

# 검사체적 유한요소법을 이용한 다축 브레이드 프리폼의 투과율 계수 예측

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## Prediction of Permeability for Multi-axial Braided Preform by Using CVFEM

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**Key Words** : permeability, braided perform, CVFEM

### ABSTRACT

Prediction of 3-D permeability tensor for multi-axial preform is critical to model and design the manufacturing process of composites by considering resin flow through the multi-axial fiber structure. In this study, the in-plane and transverse permeabilities for braided preform are predicted numerically. The flow analyses are calculated by using 3-D CVFEM(control volume finite element method) for macro-unit cells. To avoid checker-board pressure field and improve the efficiency of numerical computation, a new interpolation function for velocity is proposed on the basis of analytic solutions. Permeability of a braided preform is measured through unidirectional flow experiment and compared with the permeability calculated numerically. Unlike other studies, the current study is based on more realistic unit cell and prediction of permeability is improved.

### 1. Introduction

Resin transfer molding (RTM) is an efficient and frequently used process for producing fiber reinforced polymer composite products which have the small structures of simple shape or large structures with complex shape. Permeability is essential in the design and operation of process and its determination can be divided into three methods, which are experimental measurement, analytical, and numerical predictions through Darcy's law. The experimental measurements often require a large number of carefully controlled experiments which generally have low predictive capability. The analytical prediction using Kozeny-Carman equation is very simple but has a weakness on strongly anisotropic preforms. As a consequence, many efforts to predict numerically permeability have been conducted for the past few years. But most of them have been carried out on unidirectional preforms which are apart from real situation.

In this study, the permeability for multi-axial braided perform is computed by using CVFEM(control volume

finite element method). Unlike past literatures in modeling microstructure for determining the permeability, the present study considers more realistic unit cell representing the three-dimensional perform structures.

### 2. Numerical method

The CVFEM(control volume finite element method) which proposed by Prakash et al. is used in order to compute flow field. The differential equations that govern steady-state, three-dimensional, incompressible, and Newtonian fluid may be cast in the following general forms.

$$\nabla \cdot \mathbf{J} = S \quad (1)$$

$$\nabla \cdot \mathbf{g} = 0 \quad (2)$$

where  $\mathbf{J}$  is diffusion flux,  $S$  is source term, and  $\mathbf{g}$  is mass flux vector,  $\rho \mathbf{v}$ .

The diffusion flux and the source term of  $x$  momentum equation are

$$\mathbf{J} = \mu \nabla u \quad \text{and} \quad S = \frac{\partial p}{\partial x} \quad (3)$$

Where  $\mu$  is Newtonian viscosity of the fluid.

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In y momentum equation, the diffusion and the source are given as follows

$$\mathbf{J} = \mu \nabla v \quad \text{and} \quad S = \frac{\partial p}{\partial y} \quad (4)$$

And in z momentum equation, these terms are similar with above equations.

$$\mathbf{J} = \mu \nabla w \quad \text{and} \quad S = \frac{\partial p}{\partial z} \quad (5)$$

with applying appropriate conservation principle to a control volume, which is described as shown in Fig. 1, integral forms of equation (1) and (2) can be obtained as follows.

$$\int_{\partial V} \mathbf{J} \cdot \mathbf{n} ds = \int_V S dV \quad (6)$$

$$\int_{\partial V} \mathbf{g} \cdot \mathbf{n} ds = 0 \quad (7)$$

where  $\partial V$  is the surface of the control volume, and  $\mathbf{n}$  is a unit outward vector normal to differential area,  $ds$ .

If velocity components and pressure were stored at same grid points and interpolated by similar functions such as linear functions, the resulting discretization equations could admit physically unrealistic checker-board type pressure fields. To overcome this problem, Prakash et al. formulated a special equal-order CVFEM which explicitly account for the influence of the pressure gradient on the velocity distribution within each element.

In present study, a new interpolation functions for velocity which first proposed on the basis of various analytic solutions by Lee in my laboratory is used to enhance the efficiency of numerical computation. The velocity and the pressure are interpolated within each control volume as following forms.

$$u = A^u x + B^u y + C^u z + D^u - \frac{1}{\mu} \frac{\partial p}{\partial x} \left[ x - \frac{1}{4}(y^2 + z^2) \right] \quad (8)$$

$$v = A^v x + B^v y + C^v z + D^v - \frac{1}{\mu} \frac{\partial p}{\partial y} \left[ y - \frac{1}{4}(z^2 + x^2) \right] \quad (9)$$

$$w = A^w x + B^w y + C^w z + D^w - \frac{1}{\mu} \frac{\partial p}{\partial z} \left[ z - \frac{1}{4}(x^2 + y^2) \right] \quad (10)$$

$$p = A^p x + B^p y + C^p z + D^p \quad (11)$$

Discretization equations are obtained by substituting above equations into the control volume integration equations and assembling them appropriately. Also the contribution of boundary conditions is calculated and then added to the whole discretization equations.

$$a_i^u u_i = \sum_n a_n^u u_n + c_i^u + \sum_n \lambda_n^u p_n \quad (12)$$

$$a_i^v v_i = \sum_n a_n^v v_n + c_i^v + \sum_n \lambda_n^v p_n \quad (13)$$

$$a_i^w w_i = \sum_n a_n^w w_n + c_i^w + \sum_n \lambda_n^w p_n \quad (14)$$

$$a_i^p p_i = \sum_n a_n^p p_n + b_i^p \quad (15)$$

where  $i$  is a typical node and  $n$  represents

neighboring nodes around  $i$ .

The above discretization equations which form the coupled set of algebraic equations are solved simultaneously by using iteration solvers. When the proper limit of convergence can't be obtained, the corresponding under-relaxation method is used as a means of settlement.

### 3. Experiments

Though a 3-D circular braiding machine, the braided perform having a repeating structure per pitch length is made directly. The circular braided perform is composed of twenty four macro-unit cells in circumference and the macro-unit cell has five inner unit cells and two surface unit cells as shown in Fig. 2. It is assumed that the cross-section of yarn is ellipse and the path of yarn is spline functions. Also, in order to compare with experimental results, curved unit cells is regarded as straightened unit cells.

### 4. Flow analysis for unit cell

By using numerical method presented above, the fields of velocity and pressure are solved for macro-unit cell and total flow rate,  $Q$  is calculated. Then the permeability for the macro-cell is obtained through following equation.

$$K = \frac{Q\mu}{A\Delta p} \quad (16)$$

where  $A$  is the cross-sectional area. Pressures are imposed at the inlet and outlet planes of the unit cell and periodic boundary conditions and on other planes, a proper boundary conditions is utilized. Also the velocities on yarn surfaces are given as zero.

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Fig. 2. Macro-unit cell of braided preform

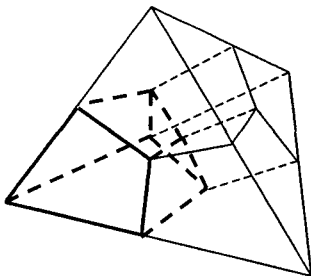


Fig. 1. Division of tetrahedral element into parts of polyhedral control volume.