Research on Digital Complex-Correlator of Synthetic Aperture Radiometer: theory and simulation result

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Abstract—A new digital correlator for an airborne synthetic aperture radiometer was designed in order to replace the conventional analog correlator unit which will become very complicated while the number of channels is increasing. The digital correlator uses digital IQ demodulator instead of the intermediate frequency (IF) phase shifter to make the correlation processing performed digitally at base band instead of analogly at IF. This technique has been applied to the digital receiver in softradio. The down-converted IF signals from each pair of receiver channels become low rate base-band digital signals after under-sampled, Digitally Down-Converted (DDC), decimated and filtered by FIR filters. The digital signals are further processed by two digital multipliers (complex correlation), the products are integrated by the integrators and finally the outputs from the integrators compose of the real part and the imaginary part of a sample of the visibility function. This design is tested by comparing the results from digital correlators and that from analog correlators. They are agreed with each other very well. Due to the fact that the digital correlators are realized with the help of Analog-Digital Converter (ADC) chips and the FPGA technology, the realized volume, mass, power consumption and complexity turned out to be greatly reduced compared with that of the analog correlators. Simulations show that the resolution of ADC has an influence on the synthesized antenna patterns, but this can be neglected if more than 2bit is used.

Key word aperture synthesis, radiometer, under-sampling, digital down converter, digital correlation

Introduction

This paper investigates a new method to simplify the hardware complexity of cross-correlator unit of synthetic aperture radiometer system. The aperture synthesis technique is borrowed from radio astronomy where it has been used to create large effective apertures from a few small antennas. An interferometric radiometer measures the correlation between the analytic signals collected by different antennas [$v_1(t)$ and $v_2(t)$]. These correlations provide the samples of the so-called visibility function.

For remote sensing, Fourier synthesis is one of the most common aperture synthesis techniques, the forward motion of the satellite will be combined with 1-D cross track synthesis to produce the 2-D image[1,6]. It uses interferometers to measure the Fourier transform of the brightness temperature map of the scene, and the scene itself is reconstructed by inverting the sampled transform. Interferometer measurements are made by complex cross-correlating the output signals of two spatially separated small antennas that have a overlapping field of view(FOV). The correlation is called the visibility of the interference, and it is a function of the distance between the two antennas. The visibility function is the Fourier transform of the image where the transform variables are the distance between the antennas and the angle across the FOV. By sampling the interference along a variable baseline these systems achieve high resolution with very few small antennas. Resolution is only limited by the largest practical baseline. This technique has also been researched in China for several years.[2,7]

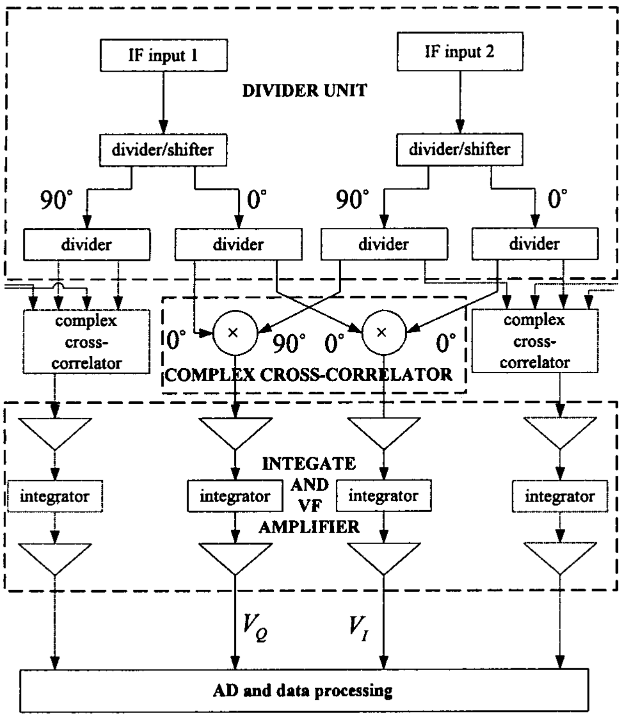

Fig.1 diagram of complex cross-correlation of input 1 and input 2

The correlation interferometer is also the basic building block of Fourier synthesis radio telescopes. Detailed analysis of it can be found in radio astronomy literature (Pawsey, 1955), (Kraus,1966). A complex-correlation interferometer suitable for synthesis in one dimension is shown in figure 1. The antennas share a common FOV which appears to the radiometer as a brightness temperature distribution $T_\Omega(\theta)$. For simplicity only the 1-D case is depicted, and the antennas are assumed to have identical and isotropic radiation patterns. Conventionally, the correlation electronics consists of the IF band pass filters of center frequency f_c and bandwidth B, one $90°$ phase shifter, two multipliers and two low pass filters.

With the help of modern IC and our new design of digital correlator, the realized volume, mass, power consumption and complexity can be greatly reduced compared with that of the analog correlators.

Analog complex correlation

The incident electric field to antenna 1 can be treated as a band limited process, ε_{RF} ,as follows,

$$v_1(t) = \int_{-\pi/2}^{\pi/2} \varepsilon_{RF}(\theta,t)d\theta \qquad (1)$$

The input at antenna number 2 comes from the same incident fields, but because of a difference in path length it follows that,

$$v_2(t) = \int_{-\pi/2}^{\pi/2} \varepsilon_{RF}(\theta,t-\Delta r/c)d\theta \qquad (2)$$

The quantity ε_{RF} can be represented as a single side band signal modulating a carrier frequency f_c, such that,

$$\varepsilon_{RF}(\theta,t) = \varepsilon(\theta,t)\cos[2\pi f_c t + \Phi(\theta,t)] \qquad (3)$$

where ε is the base-band signal with spectrum as same the shape as the RF signal, and $\Phi(\theta,t)$ is the random phase of ε_{RF} with respect to the carrier.

The $90°$ phase shifter , the multipliers, and the low pass filters of figure 1 form approximations to the cross-correlations of v_1 and v_2. After applying some trigonometric identities, and then noting that sum frequencies resulting from such identities will be rejected by the low pass filters, the conventional synthetic aperture radiometers cross-correlation results in that

$$V_I(\Delta r)=\int_{-\pi/2}^{\pi/2} T_\Omega(\theta)\cos(2\pi f_c \Delta r/c)d\theta$$
$$V_Q(\Delta r)=\int_{-\pi/2}^{\pi/2} T_\Omega(\theta)\sin(2\pi f_c \Delta r/c)d\theta \quad (4)$$

digital complex correlator theory

The digital complex cross-correlator has been used in Helsinki University of Technology Radiometer (HUTRAD) and HYDROSTAR without showing the calculations. Here we start with the general digital receiver calculations.

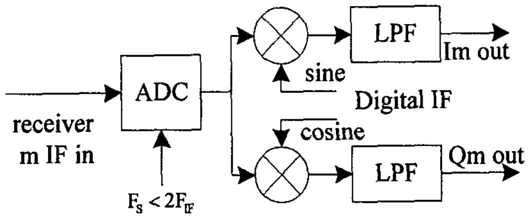

Fig.2 diagram of digital IQ demodulator

Here we use digital IQ demodulator instead of the intermediate frequency (IF) phase shifter to make the correlation processing performed digitally at base band instead of analogly at IF, its diagram is given in figure 2. After the under-sampling by ADC, the down-converted IF signal from receiver m is digitalized and its spectrum is down-converted to a lower digital intermediate frequency (DIF), the DIF is determined by both the IF center frequency f_c and the A/D sampling clock frequency f_s. Then the DIF signal is IQ demodulated, decimated and filtered by FIR filter to form base band digital in-phase and quadrature signal[5]. The digital base band IQ signals from each pair of receivers are further processed by a complex correlator (two digital multipliers), the outputs are integrated by the integrators and finally the outputs from the integrators compose of the real and the imaginary part of a sample of the visibility function.

The difference between analog correlation and digital correlation is, the analog correlation is analogly performed at IF, but the digital correlation is digitally performed at base band.

Except the ADC, the digital correlator, including DDC, decimation, FIR filters and complex correlators of all eight channels, is implemented entirely with a single FPGA chip.

The input RF signals to receiver 1 and receiver 2 are the same as equation 1 and 2, then send them to IQ demodulator respectively. The outputs from IQ demodulator 1 are

$$\begin{aligned}V_{1I}(\Delta r)&=\int_{-\pi/2}^{\pi/2}\varepsilon(\theta,t)\cos[2\pi f_{DIF}t+\Phi(\theta,t)]d\theta \\ &\cdot\cos(2\pi f_{DIF}t)\\ &=\int_{-\pi/2}^{\pi/2}\varepsilon(\theta,t)\cos[2\pi f_{DIF}t+\Phi(\theta,t)]\\ &\cdot\cos(2\pi f_{DIF}t)d\theta\\ &=\frac{1}{2}\int_{-\pi/2}^{\pi/2}\varepsilon(\theta,t)\cos[\Phi(\theta,t)]d\theta\end{aligned} \quad (5)$$

$$\begin{aligned}V_{1Q}(\Delta r)&=\int_{-\pi/2}^{\pi/2}\varepsilon(\theta,t)\cos[2\pi f_{DIF}t+\Phi(\theta,t)]d\theta \\ &\cdot\sin(2\pi f_{DIF}t)\\ &=\int_{-\pi/2}^{\pi/2}\varepsilon(\theta,t)\cos[2\pi f_{DIF}t+\Phi(\theta,t)]\\ &\cdot\sin(2\pi f_{DIF}t)d\theta\\ &=-\frac{1}{2}\int_{-\pi/2}^{\pi/2}\varepsilon(\theta,t)\sin[\Phi(\theta,t)]d\theta\end{aligned} \quad (6)$$

$$\begin{aligned}V_{2I}(\Delta r)&=\int_{-\pi/2}^{\pi/2}\varepsilon(\theta',t)\cos[2\pi f_c(t-\frac{\Delta r}{c})+\Phi(\theta',t)]d\theta' \\ &\cdot\cos(2\pi f_c t)\\ &=\int_{-\pi/2}^{\pi/2}\varepsilon(\theta',t)\cos[2\pi f_c(t-\frac{\Delta r}{c})+\Phi(\theta',t)]\\ &\cdot\cos(2\pi f_c t)d\theta'\\ &=\frac{1}{2}\int_{-\pi/2}^{\pi/2}\varepsilon(\theta',t)\cos[\Phi(\theta',t)-2\pi f_c\frac{\Delta r}{c}]d\theta'\end{aligned} \quad (7)$$

$$
\begin{aligned}
V_{2Q}(\Delta r) &= \int_{-\pi/2}^{\pi/2} \varepsilon(\theta',t)\cos[2\pi f_c(t-\frac{\Delta r}{c})+\Phi(\theta',t)]d\theta' \\
&\cdot \sin(2\pi f_c t) \\
&= \int_{-\pi/2}^{\pi/2} \varepsilon(\theta',t)\cos[2\pi f_c(t-\frac{\Delta r}{c})+\Phi(\theta',t)] \\
&\cdot \sin(2\pi f_c t)d\theta' \\
&= -\frac{1}{2}\int_{-\pi/2}^{\pi/2} \varepsilon(\theta',t)\sin[\Phi(\theta',t)-2\pi f_c\frac{\Delta r}{c}]d\theta' \qquad (8)
\end{aligned}
$$

Where $V_{1I}(\Delta r)$ and $V_{1Q}(\Delta r)$ are the in-phase and quadrature signals output from IQ demodulator 1, and $V_{2I}(\Delta r)$ and $V_{2Q}(\Delta r)$ are the outputs from IQ demodulator 2, f_{DIF} is the digital intermediate frequency.

The cross-correlation of v_1 and v_2 can be formed as

$$
\begin{aligned}
V'_I(\Delta r) &= < V_{1I}(\Delta r), V_{2I}(\Delta r) > \\
&= < \frac{1}{2}\int_{-\pi/2}^{\pi/2} \varepsilon(\theta,t)\cos[\Phi(\theta,t)]d\theta \\
&, \frac{1}{2}\int_{-\pi/2}^{\pi/2} \varepsilon(\theta',t)\cos[\Phi(\theta',t)-2\pi f_c\frac{\Delta r}{c}]d\theta' > \\
&= \frac{1}{2}\int_{-\pi/2}^{\pi/2}\int_{-\pi/2}^{\pi/2} \frac{1}{2} < \varepsilon(\theta,t), \varepsilon(\theta',t) > \\
&.\cos[\Phi(\theta,t)]\cos[\Phi(\theta',t)-2\pi f_c\frac{\Delta r}{c}]d\theta d\theta' \qquad (9)
\end{aligned}
$$

The quantity $\varepsilon(\theta,t)$ is statistically independent of $\varepsilon'(\theta,t)$ for $\theta \neq \theta'$, since they originate from independent thermal sources and are of zero mean. The expectation of their product will, therefore, vanish for $\theta \neq \theta'$, and the double integral will collapse into a single integral. Both the phase shifter and the time delay of Δr /c in equation (9) are presumed to be insignificant on the time scale of the low-pass signals ε and Φ. These time shifts will only be significant with respect to the carrier frequency. Then noting that the low pass filter will reject sum frequencies resulting from such identities.

Also, the quantity $\frac{1}{2} < \varepsilon(\theta,t), \varepsilon(\theta',t) >$ is recognized as the power in the RF signal and it is proportional to brightness temperature, $T_\Omega(\theta)$, gives the following result from equation (9).

$$
V'_I(\Delta r) = \frac{1}{4}\int_{-\pi/2}^{\pi/2} T_\Omega(\theta)\cos(2\pi f_c \Delta r/c)d\theta \qquad (10)
$$

Then after the same steps, gives

$$
V'_Q(\Delta r) = \frac{1}{4}\int_{-\pi/2}^{\pi/2} T_\Omega(\theta)\sin(2\pi f_c \Delta r/c)d\theta \qquad (11)
$$

Note that $V'_I(\Delta r) = V_I(\Delta r)$ and $V'_Q(\Delta r) = V_Q(\Delta r)$, it indicates that the results from analog cross-correlation are equal to the results from digital cross-correlation, but with different constant coefficients.

Digital cross-correlation simulations

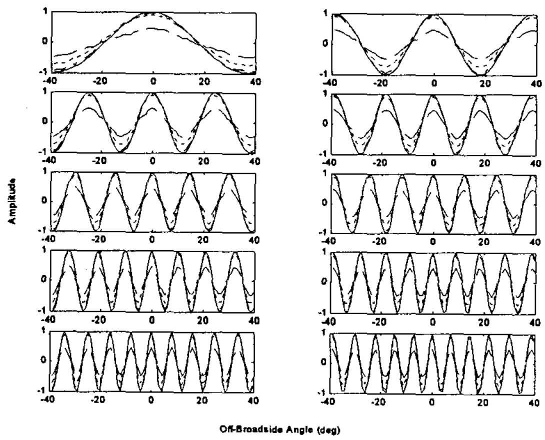

Fig.3. Comparison of basis functions resulted from different baseline and ADC levels levels

Figure 3 illustrates the spatial impulse responses, or the basis functions, of some digital correlators with different baselines. The baselines are from Δu to

$10\ \Delta u$ in our simulations. The dashed line is corresponding basis functions with 2 levels (1bit) digital correlation, the dotted line is 4 levels (2bits), the dash-dot line is 8 levels (3bits) and the solid line is the analog correlation result. Note that the magnitude of basis functions are effected by the A/D converter but when the ADC resolution is more than 2bits, there are little differences between the basis functions resulted from digital correlation and analog correlation, so the effect can be neglected when employing more than 2bits ADC.

The synthesized antenna patterns from each group of basis functions are plotted in figure 4. It is assumed that the patterns of all small antenna elements are the same, and their half power beam width are 80° .Because we use $10\Delta u$ as the maximum baseline, though the resulted patterns have an $8°$ half power beam width. Fig.4 shows that ADC resolution will affect the max amplitude of the synthesized antenna pattern but have no effect on the half power beam width. Furthermore, when the IF signal is digitalized by ADC with more than 2bits resolution, as our simulation indicated, the result from analog correlation and that from digital correlation are agreed with each other very well.

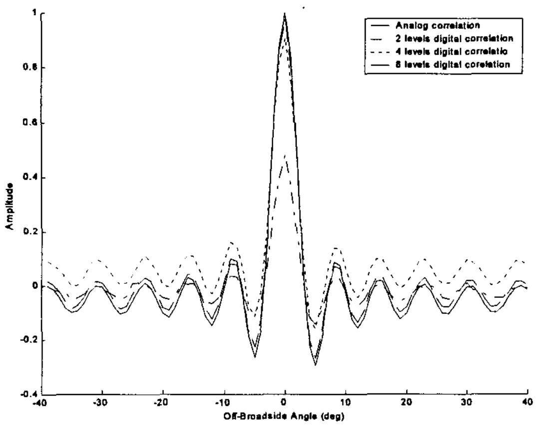

Fig.4 synthesized antenna patterns

Conclusion

Conventional synthetic aperture radiometer's analog correlator unit will become very complicated while the number of channels is increasing. This expense is one of the main difficulties while fine resolution is acquired in spaceborne radiometer. Because the resolution depends on the maximum baseline of the thinned array radiometer, and the maximum baseline is depends on the number of channels for a minimum redundancy configuration. This drawback cannot be overcome with conventional analog correlator. Our new digital correlator uses digital IQ demodulator instead of the intermediate frequency (IF) phase shifter to make the correlation processing performed digitally at base band. The down-converted IF signals from each pair of receiver channels become low rate base-band digital signals after under-sampled, Digitally Down-Converted, decimated and filtered by FIR filters. Two digital multipliers further process the digital signals, the outputs are integrated by the integrators and finally the outputs from the integrators compose of the real and imaginary part of a sample of the visibility function. Due to the fact that the digital correlators are realized with the help of ADC chips and the high density FPGA technology, the realized volume, mass, power consumption and complexity turned out to be greatly reduced compared with that of the analog correlators. Simulations show that the accuracy of ADC has an influence on the synthesized antenna patterns, but this can be neglected if more than 2bit ADC is used.

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