the surface normal results in surface evolving speed that is proportional to the mean curvature of the image surface (El-Fallah and Ford, 1997). This model has the advantage of having the mean curvature diffusion (MCD) render invariant magnitude, thereby preserving structure and locality. By coupling the min/max flow to the surface diffusion model controlled by the surface's normal magnitude and smoothness, noise is eliminated and thin edges are preserved more efficiently. In this paper, we propose methods of automatic parameter selection and implementation for the MCD model coupled to min/max flow.

2. MCD model coupled to min/max Flow

Let us consider the image as a 3-D surface, that is $z = I(x, y)$. The following function

$$g(x,y,z) = z - I(x,y) \tag{1}$$

characterizes the image, while the surface S is defined implicitly by

$$S : g(x, y, z) = 0. \tag{2}$$

The gradient $|\nabla g|$ represents the nonvanishing normal vector field over the entire surface (El-Fallah and Ford, 1994) having the magnitude

$$|\nabla g| = \sqrt{1 + I_x^2 + I_y^2} = \sqrt{1 + |\nabla I|^2}. \tag{3}$$

The diffusion of g is modeled by

$$\frac{\partial g}{\partial t} = \nabla \cdot (C \nabla g). \tag{4}$$

In the MCD model coupled to min/max flow, the diffusion coefficient is the inverse of the surface gradient magnitude and the quadratic variation Q given by equation (6)

$$C = \frac{1}{|\nabla g|} = \frac{1}{\sqrt{1 + A^2(t)(|\nabla I|^2 + Q)}} \tag{5}$$

where A is a function of the change in the surface area.

$$Q = I_{xx}^2 + 2I_{xy}^2 + I_{yy}^2. \tag{6}$$

Note that the quadratic variation of the surface is in general large at the thin edges, hence taking the values of function Q of I into consideration will help reduce fast diffusion at the thin edges.

By combining the min/max flow proposed by Malladi and Sethian (Malladi and Sethian, 1996), we modified the MCD model with the diffusion coefficient as follows:

$$\frac{\partial g}{\partial t} = F(H) = \begin{cases} 2H & \text{if } |\nabla I| < T_G \\ \left.\begin{matrix} \max(2H, 0) & \text{if } Average(x,y) < T_M \\ \min(2H, 0) & \text{otherwise} \end{matrix}\right\} & \text{otherwise} \end{cases} \tag{7}$$

where T_G is a threshold based on the local gradient magnitude, $Average(x, y)$ is the average of all pixel grey values in a small window centered at (x,y) and T_M is the average value of the intensity obtained in the direction perpendicular to the gradient (Ye and Lee, 2001).

3. Parameter selection

Two parameters are required to be fixed, namely T_G and iteration number n. The parameter T_G allows to control the amount of smoothing effect and is fixed using the noise estimator described by Canny (Canny, 1986), i.e., a histogram of the absolute values of the gradient throughout the image is computed and T_G is set equal to the 90% value of its integral.

To determine iteration number automatically, we make use of an assumption: a fraction of the image is composed of homogeneous regions. The image is subdivided into nonoverlapping blocks of the same size. We define a local homogeneity measure as the average of gradient magnitudes in each block and then measured it in each block. The blocks are sorted in order of decreasing homogeneity. We use only a fraction of the blocks with high homogeneity measure to represent the homogeneous regions of the image.

The noise in homogeneous regions will be reduced after some iterations and diffusion coefficient C gradually reaches 1. Denote by N_1 the number of all pixels in homogeneous regions in the image. We use diffusion coefficient C to test whether the neighbourhood of a pixel is smooth enough. At a pixel P, if $C < T_G$, then the neighbourhood is considered smooth; otherwise not smooth enough. The threshold T_G is obtained using the gradient and the quadratic variation. If the difference between the intensity of center pixel and that of its neighbourhood pixels in 3×3 image region is 2, we can compute the threshold T_G as follows:

$$|\nabla I|^2 = I_x^2 + I_y^2 = 2^2 + 2^2 = 8,$$

$$Q = I_{xx}^2 + 2I_{xy}^2 + I_{yy}^2 = (-4)^2 + 2 \times 0 + (-4)^2 = 32,$$

$$T_S = \frac{1}{|\nabla g|} = \frac{1}{\sqrt{1 + |\nabla I|^2 + Q}} = \frac{1}{\sqrt{1 + 8 + 32}} = \frac{1}{\sqrt{41}}.$$

The ratio $r(n) = N_2(n)/N_1(n)$, where N_2 is the number of pixels that meet $C < T_G$, gradually increases with iteration number. If the change in slope of $r(n)$, $D(n)$ given by equation (8), is small enough, diffusion stops.

$$D(n) = \big|\, |r(n) - r(n-k)| - |r(n-k) - r(n-2k)| \,\big| \tag{8}$$

4. Implementation

Mean curvature diffusion model in the case of 2D discrete signal is modeled by the equation

$$\frac{\partial I(x,y,t)}{\partial t} = \frac{\partial}{\partial x}\left[c(x,y,t)\frac{\partial}{\partial x}I(x,y,t)\right] + \frac{\partial}{\partial y}\left[c(x,y,t)\frac{\partial}{\partial y}I(x,y,t)\right] \quad (9)$$

where $I(x,y,t)$ is the image being diffused. A Taylor series expansion of I around $t = t_0$ is

$$I(x,y,t_0+\Delta t) = I(x,y,t_0) + \Delta t\frac{\partial I(x,y,t_0)}{\partial t} + \Lambda \quad (10)$$

Approximating this expansion by ignoring the higher order terms and substituting for the derivative using (9), we obtain

$$I(x,y,t_0+\Delta t) = I(x,y,t_0) + \Delta t\left\{\frac{\partial}{\partial x}\left[c(x,y,t)\frac{\partial}{\partial x}I(x,y,t)\right] + \frac{\partial}{\partial y}\left[c(x,y,t)\frac{\partial}{\partial y}I(x,y,t)\right]\right\} \quad (11)$$

Consider the center pixel (x_i, y_i) and the four adjacent pixels (x_{i+1}, y_i), (x_{i-1}, y_i), (x_i, y_{i+1}) and (x_i, y_{i-1}). Finally we obtain

$$\begin{aligned} I(x_i,y_i,t_0+\Delta t) &\approx I(x_i,y_i,t_0) - \frac{1}{2}\Delta t I(x_i,y_i,t_0)\cdot \\ &\{c(x_{i+1},y_i,t_0)+c(x_{i-1},y_i,t_0)+c(x_i,y_{i+1},t_0)+c(x_i,y_{i-1},t_0)\} \\ &+\frac{1}{2}\Delta t\{c(x_{i-1},y_i,t_0)I(x_{i-1},y_i,t_0)+c(x_{i+1},y_i,t_0)I(x_{i+1},y_i,t_0) \\ &+c(x_i,y_{i-1},t_0)I(x_i,y_{i-1},t_0)+c(x_i,y_{i+1},t_0)I(x_i,y_{i+1},t_0)\} \end{aligned} \quad (12)$$

From equation (12) the filter kernel $M^{t_0}_{x_i,y_i}$ to be used at time t is

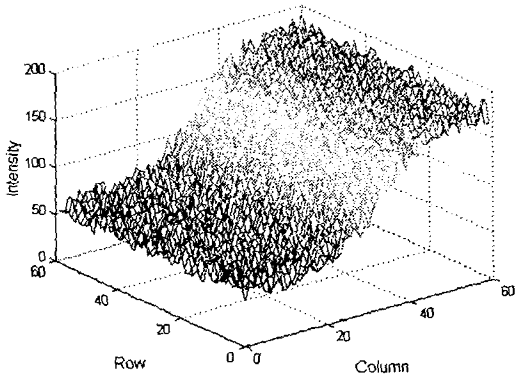
$$M^{t_0}_{x_i,y_i} = \frac{\Delta t}{2}\begin{bmatrix} 0 & c_2(t_0) & 0 \\ c_3(t_0) & \left(\frac{2}{\Delta t} - \sum_{i=1}^{4} c_i(t_0)\right) & c_1(t_0) \\ 0 & c_4(t_0) & 0 \end{bmatrix} \quad (13)$$

where

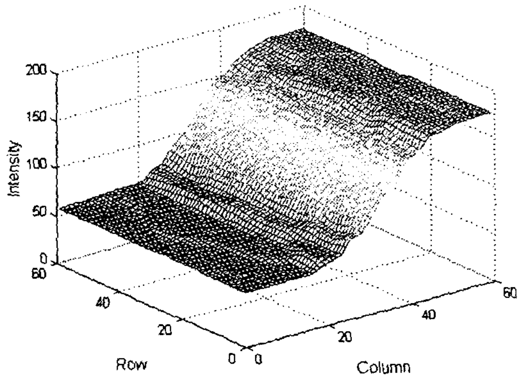
$c_1 = c(x_i, y_{i+1}, t_0)$, $c_2 = c(x_{i-1}, y_i, t_0)$, $c_3 = c(x_i, y_{i-1}, t_0)$, $c_4 = c(x_{i+1}, y_i, t_0)$.

5. Experimental results

The mean curvature diffusion algorithm was applied to a noisy edge image. The noisy edge in Fig. 1 is a rough surface of high mean curvature. The mean curvature diffusion algorithm reduces mean curvature on small patches by spatial averaging.



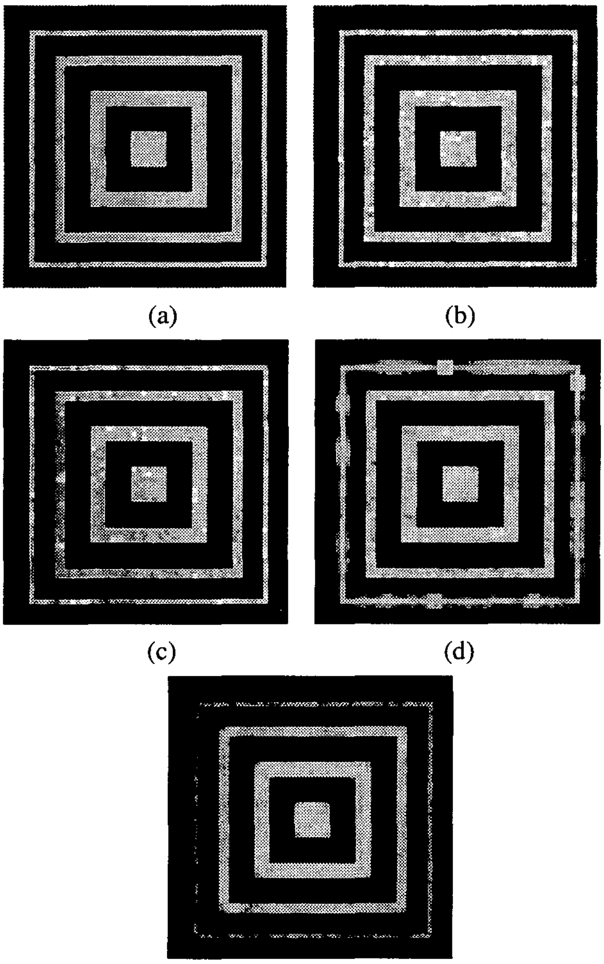
(a)



(b)

Fig. 1. (a) 3-d display of noisy edge image (b) the result of applying mean curvature diffusion to the noisy edge image.

To show the performance of the proposed algorithm we chose the thin step edge image (Fig. 2). We added 8% uniform noise to the original image. Fig. 2 compares the results of filtering using the proposed method, anisotropic diffusion proposed by Perona and Malik (Perona and Malik, 1990), and conventional mean curvature diffusion. We see that using the proposed method, thin edges were well preserved. Table 1 shows the PSNR comparison of all the methods tested. This clearly indicates that the proposed method is superior to other algorithms in preserving thin edges.



(a) (b) (c) (d) (e)

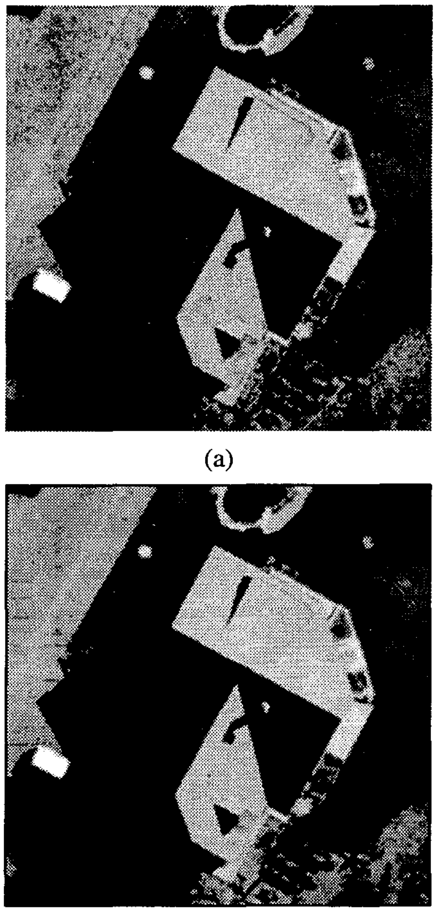
Fig. 2. (a) Original thin edge image (b) noisy image with uniform noise of 8% (c) anisotropic diffusion proposed by Perona and Malik (d) conventional mean curvature diffusion (e) proposed method.

Table 1. Quantitative performance of each algorithm for thin edge image.

Algorithm	PSNR
Perona-Malik method	23.7
Conventional mean curvature diffusion	16.8
Proposed method	24.3

We applied the proposed algorithm to aerial image as shown in Fig. 3. We see that using the proposed method, edges are well preserved and the small variations in homogeneous regions are well removed. The image was subdivided into nonoverlapping blocks of the same size 8×8 pixels. We assumed 40 percent of the image was composed of homogeneous regions. We stopped the diffusion when $D(n) < 0.0001$.

Fig. 4 and Fig. 5 show the edgeness threshold T_G curve and the curve of $r(n)$ for aerial image in Fig. 3. After some iterations, noises were removed and edgeness threshold T_G and the curve $r(n)$ reached a constant value.



(a) (b)

Fig. 3. (a) Original aerial image (b) the result of applying mean curvature diffusion to the original image.

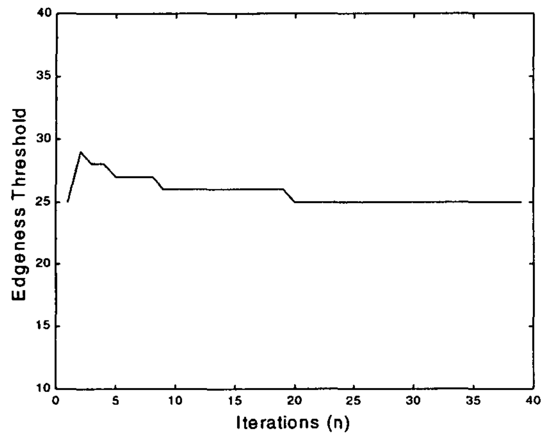


Fig. 4. Edgeness threshold T_G curve for aerial image in Fig. 3.

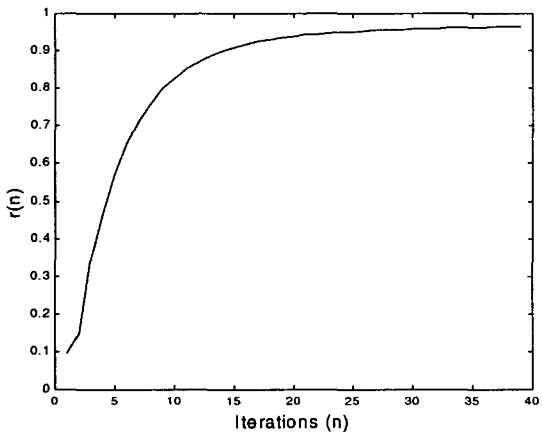


Fig. 5. The curve of $r(n)$ as a function of n.

6. Conclusions

By coupling the min/max flow to the surface diffusion model controlled by the normal surface's magnitude and smoothness, noise is eliminated and thin edges are preserved more efficiently. In this paper, we propose methods of automatic parameter selection and implementation for the MCD model coupled to min/max flow. To determine iteration number automatically, we defined a local homogeneity measure as the average of gradient magnitudes in each block and used only a fraction of the blocks with high homogeneity measure. We used diffusion coefficient to test whether the neighbourhood of a pixel is smooth enough. The algorithm has been applied to high resolution aerial images and the results show that noise is eliminated and edges are preserved after removal of noise.

Reference

Canny, J., 1986, A computational approach to edge detection, *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 8, pp. 679-698.

C.S. Ye and K. H. Lee, Anisotropic diffusion for building segmentation form aerial imagery, Proceedings of International Symposium on Remote Sensing, EMSEA and KSRS, pp. 599-604, Seogwipo, Korea, Oct. 31-Nov. 2, 2001.

El-Fallah, A.I. and Gary E. Ford, 1997, Mean curvature evolution and surface area scaling in image filtering, *IEEE Trans. Image Processing*, 6(5): 750-753.

El-Fallah, A.I. and Gary E. Ford, 1994, The evolution of mean curvature in image filtering, in Proc. *IEEE Int. Conf. Image Processing,* vol. 1, pp. 298-302.

Malladi, R. and J. Sethian, 1996, Image processing: Flows under min/max curvature and mean curvature, *Graphic. Models Image Processing,* 58(2): 127-141.

Perona, P. and J. Malik, 1990, Scale-space and edge detection using anisotropic diffusion, *IEEE Trans. Pattern Anal. Machine Intell.*, 12(7): 629-639.