Development and Evaluation of Maximum-Likelihood Position Estimation with Poisson and Gaussian Noise Models in a Small Gamma Camera

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ABSTRACT

It has been reported that maximum-likelihood position-estimation (MLPE) algorithms offer advantages of improved spatial resolution and linearity over conventional Anger algorithm in gamma cameras. The purpose of this study is to evaluate the performances of the noise models, Poisson and Gaussian, in MLPE for the localization of photons in a small gamma camera (SGC) using NaI(Tl) plate and PSPMT. The SGC consists of a single NaI(Tl) crystal, 10 cm diameter and 6 mm thick, optically coupled to a PSPMT (Hamamatsu R3292-07). The PSPMT was read out using a resistive charge divider, which multiplexes 28(X) by 28(Y) cross wire anodes into four channels. Poisson and Gaussian based MLPE methods have been implemented using experimentally measured light response functions. The system resolutions estimated by Poisson and Gaussian based MLPE were 4.3 mm and 4.0 mm, respectively. Integral uniformities were 29.7% and 30.6%, linearities were 1.5 mm and 1.0 mm and count rates were 1463 cps and 1388 cps in Poisson and Gaussian based MLPE, respectively. The results indicate that Gaussian based MLPE, which is convenient to implement, has better performances and is more robust to statistical noise than Poisson based MLPE.

1. INTRODUCTION

It has been reported that maximum-likelihood position-estimation (MLPE) algorithms offer advantages of improved spatial resolution and linearity over conventional Anger algorithm in gamma cameras [1-3]. While the fluctuation of photon measurements is more accurately described by Poisson than Gaussian statistics, the likelihood function of a scintillation event assumed to be Gaussian could be more easily implemented. Additionally, Gaussian method might provide more consistent outcomes than Poisson based MLPE because Poisson based MLPE solution is derived from the derivative of the detector response [4]. The purpose of this study is to evaluate the performances of the noise models, Poisson and Gaussian, in MLPE for the localization of photons in a small gamma camera (SGC) using NaI(Tl) plate and position-sensitive photo-multiplier tube (PSPMT).

2. MATERIALS AND METHODS

Because of the stochastic nature of signal generation in a gamma camera, the probability of measuring a given set of output signals ($X+$, $X-$, $Y+$, $Y-$) given an event at (X_i, Y_j) in the gamma camera will be described by the probability distribution as $P(X+, X-, Y+, Y-| X_i, Y_j)$. The information from this probability can be incorporated in a position estimation scheme. The MLPE solution is achieved by maximizing the likelihood function between the predetermined light response function (LRF) and measured event signals. We have implemented two MLPE methods based on Poisson and Gaussian noise models [2,4].

2.1. Poisson based MLPE

Assuming the noise process in the detector response follows Poisson, the likelihood function of event position is given by

$$\Pr[\underline{M} \mid x] = \prod_{i=1}^{n} \frac{\mu_i(x)^{M_i} e^{-\mu_i(x)}}{M_i!} \qquad (1)$$

where M_i is ith PMT output and $\mu_i(x)$ is the mean output of the ith PMT as a function of event position. Performing the maximization with respect to the log likelihood and then the resulting approximation is

$$\frac{\partial}{\partial x} \ln \Pr[M_i \mid x]_{x=\tilde{x}} \cong \sum_{i=1}^{n} M_i \omega_i(x) = 0 \qquad (2)$$

where

$$\omega_i(x) = \left(\frac{\partial \mu_i(x)/\partial x}{\mu_i(x)} - \frac{\sum_{j=1}^{n} \partial \mu_j(x)/\partial x}{\sum_{k=1}^{n} \mu_k(x)} \right) \quad (3)$$

The ML solution is achieved by minimizing the quantity of equation (3).

2. 2. Gaussian based MLPE

Assuming the noise process in the detector response follows Gaussian, the likelihood is given by

$$\Pr[\underline{M} \mid x] = \prod_{i=1}^{n} \frac{1}{\sigma_i(x)\sqrt{2\pi}} \exp\left(-\frac{(M_i - \mu_i(x))^2}{2\sigma_i^2(x)} \right) \quad (4)$$

where M_i is ith PMT output, $\mu_i(x)$ is the mean output of the ith PMT and σ_i is standard deviation as a function of event position.

Performing the maximization with respect to the log likelihood and then the resulting approximation is

$$\ln \Pr[M_i \mid x] = -\left(\sum_{i=1}^{n} \frac{(M_i - \mu_i(x))^2}{2\sigma_i^2(x)} + \sum_{i=1}^{n} \ln \sigma_i(x) \right) \quad (5)$$

The ML solution is achieved by minimizing the quantity inside of bracket in equation (5).

2. 3. Experimental Configuration

LRFs for a detector were determined by experiment [5]. Our SGC consists of a single NaI(Tl) crystal, 10 cm in diameter and 6 mm thick, optically coupled to a single PSPMT (Hamamatsu R3292-07). The PSPMT was read out using a resistive charge divider, which multiplexes 28(X) by 28(Y) cross wire anodes into four channels. To characterize LRFs, data were collected on 20 x 20 evenly spaced points on a 10 cm–diameter crystal using an X-Y stepper-motor stage and collimated Tc-99m source. Then LRFs were generated using Gaussian fitting and bilinear interpolation. The bilinear interpolation was applied to

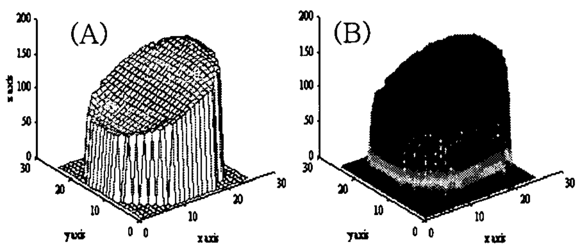

Fig. 1. Acquired light response function (LRF) of X+. (A) Experimentally generated LRF by moving a point source on a 5 mm step grid, (B) Bilinearly interpolated LRF with 0.5 mm resolution.

a coarse set of LRF samples acquired from the experiment to obtain the LRF with 0.5 mm search resolution. Figure 1 shows the experimentally generated coarse and bilinearly interpolated LRFs. Event positions were determined by ML solution using the generated LRFs during image acquisition. The spatial resolution, linearity, flood field uniformity and count rate of the SGC were measured based on the NEMA protocol for the two MLPE methods. A general-purpose parallel-hole collimator, Tc-99m point and flood source were used.

3. RESULTS AND DISCUSSIONS

The collimated source images obtained at 21 locations are illustrated in Figure 2. The summary of measured spatial resolution, linearity, uniformity and count rate of SGC are shown in Tables 1, 2 and 3 for Poisson and Gaussian based MLPE. The averaged system resolutions estimated by Poisson and Gaussian

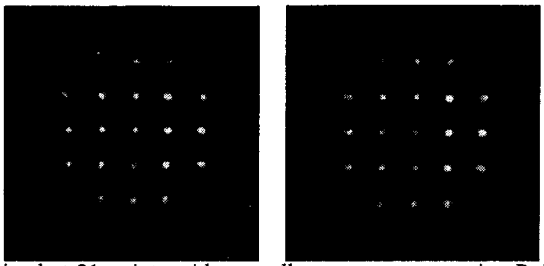

Fig. 2. Collimated source images obtained at 21 points with a small gamma camera using Poisson (left) and Gaussian (right) based MLPE.

Table 1. Summary of spatial resolution (mm) of Poisson based MLPE at 21 locations in the small gamma camera. At each location, FWHM values in x- and y-directions are given.

	X:3.87 Y:5.13	X:3.86 Y:5.33	X:3.84 Y:5.41	
X:5.15 Y:3.86	X:4.39 Y:3.78	X:3.68 Y:4.06	X:3.65 Y:3.98	X:6.29 Y:4.41
X:4.90 Y:3.84	X:4.64 Y:3.50	X:3.59 Y:4.02	X:3.80 Y:4.06	X:4.82 Y:4.13
X:5.22 Y:4.04	X:3.90 Y:3.92	X:3.66 Y:4.30	X:3.38 Y:4.14	X:6.27 Y:4.59
	X:4.06 Y:6.09	X:3.66 Y:5.46	X:3.86 Y:5.38	

Table 2. Summary of spatial resolution (mm) of Gaussian based MLPE at 21 locations in the small gamma camera. At each location, FWHM values in x- and y-directions are given.

	X: 3.42 Y:4.96	X: 3.77 Y:5.00	X: 3.83 Y:5.21	
X: 4.53 Y:4.04	X: 3.83 Y:3.84	X: 4.09 Y:4.00	X: 3.39 Y:3.36	X: 5.12 Y:4.02
X: 4.09 Y:3.58	X: 3.76 Y:3.49	X: 3.86 Y:3.74	X: 3.86 Y:3.54	X: 4.59 Y:3.77
X: 4.78 Y:4.21	X: 3.35 Y:3.72	X: 3.83 Y:3.99	X: 3.27 Y:3.66	X: 4.72 Y:4.33
	X: 3.95 Y:5.48	X: 3.75 Y:5.06	X: 4.27 Y:5.41	

Table 3. Summary of linearity, integral uniformity and count rate results measured with Poisson and Gaussian based MLPE. Linearity values are the standard deviation of the peaks of profiles.

	Poisson based MLPE	Gaussian based MLPE
Horizontal Linearity	1.5 mm	1.0 mm
Vertical Linearity	0.1 mm	0.2 mm
Integral Uniformity	29.7%	30.6%
Count Rate	1463 cps	1388 cps

based MLPE were 4.3 mm and 4.0 mm, respectively. Integral uniformities were 29.7% and 30.6%, linearities were 1.5 mm and 1.0 mm and count rates were 1463 cps and 1388 cps in Poisson and Gaussian based MLPE, respectively. Gaussian based MLPE shows slightly better performances than Poisson based MLPE, especially at the edge of the detector FOV. The results indicate that Gaussian based MLPE, which is convenient to implement, has better spatial resolution and linearity performance over the Poisson based MLPE and is more robust to statistical noise than Poisson based MLPE.

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