A Practical Method for Estimating High-Energy X-Ray Spectra Using the Iterative Perturbation Principle of Waggener

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ABSTRACT

We have developed a practical method for estimating high-energy x-ray spectra using measured attenuation curves. This method is based on the iterative perturbation technique proposed by Waggener et al. The principle is to minimize the difference between the measured and calculated transmission curves. The experimental study was made using 4 MV, 10 MV, and 15 MV x-ray beams. It has been found that the spectrum varies strongly with the off-axis distance.

Keywords: High-energy x-ray spectra, iterative perturbation technique, transmission analysis, attenuation measurement.

1. INTRODUCTION

Obtaining the x-ray beam spectrum as a function of off-axis distance is very important for accurate dose calculations especially when using convolution methods, because they perform dose calculations using the primary beam intensity as a parameter in media. It seems reasonable to evaluate attenuation of the primary beam intensity in media using the incident x-ray spectrum itself as a function of the off-axis distance. Waggener et al.[1] reported a method for estimating 25 kVp to 18 MV x-ray spectra using measured transmission curves. This method is based on an iterative perturbation principle, which is to minimize the difference between the measured and calculated transmission curves. The calculated transmission curve is reproduced from an initial spectrum. They took only one cycle for estimating a spectrum, employing a large number of measured transmission data, which is equal to the number of energy bins. It has been found that this reproduction method is poor when the number of measured transmission data is relatively small. In this paper, we propose a practical method for obtaining high-energy x-ray spectra. It is accurate even when the number of measured transmission data is relatively small. It repeats the iterative perturbation technique, employing successive initial spectra during multiple cycles. The optimum spectrum is chosen to minimize the difference between the measured and calculated transmission curves during the repeated cycles, also utilizing high-density material attenuation curves.

2. THEORETICAL BACKGROUND

It is assumed that in-air chamber readings obtained using a build-up cap ensuring electronic equilibrium express relative collision kerma data of the build-up cap material (the cap material collision kerma). For the x-ray spectrum to be derived from a transmission curve, we set minimum and maximum photon energies (E_{min} and E_{max}), where E_{min} is determined by trial and error and E_{max} is determined using the accelerating voltage. N=1,2,3,...N_{max} denotes the energy bin. Let $\Psi(N)$ be the fluence of the cap material collision kerma per unit energy at an energy of $E(N)$ with energy bin $\Delta E(N)$. Let $\mu(N)$ be the linear attenuation coefficient of the filter material at photon energy $E(N)$. Let the values of $\mu(N_{min})$ and $\mu(N_{max})$ be evaluated for the photon energies of E_{min} and E_{max}, respectively. This procedure is taken to enhance the effects of the photon energies of E_{min} and E_{max} on the transmission calculations. For a given transmission curve, K=1,2,3,...,K_{max} denotes the transmission point. Then the transmission of the cap material collision kerma through a filter thickness of $L(K)$ can be formulated as

$$T(K)=\sum_{N=1}^{N\max}\Psi(N)\,\Delta E(N)\exp[-\mu(N)L(K)]. \quad (1)$$

Let T_{meas} and T_{calc} denote the measured and calculated transmission curves, respectively. we perform the process of

minimizing the difference between the measured and calculated transmission curves using the equation,

$$\%\,\mathrm{dif} = \frac{100}{K\max}\sum_{K=1}^{K\max}\frac{\left|\,T\mathrm{calc}(K) - T\mathrm{meas}(K)\,\right|}{T\mathrm{meas}(K)}, \tag{2}$$

where

$$T_{\mathrm{calc}}(K)=\sum_{N=1}^{N\max} \blacksquare(N)\square E(N)\exp[-\mu(N)L(K)]. \tag{3}$$

This is done by forming three values of %dif at each $\blacksquare(N)$ for a positive, negative, or no perturbation of $\blacksquare(N)$. Thus for

$$\square\blacksquare(N)=\blacksquare(N)/2^{k}, \quad k=1,2,3,\ldots\ldots,k_{\max}, \tag{4}$$

(i)

$$\%\mathrm{dif1} = \frac{100}{K\max}\sum_{K=1}^{K\max}\frac{\left|\,T\mathrm{calc},1(K) - T\mathrm{meas}(K)\,\right|}{T\mathrm{meas}(K)}, \tag{5}$$

where

$$\mathrm{T}_{\mathrm{calc},1}(K)=\sum_{N=1}^{N\max} \blacksquare_1(N)\square E(N)\exp[-\mu(N)L(K)] \tag{6}$$

$$\blacksquare_1(N)=\blacksquare(N)+\square\blacksquare(N); \tag{7}$$

(ii)

$$\%\mathrm{dif2} = \frac{100}{K\max}\sum_{K=1}^{K\max}\frac{\left|\,T\mathrm{calc},2(K) - T\mathrm{meas}(K)\,\right|}{T\mathrm{meas}(K)}, \tag{8}$$

where

$$\mathrm{T}_{\mathrm{calc},2}(\mathrm{K})=\sum_{N=1}^{N\max} \blacksquare_2(N)\square E(N)\exp[-\mu(N)L(K)] \tag{9}$$

$$\blacksquare_2(\mathrm{N})=\blacksquare(\mathrm{N}); \tag{10}$$

(iii)

$$\%\mathrm{dif3} = \frac{100}{K\max}\sum_{K=1}^{K\max}\frac{\left|\,T\mathrm{calc},3(K) - T\mathrm{meas}(K)\,\right|}{T\mathrm{meas}(K)}, \tag{11}$$

where

$$T_{\mathrm{calc},3}(K)=\sum_{N=1}^{N\max} \blacksquare_3(N)\square E(N)\exp[-\mu(N)L(K)] \tag{12}$$

$$\blacksquare_3(N)=\blacksquare(N)-\square\blacksquare(N). \tag{13}$$

As a new value for $\blacksquare(N)$, we select the one which yields the smallest %dif among the three perturbations. This procedure is performed separately for each of N=1,2,3,…,$N_{\max}$. Thus we obtain a raw set of fluences and then smooth it using

$$@\Psi'(@N) = \frac{W_1}{W_1 + W_2}(Y_2 - Y_1) + Y_1, \tag{14}$$

@

with W_1=[$E(N)$–$E(N$–1)]/2, W_2=[$E(N$+1)–$E(N)$]/2, Y_1=[$\blacksquare(N$–1)+$\blacksquare(N)$]/2, and Y_2=[$\blacksquare(N)$+$\blacksquare(N$+1)]/2, where $E(0)$=$E_{\min}$, $E(N_{\max}+1)$=$E_{\max}$, and $\blacksquare(0)$=$\blacksquare(N_{\max}+1)$=0. In this paper, the starting value for $\blacksquare(N)$ or the pre-spectrum was constructed using the equation

$$\blacksquare(N)\propto\sin[\pi(E(N)-E_{\min})/(E_{\max}-E_{\min})]. \tag{15}$$

3. CALCULATION LOOP

Attenuation measurements are performed using two types of filter materials for a given x-ray beam. One is a low Z material (like water, acrylic, polystyrene, aluminum, etc.), used for x-ray spectra estimation. The other is a high Z material (like lead, etc.), used only for verifying the x-ray spectra estimation. The calculation loop of the computer program is as follow:

(i) Set minimum and maximum photon energies (E_{min} and E_{max}) and energy bin widths ($\Delta E(N)$).

(ii) Set a pre-spectrum, as expressed by Eq. (15).

(iii) Start the cycle of Eqs. (4)-(14).

(iv) Pick up a set of $\Psi'(N)$ fluences performing the smallest %dif during the procedures for k=1,2,3,.....,$k_{max.}$ (the smallest %dif repeats when k is beyond a certain value for each procedure). Use it as a new pre-spectrum and go to stage (iii). While repeating the cycle, on the other hand, pick up a set of $\Psi'(N)$ fluences which yields the smallest %dif (the new value does not appear when the repeated number of the cycle is beyond a certain value), and then go to stage (v).

(v) Calculate the attenuation curve for a high Z material (like lead) using the set of $\Psi'(N)$ fluences and compare it with a measured one. If the difference between the curves is within a certain limit, take it as a final set of $\Psi'(N)$ fluences and go to stage (vi). If not, use it as a pre-spectrum and go to stage (iii).

(vi) Put $\Psi_{cap}(N)=\Psi'(N)$ (because the set of $\Psi'(N)$ fluences is expressed using the cap material collision kerma).

A set of photon energy fluences $\Psi(N)$ can then be re-expressed as

$$\Psi(N)=\Psi_{cap}(N)/(\mu_{en}(N)/\rho)_{cap}, \quad (16)$$

where $(\mu_{en}(N)/\rho)_{cap}$ is the mass energy absorption coefficient for the cap material at a photon energy of $E(N)$.

4. RESULTS AND CONCLUSION

The experimental study was performed using a linear accelerator producing 4, 5, and 15 MV x-rays. Acrylic and aluminum were took as low Z filter materials and lead as a high Z filter material. Attenuation measurements were made using a 0.6-cm^3 thimble chamber with acrylic buildup cap. Narrow beams were made using the upper and lower collimators and the multi-leaf collimator. The field sizes at the isocenter were about 1.6×1.6 cm^2. The values of μ and (μ_{en}/ρ) for the irrespective materials were taken from published data.[2] If the densities of the filter materials were different from standard ones, density corrections were taken into account when evaluating μ for the filter materials. Figures 1(a)-(c) show sets of photon energy fluences at R=0 cm (off-axis distance) for 4, 10, and 15 MV x-ray beams, also including published data.[3] Figures 2(a)-(c) show how the set of photon energy fluences changes with off-axis distance for 4, 10, and 15 MV x-ray beams. It has been

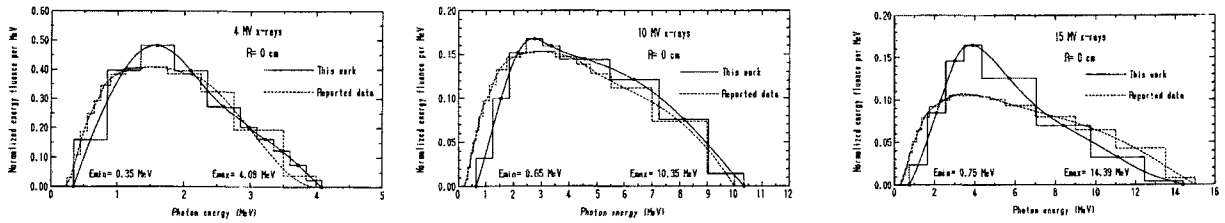

Fig. 1. Sets of photon energy fluences at R=0 cm for (a) 4 MV, (b) 10 MV, and (c) 15 MV x-ray beams. Broken lines are published data.

found that the attenuation curves as a function of off-axis distance for water, acrylic, aluminum, and lead, calculated using the photon energy fluences of Fig. 2(a)-(c), coincide well with measured attenuation curves. From the results shown it can be understood that the set of photon energy fluences changes greatly with off-axis distance.

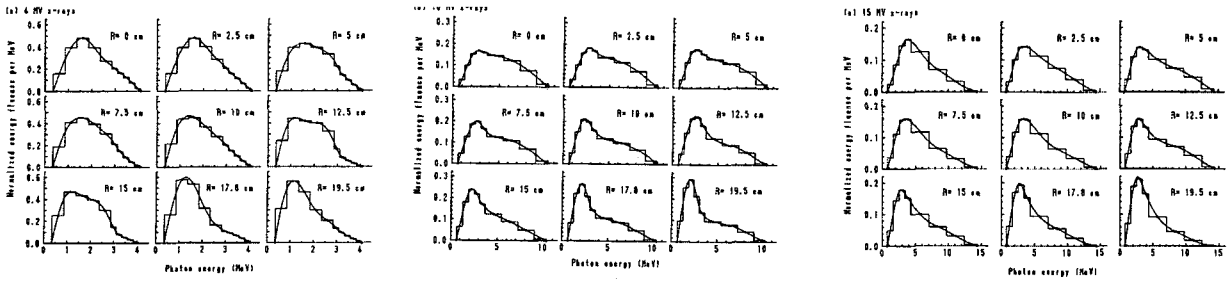

Fig. 2. Diagrams showing how the set of photon energy fluences changes with off-axis distance (R) for (a) 4 MV, (b) 10 MV, and (c) 15 MV x-ray beams.

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