

# The Noise Power Spectrum in Heavy Ion CT Based on Measurement of Residual Range Distribution

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## ABSTRACT

The relative electron density resolution was discussed by the noise power spectrum (NPS) in the heavy ion CT image. The heavy ion beam <sup>12</sup>C accelerated up to 400MeV/u by HIMAC was used in this study. The two-dimensional (2-D) NPS in the CT image was obtained from the one-dimensional (1-D) NPS of the measured residual range distribution of water phantom for single projection, and the noise variance in the CT image was calculated from 2-D NPS. The technique used in the reconstruction was the filtered back-projection method with Shepp-Logan filter. The calculated value suggests the result of our previous works using the density resolution phantom, assuming that the relative electron density resolution is twice the standard deviation. Therefore, the estimation of the noise in CT images by 2-D NPS obtained the measured residual range distribution is the useful method.

**Keywords:** heavy ion CT, noise power spectrum, residual range

## 1. INTRODUCTION

Today, the heavy ion radiotherapy is paid attention since the heavy ion beam has potentially a superior dose distribution due to physical and biological properties compared with a conventional X-ray beam, and the heavy ion radiotherapy is performed in HIMAC (Heavy Ion Medical Accelerator in Chiba). In the effective treatment planning of radiotherapy, it is necessary to obtain an accurate three-dimensional distribution of the heavy ion stopping power. Its distribution is usually calculated from X-ray CT numbers. However, it is difficult to obtain an accurate distribution of stopping power by this method because there is no exact correlation between CT numbers and stopping powers, especially for materials with high atomic numbers<sup>1)</sup>. Research of the heavy ion CT is proceeding as one of the techniques of raising the accuracy of the heavy ion radiotherapy planning<sup>2)</sup>. The heavy ion CT theoretically allows the distribution as stopping power to be obtained directly according to the Bethe-Bloch equation. Namely, if the residual range difference between a sample and a homogeneous reference material like water at each projection angle is known, CT image of the relative stopping power can be reconstructed. In the heavy ion CT based on measurement of residual range distribution, the fluctuation of the measurement value of residual range has an effect on the noise and relative electron density resolution in the reconstructed image. In this paper, we discuss of the relative electron density resolution and the heavy ion CT image by two-dimensional noise power spectrum (2-D NPS).

## 2. METHOD

### 2.1. Principle of the heavy ion CT

Stopping power is given by the Bethe-Bloch equation for heavy ion particles:

$$-\frac{dT}{dx} = \frac{4\pi z^2 e^4}{m_e c^2 \beta^2} \rho_e \left[ \ln \frac{2m_e c^2 \beta^2}{I(1-\beta^2)} - \beta^2 \right] \quad (1)$$

neglecting some of the correction terms. Here  $\rho_e$  is the electron density in the medium,  $T$  the kinetic energy of the heavy ions,  $\beta$  the velocity of the heavy ions in units of light velocity,  $z$  the atomic number of the heavy ion,  $e$  the electric charge,  $m_e$  the mass of an electron, and  $I$  the mean excitation energy of the medium. Using the Cartesian coordinates  $(X, Y)$  rotated by angle  $\theta$  with respect to the coordinates  $(x, y)$ , the range loss of the heavy ion

particles  $R_0 - \Delta R_0(X, \theta)$  is

$$R_0 - \Delta R_0(X, \theta) = \int_0^L \eta(x, y) dY. \quad (2)$$

The above equation is just the two-dimensional Radon transform of  $\eta(x, y)$ , so that the relative stopping power or electron density can be obtained by the measurement of the range loss at each rotation angle. Here  $\eta(x, y)$  the relative electron density distribution of the medium (Fig.1).

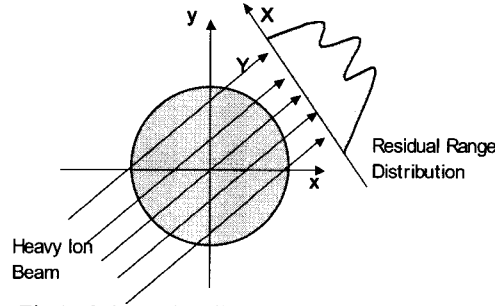


Fig.1 Schematic diagram of the residual range distribution at each angle.

## 2.2. Noise Power Spectrum in CT image

Assuming that the complete reconstruction is performed using  $m$  projections taken at equally spaced angles covering from 0 to  $\pi$  each with the same NPS  $S_p$  for the projection, the noise power spectrum  $S$  of the CT image is expressed as

$$S(f) = \left(\frac{\pi}{m}\right) |f|^{-1} \cdot |H(f)|^2 \cdot S_p(f), \quad (3)$$

where  $m/f\pi$  is the spoke density at radial frequency  $f$  and  $\pi/m$  is the proper normalization factor for the reconstruction. Then the noise variance in the reconstructed image is given as the total power

$$\sigma_r^2 = \int_0^{2\pi} d\theta \int_0^{\infty} df \cdot f \cdot S(f). \quad (4)$$

Accordingly, it is possible to estimate the relative electron density resolution in the heavy ion CT image using Eqs. 3 and 4.

## 2.3. Measurement of the residual range distribution

The heavy ion beam  $^{12}\text{C}$  used in this work was accelerated up to 400MeV/u by HIMAC operated by NIRS (National Institute of Radiological Sciences), the beam intensity  $3.6 \times 10^8$  pps and the beam-spill time width was 2.7sec. The scheme of our heavy ion CT system was shown in Fig.2. We use this system, and measured the residual range distribution of the phantom.

## 3.RESULTS AND DISCUSSION

We obtained the 2-D NPS in the CT image from 1-D NPS of the measured residual range distribution of water phantom for single projection, and calculated the noise variance in the CT image from 2-D NPS (Eq.(4)). 2-D NPS  $S(f)$  of the image reconstructed with the slice thickness 10 pixel (1 pixel=0.21mm) was shown in Fig.3. From Eq.4, the relative electron density resolution was calculated with 0.023. In our previous works, relative electron density resolution of the reconstructed image by filtered back projection technique was reported to 0.048<sup>3)</sup>. The result in this work is near to our previous one.

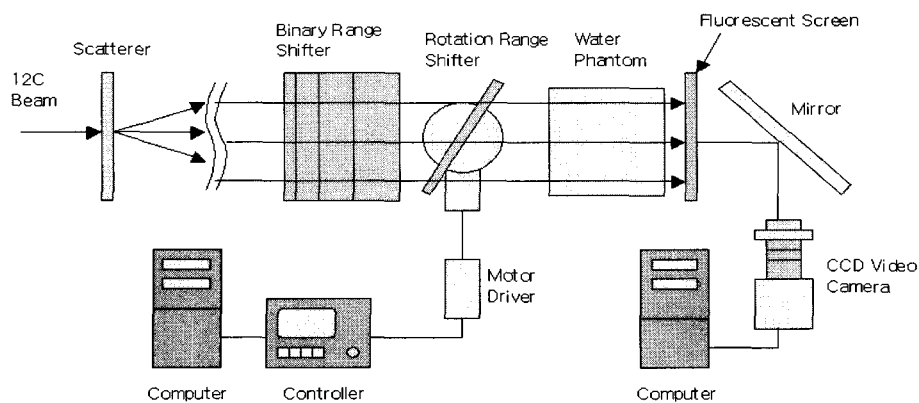


Fig.2. Schematic diagram of our heavy ion CT system

#### 4.CONCLUSION

The electron density resolution is estimated by the noise power spectrum of the single back projection. The result is consisted with the previous one. If the NPS for the projection image is obtained, the relative electron density could be estimated. Therefore the estimation of the noise in CT images by 2-D NPS obtained the measured residual range distribution is the useful method.

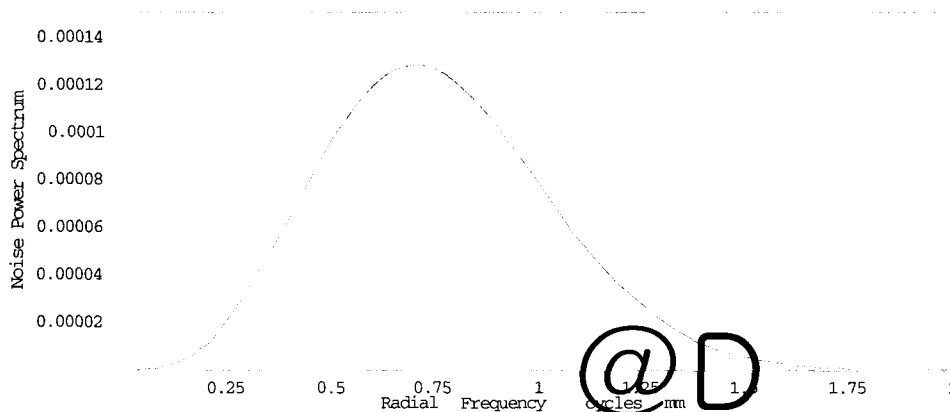


Fig.3. Calculated Noise Power Spectrum for reconstructed CT image using Filtered Back Projection method with Shepp and Logan filter.

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