A new approach on soil-structure interaction.

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Abstract :
This article summarises the traditional method of soil-structure interaction based on the modulus of subgrade reaction and shows its weakness. In order to avoid these weakness, a new soil-structure interaction model is proposed. This model considers the soil as a set of connected springs which enables interaction between springs. Its use is as simple as the traditional model but allows to define the soil properties independently from the structural properties and the loading conditions. Thus, the definition of the modulus of subgrade reaction is unnecessary as each component is defined by its own modulii (Young's modulus and shear modulus). The non-linear soil behaviour for the shear stress versus distortion is also incorporated in the model. This feature allows to pinpoint the arching effect in the ground and shows how the stresses concentrate on stiff materials. Based on these principles, three dimensional program has been developed in order to solve the difficult problem of soil improvement by inclusions (stiff or soft). Also the possibility to take into account a flexible mat and/or a subgrade layer has been implemented. Equations used in the model are developed and a parametric study of the necessary data used in the program is presented. In particular, the Westergaard modulus notion and the arching effect are analysed.

Introduction :
The current design methods for foundations and retaining structures are based on the traditional subgrade reaction coefficient theory. This soil-structure interaction model has been standardized as it allows a straightforward model thanks to efficient and rather cheap softwares.
Concerning retaining structures and foundations (piles, slab...) design, this kind of model ensures the determination of the structure's settlement as well as the applied strains. The main weakness of this method is the determination of the subgrade reaction coefficient, which depends on several parameters such as the stiffness and dimensions of the structure, the applied loads, the soils conditions...As a result, the specification becomes ambiguous.
The soil-structure interaction approach presented in this article relies on the use of a simple and efficient model, without disregard to the basic geotechnical models of soil as a much more complex material than a spring.

Subgrade reaction modulus method and its weaknesses :
The coefficient of subgrade reaction concept was developed by Westergaard (1938). It links the pressure applied to the soil, p, and the settlement, δ :

$$p = K.\delta \qquad (1)$$

where K is given in kPa/m or kPa/m^3.
In addition, the modulus of reaction, Es, is defined as the product of the reaction coefficient K, and the pressure application width B such as :

$$Es = K.B \qquad (2)$$

The modulus of reaction is given in kPa.
The coefficient of subgrade reaction is consequently simply defined from a plate-loading test, as schematically shown on Fig 1. The graph shows the settlement curve, as a function of the applied pressure.
The value of the coefficient of subgrade reaction depends then on the definition itself of this coefficient : secant coefficient, tangent coefficient or initial coefficient.
In addition, the values from the plate load test are average values : pressure is the ratio between the total stress applied to the plate and the plate area ; the settlement is the average monitored settlement.

As an example, and in order to show some parameters from which depends the subgrade reaction coefficient, this coefficient is calculated in the particular case of a circular, stiff and uniformly loaded plate. The Boussinesq formula (1885), showing the settlement, δ, as a function of the applied pressure, p :

$$\delta = \pi.p.R.(1 - \nu^2) / (2.E) \qquad (3)$$

where **R** is the circular plate radius, ν the Poisson coefficient and **E** the Young modulus of the elastic material.

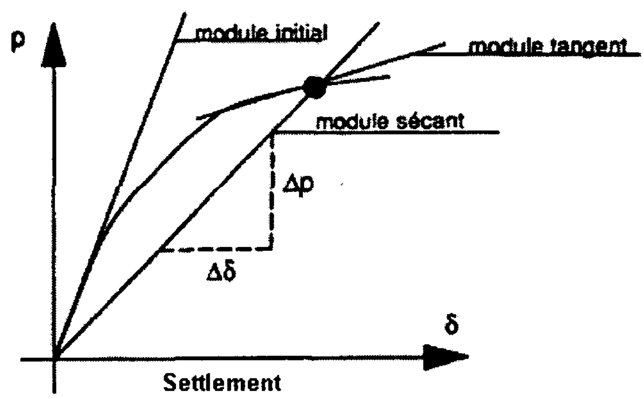

FIG 1 **Définition du coefficient de réaction.**
Definition of the coefficient of subgrade reaction.

The settlement shall be the same in equations (1) and (3), consequently, the coefficient of subgrade reaction is :

$$K = 2.E / (\pi.R.(1 - \nu^2)) \qquad (4)$$

Equation (4) shows that in the particular case of an infinitely stiff plate, the coefficient of subgrade reaction depends on the elastic properties of the soil, as well as the circular plate dimensions.
In the particular case of an infinitely flexible and uniformly loaded circular plate, the coefficient of subgrade reaction can be defined in a similar way. The settlement δ_c, at the plate centre is given by :

$$\delta_c = 2.p.R.(1 - \nu^2) / E \qquad (5)$$

then the settlement at the plate edge, δ_b, is given by :

$$\delta_b = ½ . \delta_c \qquad (6)$$

The coefficient of subgrade reaction at the plate centre, $\mathbf{K_c}$, and at the plate edge, $\mathbf{K_b}$, for an infinitely soft plate are equal to :

$$K_c = E / (2.R.(1 - \nu^2)) \qquad (7)$$

and :

$$K_b = ½ . K_c \qquad (8)$$

The comparison of equations (4) and (7) shows that the coefficient of subgrade reaction depends also on the plate stiffness..
Westergaard had already taken this phenomenon into account in the definition of a "stiffness radius", which correspond to the elastic length as far as beams are concerned. It is defined as :

$$r_0 = E_y . h^3 / 12 / (1 - \nu^2) / K \qquad (9)$$

where $\mathbf{E_y}$ *is the plate material* Young modulus ; ν, its Poisson coefficient ; **h**, its thickness and **K** the subgrade reaction coefficient.

It is important to note that in all the different formulations, the coefficient of subgrade reaction is proportional to the inverse of the radius (or "stiffness radius"). Consequently, the stiffer the plate is, the greater its "stiffness radius" is, and so the smaller the subgrade reaction coefficient is. Terzaghi (1958) had already pointed it out ; his physical demonstration was presented by the iso-stress curves.

The determination of the subgrade reaction coefficient is not straightforward and depends on several parameters ; in practice, one must fix some empirical rules for the determination of the subgrade reaction coefficient, based on the experience. In France, for shallow / deep foundations and retaining structures, the coefficient of subgrade reaction is mainly based on the pressuremetric modulus. Other common formulations are currently used, *depending on the type of structure* (pile or retaining structure), and on the kind of strain (tensioning of anchor, excavation...), as well as the speed of loading. The most common are given here :

The basic formula for the coefficient of subgrade reaction consists of the direct use of the pressuremetric test results. It gives the coefficient of subgrade reaction for an cylindrical shaped area, with a radius R :

$$K = E_m / [(1 + \nu).R] \quad (10)$$

where $\mathbf{E_m}$ is the pressuremetric modulus.
Whereas the soil surrounding the excavation becomes plastic, in a purely elastic material, the coefficient of subgrade reaction is equal to :

$$K = E / [(1 + \nu).R] \quad (11)$$

where **E** is the Young's modulus of the elastic material.
Concerning shallow foundations, with length, L, width, B, the coefficient of subgrade reaction is extracted from the pressuremetric formulas :

$$K = 9.E_m / (2.B_0.(\lambda_d.B / B_0)^\alpha + \alpha.\lambda_c.B) \quad (12)$$

where $\mathbf{B_0}$ is the reference width ; λ_d and λ_c are shape coefficients, function of the ratio L/B.

Concerning piles, with a diameter, B, larger than 0,60m, the modulus of reaction under horizontal strain is equal to :

$$K = 18.E_m / [4(2{,}65.(B/B_0)^\alpha . B/B_0) + 3.\alpha] \quad (13)$$

where $\mathbf{B_0}$ is the reference diameter, equal to 0,60m ; α the rheologic coefficient and $\mathbf{E_m}$ the pressuremetric modulus.

When the pile diameter is smaller or equal to 0,60m ; the modulus of modulus is :

$$Es = 18.E_m / [4(2{,}65^\alpha) + 3.\alpha] \quad (14)$$

For retaining structures, the determination of the coefficient of subgrade reaction, several formulas are commonly used, the most used are the following :
- Ménard-Bourdon (1965) :

$$K = E_m / [\alpha.a/2 + 0{,}133.(9.a)^\alpha] \quad (15)$$

where **a** is linked to the passive reaction length,

- Balay (1980) :

$$K = r.s.4.E_m/L_0 \quad (16)$$

with :

$$L_0 = (E.I/E_m)^{1/3}$$

where **r** and **s** take into account the geometry as well as the setting up method.

The pressuremetric method can also be used so as to give some correlations between the horizontal reaction modulus and the cone resistance, $\mathbf{q_c}$. For reference, the most commonly used formula is :

$$Es = 4{,}5.q_c \quad (17)$$

Several authors like Terzaghi (1955), Rowe (1957), Gigan (1989), have given some empirical relationships in order to estimate the coefficient of subgrade reaction according to several parameters such as the undrained shear strength, the depth, the embedment...For the definition of the horizontal coefficients of reaction for retaining structures, Chadeisson (1975) gave a commonly used formulation presented by graphs, where the reaction coefficient is linked to the friction angle and soil cohesion.
Generally, Schmitt (1984) experimentally demonstrated the common underestimation of reaction coefficients defined by the traditional methods, when strains are small. This assessment is the same as the one for the Young's modulus.

Equation (1) shows that the settlement of the soil, loaded under p(x), with **x** the abscissa, is proportional to the loading. The settlement of the soil, s(x), can be written :

$$S(x) = p(x) / K \qquad (18)$$

Fig 2 represents the settlement curve of a soil defined with a coefficient of subgrade reaction, under a uniformly loaded and flexible plate. It compares it to the settlement curve of an elastic material, under the same loading.

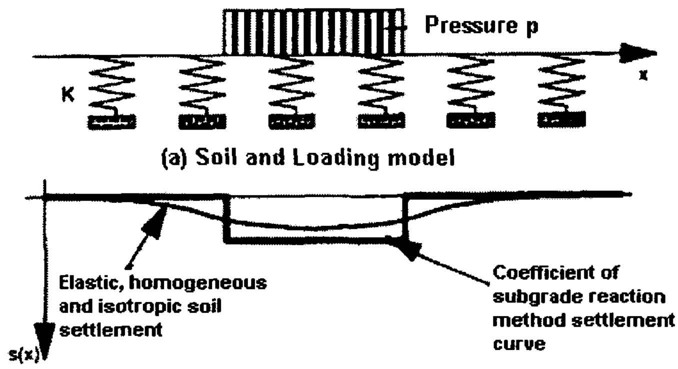

FIG 2 **Exemple de chargement et déformée selon le modèle pris en compte.**
Example of settlement curves.

It appears on this graph that the coefficient of subgrade reaction model soil does not undergo any settlement outside the loaded area, contrary to the homogenous, elastic and isotropic model.
Moreover, the coefficient of subgrade reaction model settlement curve is discontinuous and far away from the real settlement, whatever the reaction modulus value.
These empirical formula make the designer's work a lot easier, however it underemphasizes the more complex characteristics of the soil. In order to consider the soil, a simple interaction model is developed.

The soil-structure interaction model : GATEM
The weakness of the coefficients of subgrade reaction method is that it does not link the adjacent springs. This new approach consists of connecting the adjacent springs with some contact elements and to establish an interaction law.
In order to define the necessary equations, let's take two springs, as shown in Fig 3.

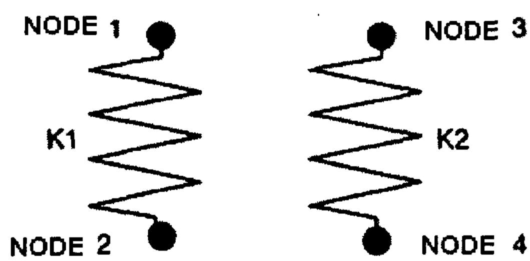

FIG 3 **Interaction entre deux ressorts adjacents.**
Adjacent springs interaction.

These springs represents some soil columns, as shown in Fig 4. The basic stiffness matrix for each string is :

$$K_{c1} = \frac{ES}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad (19)$$

Where le ratio **ES/L** represents the stiffness **K** of the spring. E is the soil's Young's modulus ; **S** soil column section and **L**, its length.

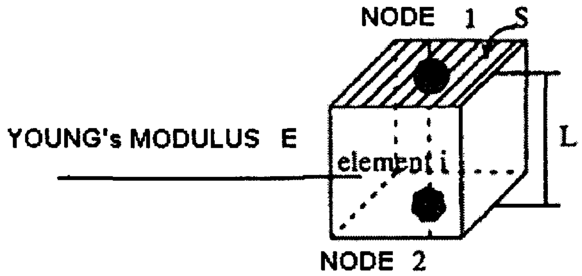

FIG 4 **Représentation de l'élément de sol représenté par le ressort de rigidité K.**
Soil column modeled as a spring.

In addition to the axial stiffness , there is also an interaction between the neighbouring element i and j, as shown in Fig 5.
In order to define the correct equations, let's consider a linear-elastic behaviour between these two elements.
Let's take **p**, the contact width between these two elements and **L** the contact height ; the relative displacement between the top nodes 1 and 3, as well as between the bottom nodes, is defined by :

$$-\Delta u_1 = u_3 - u_1$$
$$-\Delta u_2 = u_4 - u_2$$

where $\mathbf{u_i}$ is the displacement of node i. If we name **k** the stiffness which links the shear stress, τ, to the relative displacement **Δu** :

$$\tau = k.\Delta u \qquad (20)$$

The basic stiffness matrix for the elastoplastic behaviour of the element in the XOZ or YOZ plane is :

$$K_e = C.\begin{bmatrix} 2 & 1 & -2 & -1 \\ 1 & 2 & -1 & -2 \\ -2 & -1 & 2 & 1 \\ -1 & -2 & 1 & 2 \end{bmatrix}$$

with :

$$C = p.L.k / 6$$

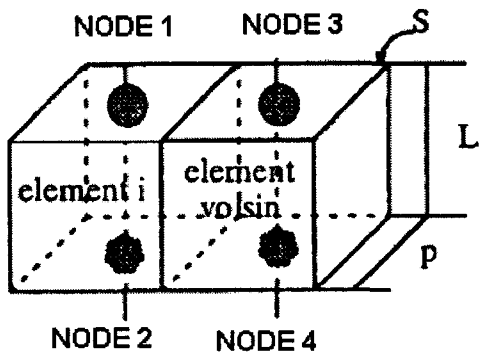

FIG 5 **Représentation de deux éléments adjacents et numérotation.**
Adjacent soil elements.

In the case of an elastoplastic behaviour of the contact elements, the stiffness matrix is similar to the previous one, however **k** is taken as an average value of the values at the top and bottom nodes. In this case, the matrix is calculated thanks to a secant modulus. Besides, this approach enables to keep a symmetrical matrix, which simplifies the calculations of the equations, as well as the data storage.

Basic applications of this model :
This model has been included in the GATEM software, which deals with the soft soil improvement by inclusions. Its aim is to solve the following problems :

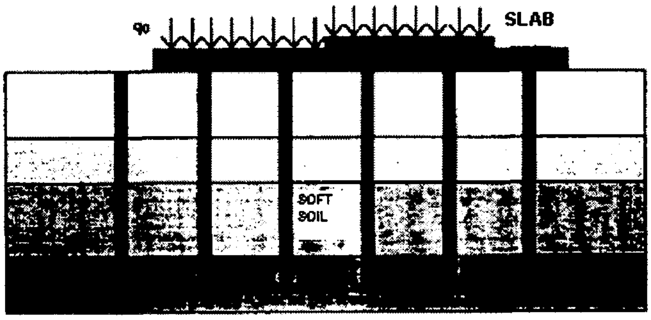

FIG 6 **Champs d'application du programme GATEM.**
Problems dealt with by GATEM.

In particular, the use of stiff or soft inclusions, like piles, jet grouted piles, stone columns...can be better analysed in order to emphasize their role in the soft soil improvement.
Two examples will be displayed, related to the determination of the Westergaard modulus, as well as the arching effect that occurs within the existing stiff materials.
The first example consists of a plate-loading test, in which the plate dimensions are variable parameters. The following example deals with a soil area of 25m * 25m, made of a 10m thick layer, topped by a 0,50m thick backfill layer.

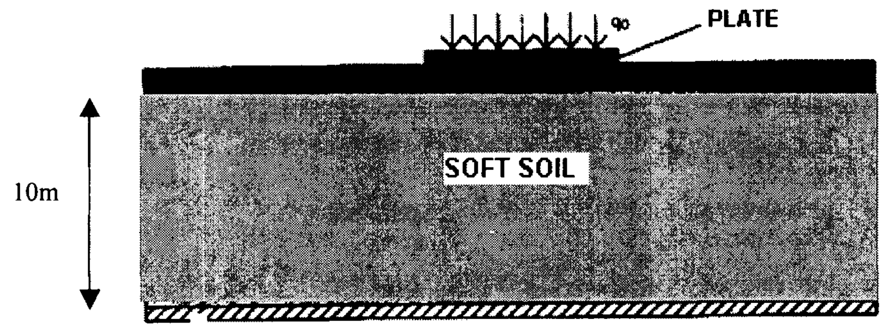

FIG 7 **Géométrie de l'exemple 1.**
Geometry for example 1.

The soil properties for the two layers taken into account are given in the Table 1.

TAB 1 **Caractéristiques mécaniques prises en compte pour l'exemple 1.**
Soil properties for example 1.

	Thickness m	E kPa	τ max kPa	Uplas m
Remblai	0,50	70	100	0,01
Argile	10,0	10	10	0,01

The behaviour at the soil columns interface is considered as elastoplastic. Specifically, the shear stress varies as a function of the relative displacement, as shown in Fig 8.

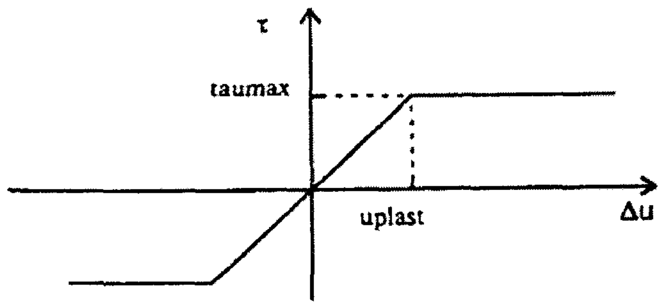

FIG 8 **Relation entre les contraintes de cisaillement et le déplacement relatif à l'interface sol/sol.**
Stress-strain law for the contact element.

1,0m ; 3,0m ; 5,0m ; 15,0m and 25,0m square plates are loaded with a 70 kPa strain and the evolution of the coefficient of subgrade reaction as a function of the plate dimensions is given in Fig 9.

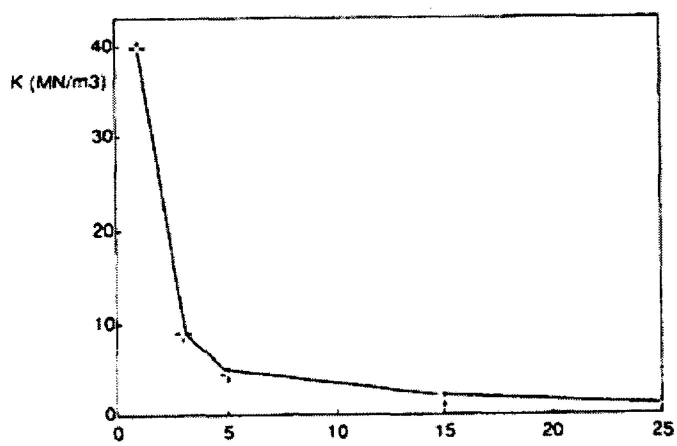

FIG 9 **Évolution du coefficient de réaction en fonction des dimensions de la zone chargée.**
Coefficient of subgrade reaction as a function of plate width.

As the elastic behaviour based formulas suggests it, the larger the loaded area is, the smaller the subgrade reaction coefficient becomes. Besides, when the loaded area gets larger, the coefficient of subgrade reaction tends to the following value :

$$K = E_y / H \quad (21)$$

where $\mathbf{E_y}$ is the modulus of the element soil + backfill (harmonic average), and **H** corresponds to the global thickness. For the example 1, the average Young modulus is equal to 10,42 Mpa, and the modulus of subgrade reaction is consequently equal to 0,99 MN/m^3.
The second example deals with the determination of the transfer of stress on the stiff inclusions like piles, when they are used to strengthen cohesive soil. The geometry of this particular case is shown in Fig 10, and the soil properties for this example are given in Tab 2.

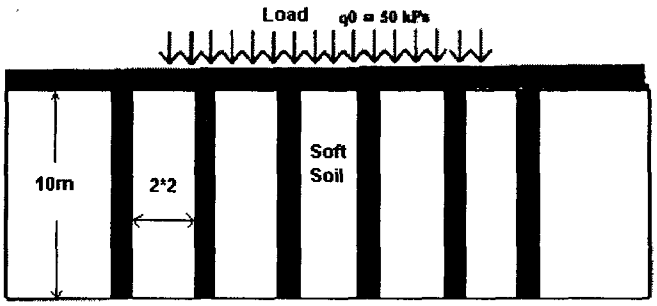

FIG 10 **Géométrie de l'exemple 2.**
Geometry for example 2.

The soil is uniformly loaded with 50kPa. The cohesive soil is strengthened by concrete inclusions embedded in the substratum. The spacing of these 0,27m diameter inclusions is 2,0m by 2,0m.
The created negative friction on the piles will discharge the surrounding soil while the piles get loaded up to the nil negative friction depth. As a consequence of this load transfer, the settlement of the cohesive soil layer will be reduced. In this example, the nil negative friction level is situated at the bottom of the cohesive soil layer, as it will be demonstrated later on.
In this example, one assumes that the soil-inclusion interaction is the same, whatever the shear strength sign (positive or negative friction). The inclusions' Young's modulus is fixed to 10000 MPa.

TAB 2 **Caractéristiques mécaniques prises en compte pour l'exemple 2.**
Soil characteristics for example 2.

		Ey MPa	τ_{max} (kPa)	U_{plas} (m)	τ_{max} (kpa)	U_{plas} m)
Remblai	0,50	70	100	0,01	100	0,01
Argile	10,0	6	50	0,01	30	0,01

The results are shown on Fig 11 and 12. Fig 11 shows the surface settlement under a 50kPa load, and compares it to the settlement without any soil improvement. The settlement is consequently divided by about 3,3 ; which corresponds to a common value for this kind of soil improvement.

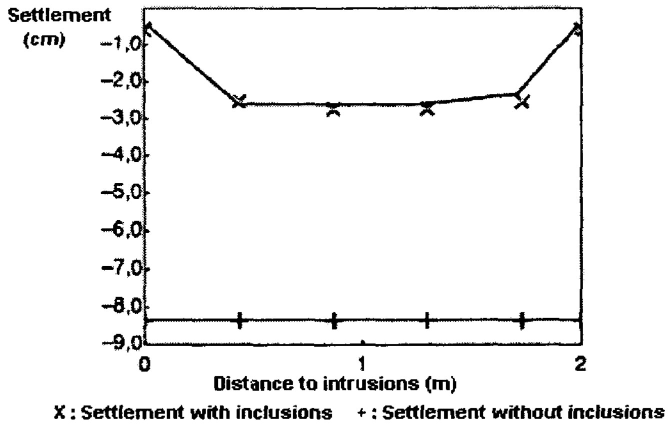

FIG 11 **Tassement en surface en fonction de la distance à l'inclusion.**
Surface settlement.

In the same way, the stress change in the inclusion, as a function of the depth, is given in Fig 12. There, it appears that the deeper it is, the more the inclusion gets loaded, due to the negative friction effect.
The phenomenon of unloading the soil, is represented in Fig 13 for several distances from the inclusion. Starting with a stress of 0,05 MPa at the surface, the deeper it is, the more the vertical stress decreases ; moreover the greater the distance from the inclusion is, the more the vertical stress increases. This phenomenon is compatible with the theory of Combarieu.

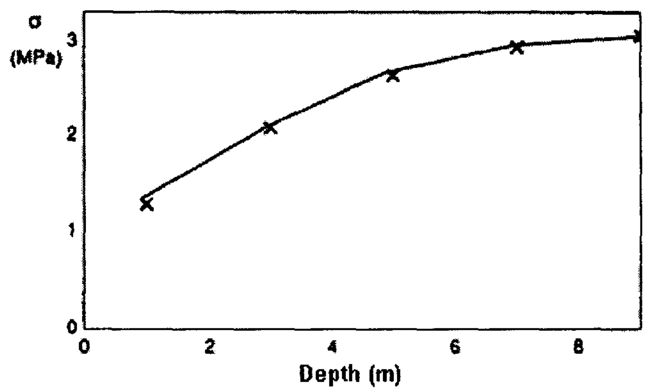

FIG 12 **Contrainte dans les inclusions en fonction de la profondeur.**
Compressive stress in the inclusion.

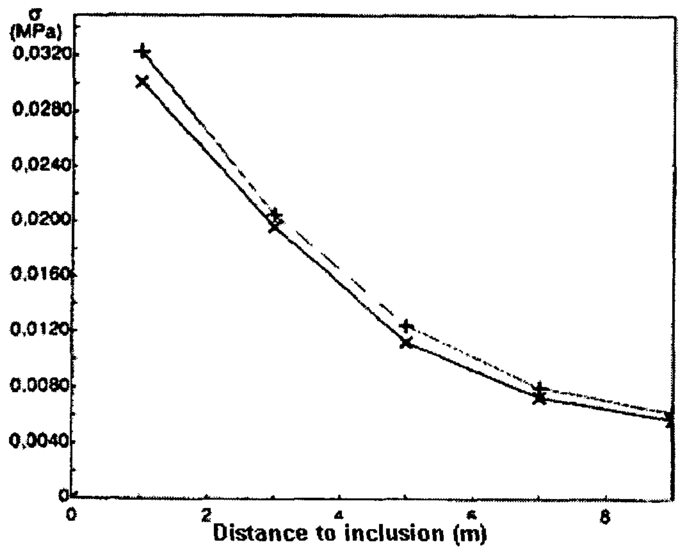

FIG 13 **Contrainte dans le sol en fonction de la profondeur pour différentes distances à l'inclusion.**
Vertical stress in the soil as a function of the distance to the inclusion.

Conclusion :
The soil-structure interaction model presented in this article allows to better represent the behaviour of soil compared to the traditional subgrade reaction modulus method. The foreseen notions modifications are quite slight and should consequently be easily integrated in the continuity of design offices work, without disrupting their traditional approaches. This more realistic method is halfway between the too simple modulus of subgrade reaction method and the more complicated finite element analysis method. Furthermore, this approach applied to the difficult problem of soil improvement, allows to quantify the carried out improvement. This model can be applied to retaining structures design, and will certainly allow to consider the arching effects currently totally disregarded.

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