Representing Topological Relationships for 3-Dimensional Spatial Features

Seong-Ho Lee, Kyong-Ho Kim, Sung-Soo Kim, and Kyung-Ok Kim
Spatial Information Technology Center,
ETRI-Computer & Software Research Laboratory
161 Gajeong-Dong, Yuseong-Gu, Daejon, 305-350, KOREA
Email: {sholee, kkh, sungsoo, kokim}@etri.re.kr

Abstract

One of the fundamental components important to the analysis of spatial objects is to represent topological relationships between spatial features. Users of geographic information systems retrieve a lot of objects from spatial database and analyze their condition by means of topological relationships. The existing methods that represent these relationships have the disadvantage that they have limited information in $\Re^2$. In this paper, we represent and define the topological relationships between 3-dimensional spatial objects using the several representing methods of 2-dimensional features. We use the diverse representing methods, which include the 4-, 9-intersection, dimension extended and calculus-based method. Furthermore, we discuss OGC's topological relationships and operators for 3-dimensional spatial data.

1. Introduction

The spatial queries like "Does this highway *cross* that city?" or "Find all public offices that are *within* 1km of this city hall" are general and widespread queries in geographic information systems(GISs). Topological properties of spatial objects commonly used in GIS are the primary information which users of such systems need to deal with objects[7], that is, the topological relationships between geographic objects can provide the users with the various spatial analysis services.

Spatial query languages need formal definitions to describe algorithms to decide relationships. Many GIS applications define their own spatial query languages. Most of these languages include some topological operators among a set of spatial operators. And furthermore, these languages do not pay enough attention to the definition and representation of topological issues.

Existing GISs access and handle 2-dimensional spatial data models such as point, line, and area that consist of a set of coordinates. Development of remote sensing, computer graphics, and image processing technologies makes possible the acquisition of complex spatial data including height of buildings. Due to this reason users of the GIS want to use spatial data in 3-dimensional space as well as 2-dimensional space. This request needs to extend and add spatial abstract data models so that they can apply to each process among acquiring, processing, analyzing, and service providing 3-dimensional spatial information.

In this paper, we first describe briefly the existing representing methods of topological relationships including the 4-intersection method(4IM)[2], 9-intersection method(9IM)[5], dimension extended method(DEM)[3], and calculus-based method(CBM)[3]. We then define and represent topological relationships between 3-dimensional spatial objects using above diverse methods.

The rest of the paper is organized as follows: Section 2 contains general definitions. Section 3 recalls the various representing methods. In section 4, we describe some topological relationships among 3-dimensional spatial data and several examples of usage of them. The final section summarizes the results.

2. General Definitions

In 2-dimensional space, each node that consists of general geometries has two values(x, y). The subjects of the relationships are the *simple points*, *lines*, and *areas*. From now, in 3-dimension, each object has a height value(z)and an additional subject is *simple solid*. This solid features are only connected solids without a hole or concave. All kinds of geometric objects are closed sets and connected.

The boundary(∂) and the interior($^{\circ}$) of features are used in [2] and the exterior($^{-}$) is used in [5] for describing the topological relationships. In addition to these, the dimension(*dim*) of objects is defined in [3]. The boundary, interior, exterior, dimension of a feature λ are respectively denoted by $\partial\lambda$, λ°, λ^{-}, and $dim(\lambda)$. It is defined for each of the feature types as follows :

(1) ∂P: the boundary of a point feature P is empty;

(2) ∂L: the boundary of a line feature L is a set containing the two end-points of L;

(3) ∂A : the boundary of an area feature A is a circular line consisting of all the accumulation points of the area;

(4) ∂S: the boundary of a solid feature S is a circular area consisting of all the accumulation points of the solid;

(5) $\lambda^{\circ} = \lambda - \partial\lambda$ (6) $\lambda^{-} = \Re^3 - \lambda$

(7) $dim(\partial L) = 0$, $dim(L^{\circ}) = 1$, $dim(\partial A) = 1$,
$dim(A^{\circ}) = 2$, $dim(\partial S) = 2$, $dim(S^{\circ}) = 3$.

3. Representing Methods of Topological Relationships

Many methods that classify topological relationships have been studied in the last decade. There are the 4-intersection method, the 9-intersection method, the dimension extended method, the calculus-based method, and etc. These methods support descriptions of space in terms of the 2-dimensional spatial objects primitives and the spatial relationships between such primitives. First of all, we survey these methods in this section.

3.1 The 4-intersection method

The 4IM was described for classifying topological relationships between one-dimensional intervals of $\Re^1$ in [2] and this method was adopted between area features of $\Re^2$ in [5]. There are six major groups of binary relationships: point/point, point/line, point/area, line/line, line/area, and area/area. In the 4IM, the relationships are based on the intersection of the boundaries and interior of two features λ_1, λ_2. Each intersection may be empty(ϕ) or non-empty($\neg\phi$). Each case is represented by a matrix(2×2) of values.

$$M = \begin{pmatrix} \partial\lambda_1 \cap \partial\lambda_2 & \partial\lambda_1 \cap \lambda_2^{\circ} \\ \lambda_1^{\circ} \cap \partial\lambda_2 & \lambda_1^{\circ} \cap \lambda_2^{\circ} \end{pmatrix}$$

Table 1. The example of area(λ_1)/line(λ_2) situations

Cases					...
4IM Matrix	$\begin{pmatrix} \phi & \phi \\ \phi & \phi \end{pmatrix}$	$\begin{pmatrix} \phi & \neg\phi \\ \phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} \phi & \phi \\ \neg\phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} \phi & \neg\phi \\ \phi & \phi \end{pmatrix}$	...
9IM Matrix	$\begin{pmatrix} \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} \phi & \neg\phi & \neg\phi \\ \phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} \phi & \neg\phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{pmatrix}$	...

3.2 The 9-intersection method

The 9IM is an extended method of 4IM. This is based on the exterior of features, besides boundary and interior[3]. Therefore, the following matrix of nine sets represents each case.

$$M = \begin{pmatrix} \partial\lambda_1 \cap \partial\lambda_2 & \partial\lambda_1 \cap \lambda_2^{\circ} & \partial\lambda_1 \cap \lambda_2^{-} \\ \lambda_1^{\circ} \cap \partial\lambda_2 & \lambda_1^{\circ} \cap \lambda_2^{\circ} & \lambda_1^{\circ} \cap \lambda_2^{-} \\ \lambda_1^{-} \cap \partial\lambda_2 & \lambda_1^{-} \cap \lambda_2^{\circ} & \lambda_1^{-} \cap \lambda_2^{-} \end{pmatrix}$$

For example, table 1 gives a pictorial representation of several relationships and shows the matrices applied to 4IM and 9IM.

3.3 The dimension extended method

This method considers the dimension of the intersections, that is, the DEM calculates the dimension values about the result from 4IM. The dimension of the four intersection sets assume the values: -, 0, 1, 2. This method can be represented such as:

$$M = \begin{pmatrix} dim(\partial\lambda_1 \cap \partial\lambda_2) & dim(\partial\lambda_1 \cap \lambda_2^\circ) \\ dim(\lambda_1^\circ \cap \partial\lambda_2) & dim(\lambda_1^\circ \cap \lambda_2^\circ) \end{pmatrix}$$

Table 2 shows a matrix of dimension sets between area/area, area/line, and line/line groups as result of the DEM.

Table 2. DEM for several relationship groups

groups	Representing relationships
area/area	$\begin{pmatrix} dim(\partial A_1 \cap \partial A_2) & dim(\partial A_1 \cap A_2^\circ) \\ dim(A_1^\circ \cap \partial A_2) & dim(A_1^\circ \cap A_2^\circ) \end{pmatrix} = \begin{pmatrix} \{-,0,1\} & \{-,1\} \\ \{-,1\} & \{-,2\} \end{pmatrix}$
area/line	$\begin{pmatrix} dim(\partial A_1 \cap \partial L_2) & dim(\partial L_1 \cap L_2^\circ) \\ dim(A_1^\circ \cap \partial L_2) & dim(L_1^\circ \cap L_2^\circ) \end{pmatrix} = \begin{pmatrix} \{-,0\} & \{-,0,1\} \\ \{-,0\} & \{-,1\} \end{pmatrix}$
line/line	$\begin{pmatrix} dim(\partial L_1 \cap \partial L_2) & dim(\partial L_1 \cap L_2^\circ) \\ dim(L_1^\circ \cap \partial L_2) & dim(L_1^\circ \cap L_2^\circ) \end{pmatrix} = \begin{pmatrix} \{-,0\} & \{-,0\} \\ \{-,0\} & \{-,0,1\} \end{pmatrix}$

3.4 The calculus-based method

The CBM[5] represented the topological relationships among the spatial objects using formal definitions for five relationships and for boundary operators. In the following, an Object-Calculus fact involving a topological relationship is on the left side of the equivalence sign and its definition in the form of a point-set expression is given on the right side.

Definition 1. The *touch* relationship (it applies to every group except the point/point case) :

$$\langle \lambda_1, touch, \lambda_2 \rangle \Leftrightarrow (\lambda_1^\circ \cap \lambda_2^\circ = \phi) \wedge (\lambda_1 \cap \lambda_2 \neq \phi)$$

Definition 2. The *in* relationships (it applies to every group) :

$$\langle \lambda_1, in, \lambda_2 \rangle \Leftrightarrow (\lambda_1 \cap \lambda_2 = \lambda_1) \wedge (\lambda_1^\circ \cap \lambda_2^\circ \neq \phi)$$

Definition 3. The *cross* relationship (it applies to line/line and line/area groups)

$$\langle \lambda_1, cross, \lambda_2 \rangle \Leftrightarrow (dim(\lambda_1^\circ \cap \lambda_2^\circ) = max(dim(\lambda_1^\circ), dim(\lambda_2^\circ)) - 1) \wedge (\lambda_1 \cap \lambda_2 \neq \lambda_1) \wedge (\lambda_1 \cap \lambda_2 \neq \lambda_2)$$

Definition 4. The *overlap* relationship (it applies to area/area and line/line groups)

$$\langle \lambda_1, overlap, \lambda_2 \rangle \Leftrightarrow (dim(\lambda_1^\circ) = dim(\lambda_2^\circ) = dim(\lambda_1^\circ \cap \lambda_2^\circ)) \wedge (\lambda_1 \cap \lambda_2 \neq \lambda_1) \wedge (\lambda_1 \cap \lambda_2 \neq \lambda_2)$$

Definition 5. The *disjoint* relationship (it applies to every group)

$$\langle \lambda_1, disjoint, \lambda_2 \rangle \Leftrightarrow \lambda_1 \cap \lambda_2 = \phi$$

Definition 6. The boundary operator *b* for an area feature *A*: The pair (*b*, *A*) returns the circular line ∂A

Definition 7. The boundary operators *f*, *t* for a line feature *L*: The pairs (*L*, *f*) and (*L*, *t*) return the two point features corresponding to the set ∂L.

3.5 The DEM plus the 9IM

In [7], the new method, DE+9IM, takes into account the dimension of the intersection of boundaries, interiors, and exteriors between two spatial objects. A case of such a method will be indicated by a matrix:

$$M = \begin{pmatrix} dim(\partial\lambda_1 \cap \partial\lambda_2) & dim(\partial\lambda_1 \cap \lambda_2^\circ) & dim(\partial\lambda_1 \cap \lambda_2^-) \\ dim(\lambda_1^\circ \cap \partial\lambda_2) & dim(\lambda_1^\circ \cap \lambda_2^\circ) & dim(\lambda_1^\circ \cap \lambda_2^-) \\ dim(\lambda_1^- \cap \partial\lambda_2) & dim(\lambda_1^- \cap \lambda_2^\circ) & dim(\lambda_1^- \cap \lambda_2^-) \end{pmatrix}$$

In this method, the dimension of the 9 intersection sets can assume the values {-, 0, 1, 2} since the available features are point, line, and area in $\Re^2$.

4. Representing Topological Relationships for 3D Spatial Features

OGC[7] named the operators for topological relationships on Geometry class : *Equal*, *Disjoint*, *Intersects*, *Touches*, *Crosses*, *Within(in)*, *Contains*, *Overlaps*. These relationships were defined under the two-dimensional space. In this section, we give examples and represent above several relationships between 3-dimensional spatial features.

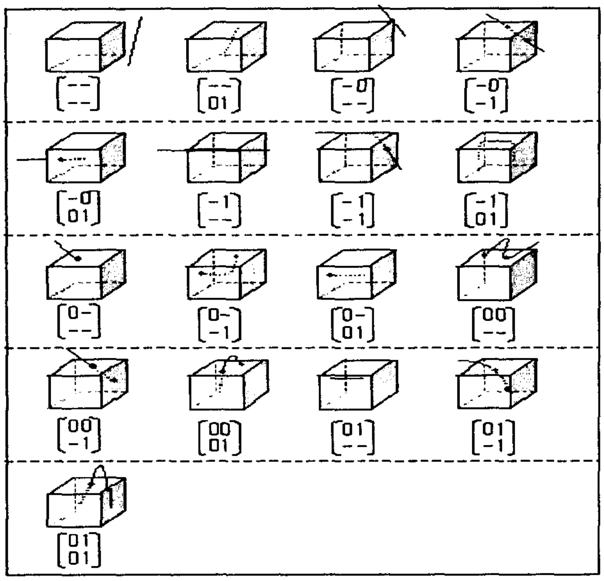


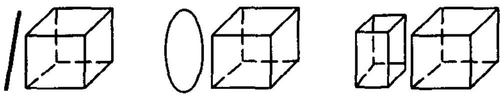
Figure 1. The solid/line and solid/solid cases in $\Re^3$

Figure 1 illustrates the real cases of the line/solid and solid/solid groups. In each case, the four values represent the dimension of the four intersection sets. The left of figure 1(a) shows the relationships between solid(λ_1) and line(λ_2), and the right(b) represents the examples between solid and solid.

In the following examples, the values in order pairs are results of intersection between line/solid, area/solid, and solid/solid groups respectively.

4.1 Disjoint

This relationship tests if the geometries are *disjoint.*

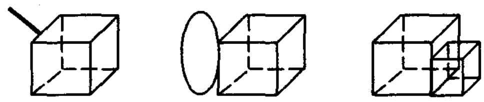


4IM	9IM	DEM	DE+9IM
$\begin{pmatrix} \phi & \phi \\ \phi & \phi \end{pmatrix}$	$\begin{pmatrix} \phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} - & - \\ - & - \end{pmatrix}$	$\begin{pmatrix} - & - & (0,1,2) \\ - & - & (1,2,3) \\ 2 & 3 & 3 \end{pmatrix}$

$\langle \lambda, disjoint, s \rangle \Leftrightarrow \lambda \cap s = \phi, \lambda \in P, L, A, S$

4.2 Touches

This relationship tests if the geometries *touch* each other. As the below figures, the point of contact between two objects can be a point, line, or area. The point/point group is an exception in this case.

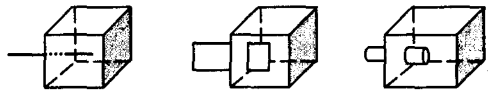


4IM	9IM	DEM	DE+9IM
$\begin{pmatrix} \neg\phi & \phi \\ \phi & \phi \end{pmatrix}$	$\begin{pmatrix} \neg\phi & \phi & \neg\phi \\ \phi & \phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} (0,1,2) & - \\ - & - \end{pmatrix}$	$\begin{pmatrix} (0,1,2) & - & (0,1,2) \\ - & - & (-,2,3) \\ 2 & 3 & 3 \end{pmatrix}$

$\langle \lambda, touch, s \rangle \Leftrightarrow (\lambda^\circ \cap s^\circ = \phi) \wedge (\lambda \cap s \neq \phi), \lambda \in P, L, A, S$

4.3 Crosses

The *Cross* relationship tests if the geometries *cross* each other.



4IM	9IM	DEM	DE+9IM
$\begin{pmatrix} \phi & \neg\phi \\ \neg\phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} \phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} - & (0,1,2) \\ (0,1,2) & (,1,2,3) \end{pmatrix}$	$\begin{pmatrix} - & (0,1,2) & (0,1,2) \\ (0,1,2) & (1,2,3) & (1,2,3) \\ 2 & 3 & 3 \end{pmatrix}$

Case $\lambda \in L$,or S, $\lambda \in S$

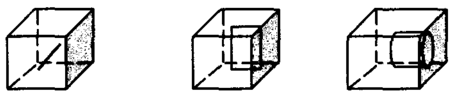
$\langle \lambda, cross, s \rangle \Leftrightarrow (dim(\lambda^\circ \cap s^\circ) < max(dim(\lambda^\circ), dim(s^\circ))) \wedge (\lambda \cap s \neq \lambda) \wedge (\lambda \cap s \neq s)$

Case λ, s$\in S$

$\langle \lambda, cross, s \rangle \Leftrightarrow (dim(\lambda^\circ \cap s^\circ) = 3) \wedge (\lambda \cap s \neq \lambda) \wedge (\lambda \cap s \neq s)$

4.4 Within(In)

This relationship tests if the given geometry is *within* another given geometry.

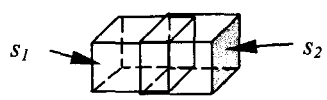


4IM	9IM	DEM	DE+9IM
$\begin{pmatrix} \phi & \neg\phi \\ \phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} \phi & \neg\phi & \phi \\ \phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} - & (0,1,2) \\ - & (1,2,3) \end{pmatrix}$	$\begin{pmatrix} - & (0,1,2) & - \\ - & (1,2,3) & - \\ 2 & 3 & 3 \end{pmatrix}$

$\langle \lambda, in, s \rangle \Leftrightarrow (\lambda \cap s = \lambda) \wedge (\lambda^\circ \cap s^\circ \neq \phi), \lambda \in P, L, A, S$

4.5 Overlaps

The *overlap* relationship tests if the geometry *overlaps* another geometries.



4IM	9IM	DEM	DE+9IM
$\begin{pmatrix} \neg\phi & \neg\phi \\ \neg\phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \\ \neg\phi & \neg\phi & \neg\phi \end{pmatrix}$	$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 3 & 3 \\ 2 & 3 & 3 \end{pmatrix}$

Case $s_1, s_2 \in S$

$\langle s_1, overlap, s_2 \rangle \Leftrightarrow (dim(s_1{}^\circ) = dim(s_2{}^\circ) = dim(s_1{}^\circ \cap s_2{}^\circ)) \wedge (s_1 \cap s_2 \neq s_1) \wedge (s_1 \cap s_2 \neq s_2)$

4.6 Contains

The *contain* relationship tests if the given geometry *contains* another given geometry. This is the converse of *within* relationship. Then this can be denoted as:

$$\langle \lambda_1, contains, \lambda_2 \rangle \Leftrightarrow \langle \lambda_2, within, \lambda_1 \rangle$$

4.7 Intersects

The *intersect* relationship tests if the geometries *intersect.* This is the opposite of *disjoint* relationship.

$$\langle \lambda_1, intersect, \lambda_2 \rangle \Leftrightarrow ! \langle \lambda_1, disjoint, \lambda_2 \rangle$$

5. Conclusions

There are diverse methods for representing spatial topological relationships in spatial queries. In this paper, we surveyed several methods for classifying topological relationships. These subjects were spatial objects on 2-dimensional space. Since it is necessary to define the methods for high-dimensions and users are interested in analyzing 3-dimensional spatial data, we showed topological relationships for that. Furthermore, we defined topological operators for 3-dimensional features through existing various methods that are the 4-intersection method, the 9-intersection method, the dimension extended method, the calculus-based method, the DE+9IM, and point-set expression.

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