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# 유체가 채워진 실린더형 공동에 의한 탄성파 공명 산란 해석

## Elastic Wave Resonance Scattering from a Fluid-filled Cylindrical Cavity

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**Key Words :** resonance scattering(공명 산란), elastic wave(탄성파), mode conversion(모우드 변환), longitudinal wave(종파), shear wave(횡파)

ABSTRACT

A new method is presented for the isolation of resonances from scattered waves for elastic wave resonance scattering problems. The resonance scattering function consisting purely of resonance information is defined. Elastic wave resonance scattering from a water-filled cylindrical cavity imbedded in an aluminum matrix is numerically analyzed. The classical resonance scattering theory and the new method compute different magnitudes and phases of the resonances from each partial wave, and therefore, their total resonance spectra are quite different. The exact $\pi$ - radians phase shifts through the resonance and anti-resonance frequencies show that the proposed method properly extracts the vibrational resonance information of the scatterer compared to resonance scattering theory.

## 1. Introduction

In this paper, the concept of the new resonance formalism, previously developed for acoustic wave scattering in Ref(1), is extended to elastic wave scattering that involves mode conversion phenomena. Elastic wave scattering from a cylindrical water-filled cavity is numerically analyzed using both the conventional resonance scattering theory (RST) and the proposed new resonance formalism. The new resonance formalism computes different magnitudes and phases of the resonances of each partial wave from previously published studies [2]. The proposed method generates physically meaningful behavior of phases compared to the conventional RST.

## 2. RESONANCE SCATTERING THEORY FOR ELASTIC WAVE SCATTERING

As shown in Fig. 1, we consider the problem of a plane elastic wave incident normally on an infinitely long cylindrical fluid-filled cavity of radius $a$ which is aligned with the $Z$ - axis.

A plane $P$ wave with unit magnitude incident along the $X$ axis can be expressed as a partial wave series,

$$\varphi_{inc} = \exp i(k_p X - \omega t)$$

$$= \sum_{n=0}^{\infty} \varepsilon_n i^n J_n(k_p r)\cos n\phi, \qquad (1)$$

where $k_p = \omega / c_p$ is the longitudinal wave number, $c_p$ is the speed of longitudinal waves in the elastic medium, $n$ is the normal mode number, $\varepsilon_n$ is the Neumann factor, and $J_n$ is the Bessel function. The time dependent term is suppressed in the right hand side of Eq. (1) because the steady state is assumed.

The scattered elastic waves have the following form of potentials:

$$\varphi_{sc} = \sum_{n=0}^{\infty} \varepsilon_n i^n A_n H_n^{(1)}(k_p r)\cos n\phi, \qquad (2a)$$

$$\psi_{sc} = \sum_{n=0}^{\infty} \varepsilon_n i^n B_n H_n^{(1)}(k_s r)\sin n\phi, \qquad (2b)$$

where $k_s = \omega / c_s$ is the transversal wave number, $c_s$ is the speed of the transverse waves in the elastic medium, and $H_n^{(1)}$ is the Hankel function of the first kind. The incident *P* wave has created an outgoing shear wave (*S* wave) of Eq. (2b), which represents a phenomenon known as mode conversion. In addition, a standing compressional wave is excited in the fluid-filled cavity given by

$$\varphi_f = \sum_{n=0}^{\infty} \varepsilon_n i^n E_n J_n(k_f r)\cos n\phi, \qquad (3)$$

where $k_f = \omega / c_f$ is the wave number in the fluid, and $c_f$ is the speed of the longitudinal waves in the fluid. The expansion coefficients $A_n, B_n$ and $E_n$ can be determined from the appropriate boundary conditions.

By applying the displacement and stress equations to the boundary conditions, we can obtain matrix equations for the expansion coefficients as follows:

$$\begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} A_n \\ B_n \\ E_n \end{pmatrix} = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}. \qquad (4)$$

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Letting

$$x_p \equiv k_p a, \quad x_s \equiv k_s a, \quad x_f \equiv k_f a, \qquad (5)$$

the matrix elements in Eq. (4) are:

$$g_{11} = x_p H_n^{(1)}(x_p),$$

$$g_{12} = n H_n^{(1)}(x_s),$$

$$g_{13} = -x_f J_n'(x_f),$$

$$g_{21} = (2n^2 - x_s^2) H_n^{(1)}(x_p) - 2x_p H_n^{(1)'}(x_p),$$

$$g_{22} = -2n H_n^{(1)}(x_s) + 2n x_s H_n^{(1)'}(x_s),$$

$$g_{23} = (\rho_f / \rho) x_s^2 J_n(x_f),$$

$$g_{31} = 2n( H_n^{(1)}(x_p) - x_p H_n^{(1)'}(x_p)),$$

$$g_{32} = x_s^2 H_n^{(1)}(x_s) + 2x_s H_n^{(1)'}(x_s) - 2n^2 H_n^{(1)}(x_s),$$

$$g_{33} = 0,$$

$$h_i = -\mathrm{Re}[g_{i1}] \,(for \;\; i = 1,2,3), \qquad (6)$$

$\rho$ and $\rho_f$ are densities of the elastic medium and the cavity fluid, respectively.

In the far-field where $r >> a$, the far-field form function $f_\infty^{pp}$ and $f_\infty^{ps}$ are defined as summations of the individual normal modes $f_n^{pp}$'s and $f_n^{ps}$'s, respectively, of scattered waves as follows:

$$f_\infty^{pp}(\phi) = \sum_{n=0}^{\infty} f_n^{pp}(\phi) = \sqrt{\frac{2}{\pi i x_p}} \sum_{n=0}^{\infty} \varepsilon_n A_n \cos n\phi, \quad (7a)$$

$$f_\infty^{ps}(\phi) = \sum_{n=0}^{\infty} f_n^{ps}(\phi) = \sqrt{\frac{2}{\pi i x_s}} \sum_{n=0}^{\infty} \varepsilon_n B_n \sin n\phi. \quad (7b)$$

For *S* wave incidence case, similarly we have

$$\psi_{inc} = \exp i(k_s X - \omega t)$$
$$= \sum_{n=0}^{\infty} \varepsilon_n i^n J_n(k_s r) \cos n\phi. \qquad (8)$$

$$\varphi_{sc} = \sum_{n=0}^{\infty} \varepsilon_n i^n C_n H_n^{(1)}(k_p r) \sin n\phi, \qquad (9a)$$

$$\psi_{sc} = \sum_{n=0}^{\infty} \varepsilon_n i^n D_n H_n^{(1)}(k_s r) \cos n\phi, \qquad (9b)$$

$$\varphi_f = \sum_{n=0}^{\infty} \varepsilon_n i^n F_n J_n(k_f r) \sin n\phi. \qquad (9c)$$

By applying the same boundary conditions, we obtain

$$\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \begin{pmatrix} C_n \\ D_n \\ F_n \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}, \qquad (10)$$

where

$$m_{11} = g_{11}, m_{12} = -g_{12}, m_{13} = g_{13}, m_{21} = g_{21},$$

$$m_{22} = -g_{22}, m_{23} = g_{23}, m_{31} = -g_{31}$$

$$m_{32} = g_{32}, \quad m_{33} = g_{33}, \text{ and}$$

$$q_i = -\mathrm{Re}[m_{i2}] \,(for \;\; i = 1,2,3).$$

By the similar procedure as the *P* wave incidence case, the far-field form functions $f_\infty^{sp}$ and $f_\infty^{ss}$ are defined as follows:

$$f_\infty^{sp}(\phi) = \sum_{n=0}^{\infty} f_n^{sp}(\phi) = \sqrt{\frac{2}{\pi i x_p}} \sum_{n=0}^{\infty} \varepsilon_n C_n \sin n\phi, \quad (11a)$$

$$f_\infty^{ss}(\phi) = \sum_{n=0}^{\infty} f_n^{ss}(\phi) = \sqrt{\frac{2}{\pi i x_s}} \sum_{n=0}^{\infty} \varepsilon_n D_n \cos n\phi. \quad (11b)$$

The scattering matrix is defined as

$$S_n = \begin{pmatrix} S_n^{pp} & S_n^{sp} \\ S_n^{ps} & S_n^{ss} \end{pmatrix} = \begin{pmatrix} 1 + 2A_n & -2C_n \\ 2B_n & 1 + 2D_n \end{pmatrix}. \qquad (12)$$

The unitarity property of the scattering matrix $S_n$, as the result of energy conservation, requires that

$$\left|S_n^{pp}\right|^2 + \left|S_n^{ps}\right|^2 = 1, \qquad (13a)$$

$$\left|S_n^{ss}\right|^2 + \left|S_n^{sp}\right|^2 = 1, \qquad (13b)$$

$$S_n^{pp\,*} \cdot S_n^{sp} + S_n^{ps\,*} \cdot S_n^{ss} = 0, \qquad (13c)$$

$$S_n^{sp\,*} \cdot S_n^{pp} + S_n^{ss\,*} \cdot S_n^{ps} = 0, \qquad (13d)$$

The far-field form functions in Eqs. (7) and (11) can be rewritten in terms of the scattering functions as

$$f_\infty^{pp}(\phi) = \sqrt{\frac{2}{\pi i x_p}} \sum_{n=0}^{\infty} \varepsilon_n \frac{1}{2}(S_n^{pp} - 1) \cos n\phi, \quad (14a)$$

$$f_\infty^{ps}(\phi) = \sqrt{\frac{2}{\pi i x_s}} \sum_{n=0}^{\infty} \varepsilon_n \frac{1}{2} S_n^{ps} \sin n\phi, \quad (14b)$$

$$f_\infty^{sp}(\phi) = \sqrt{\frac{2}{\pi i x_p}} \sum_{n=0}^{\infty} \varepsilon_n (-\frac{1}{2} S_n^{sp}) \sin n\phi, \quad (14c)$$

$$f_\infty^{ss}(\phi) = \sqrt{\frac{2}{\pi i x_s}} \sum_{n=0}^{\infty} \varepsilon_n \frac{1}{2}(S_n^{ss} - 1) \cos n\phi. \quad (14d)$$

Magnitudes and phases of one of the normal modes which make up the form functions in Eq. (14) is plotted in Fig. 2 as a function of non-dimensionalized frequency $x_p$. The density of aluminum was taken as $\rho$ = 2800 $kg \cdot m^{-3}$, and the speed of compressional and shear waves as $c_p$ = 6370 and $c_s$ = 3070 $m \cdot \sec^{-1}$, respectively. The density of water was also taken as $\rho_f$ = 1000 $kg \cdot m^{-3}$, and the compressional wave speed in water was taken as $c_f$ = 1480 $m \cdot \sec^{-1}$, In Fig. 1 the shape of the magnitude plot is seen to be fairly complicated functions of the frequency, consisting of relatively broad plateaus but punctuated by sharp maxima and minima. Resonance scattering theory claimed that the complicated shapes of each partial waves are due to the summation of the resonances of the fluid cavity and the smoothly-varying background[2]. Based on resonance scattering theory, the resonances have been obtained by subtracting the proper background from each partial waves in the references as

$$f_n^{(b)res,\nu\sigma} = f_n^{\nu\sigma} - f_n^{(b)\nu\sigma}, \quad (15)$$

where *res* represents *resonance* and $b$ in the parenthesis stands for *background.* $b$ may be $s$ for the soft background or $r$ for the rigid background. $\nu$ and $\sigma$ represent $p$ or $s$, appropriately. Eq. (15) can be written as

$$f_n^{(b)res,pp} = \sqrt{\frac{2}{\pi i x_p}}\,\varepsilon_n(A_n - A_n^{(b)})\cos n\phi, \quad (16a)$$

$$f_n^{(b)res,ps} = \sqrt{\frac{2}{\pi i x_s}}\,\varepsilon_n(B_n - B_n^{(b)})\sin n\phi, \quad (16b)$$

$$f_n^{(b)res,sp} = \sqrt{\frac{2}{\pi i x_p}}\,\varepsilon_n(C_n - C_n^{(b)})\sin n\phi, \quad (16c)$$

$$f_n^{(b)res,s} = \sqrt{\frac{2}{\pi i x_s}}\,\varepsilon_n(D_n - D_n^{(b)})\cos n\phi, \quad (16d)$$

In the present case, the soft background corresponding to an empty cavity is chosen as the proper background because the acoustical impedance of the cavity fluid is smaller than that of the elastic (aluminum) medium. Fig. 3 (dotted curves) show the "difference" curves computed by Eq. (16) using the soft background. The sharp peaks in magnitudes in these figures have been considered as the resonances of the target in the previous works[2]. We, however, want to note that the behavior of phases obtained by this "background subtraction" method is not physically explainable as can be seen in Fig. 3 although it is well known that the phase of a resonance should shift by $\pi$ radians as the frequency passes through the resonance frequency. By this reason, the phases of resonances have not usually been presented or discussed in open literatures except some limited discussions as already mentioned in Ref.1. Therefore, we can argue that Eq. (16) may not correctly isolate the resonances of targets although the magnitude plots show resonance-like features.

## 3. NOVEL FORMALISM FOR ELASTIC WAVE RESONANCE SCATTERING

The scattering functions in Eq. (12) can be expressed as

$$S_n^{\nu\sigma} = S_n^{(s)\nu\sigma} S_n^{(s)*} = S_n^{(s)\nu\sigma}\left(\frac{z_{1n}^{\nu\sigma} - F_n}{z_{2n}^{\nu\sigma} - F_n}\right), \quad (17a)$$

or

$$S_n^{\nu\sigma} = S_n^{(r)\nu\sigma} S_n^{(r)*} = S_n^{(r)\nu\sigma}\left(\frac{1/z_{1n}^{\nu\sigma} - 1/F_n}{1/z_{2n}^{\nu\sigma} - 1/F_n}\right), \quad (17b)$$

where the $z$ 's are acoustoelastic impedances which are ratios of two 2 x 2 minor determinants, and $F_n$ is the modal mechanical impedance of the fluid cavity. The superscript $s$ or $r$ in the parenthesis denotes the soft or rigid background, respectively. $S_n^{(s)\nu\sigma}$ and $S_n^{(r)\nu\sigma}$, which are the scattering functions corresponding to the soft and rigid cylinders, respectively, can be determined by Eq. (12) with the matrix equations for an impenetrable soft or rigid cylinder.

Eq. (17) states that $S_n^{\nu\sigma}$ is the product of the background $S_n^{(s)\nu\sigma}$ (or $S_n^{(r)\nu\sigma}$) and the remaining term $S_n^{(s)*}$ (or $S_n^{(r)*}$) which includes resonances. However, $S_n^{(s)*}$ and $S_n^{(r)*}$ are not pure resonance terms. $S_n^{(s)*}$ and $S_n^{(r)*}$ contain a real unit constant which hides resonances unless it is removed. To see this, $S_n^{(s)*}$ and $S_n^{(r)*}$ may be written as

$$S_n^{(s)*} = \frac{z_{1n}^{\nu\sigma} - F_n}{z_{2n}^{\nu\sigma} - F_n} = \frac{z_{1n}^{\nu\sigma} - z_{2n}^{\nu\sigma}}{z_{2n}^{\nu\sigma} - F_n} + 1 = S_n^{(s)res,\nu\sigma} + 1, \quad (18a)$$

and

$$S_n^{(r)*} = \frac{1/z_{1n}^{\nu\sigma} - 1/F_n}{1/z_{2n}^{\nu\sigma} - 1/F_n} = \frac{1/z_{1n}^{\nu\sigma} - 1/z_{2n}^{\nu\sigma}}{1/z_{2n}^{\nu\sigma} - 1/F_n} + 1 = S_n^{(r)res,\nu\sigma} + 1, \quad (18b)$$

where $S_n^{(s)res,\nu\sigma}\left(= \frac{S_n^{\nu\sigma}}{S_n^{(s)\nu\sigma}} - 1\right)$ and $S_n^{(r)res,\nu\sigma}\left(= \frac{S_n^{\nu\sigma}}{S_n^{(r)\nu\sigma}} - 1\right)$ are defined as the resonance scattering functions which consist purely of resonance information of the scatterer. The unit constant in $S_n^{(s)*}$ or $S_n^{(r)*}$ should be subtracted in order to obtain the resonances because adding a constant term to a complex quantity changes both magnitude and phase of the original complex quantity.

By the definition in Eq. (18), the resonance scattering functions can be written as

$$S_n^{(s)res,pp} = \frac{z_{1n}^{pp} - z_{2n}^{pp}}{z_{2n}^{pp} - F_n} = \frac{S_n^{pp}}{S_n^{(s)pp}} - 1 = 2\frac{A_n - A_n^{(s)}}{1 + 2A_n^{(s)}},$$

$$S_n^{(s)res,ps} = \frac{z_{1n}^{ps} - z_{2n}^{ps}}{z_{2n}^{ps} - F_n} = \frac{S_n^{ps}}{S_n^{(s)ps}} - 1 = \frac{B_n - B_n^{(s)}}{B_n^{(s)}},$$

$$S_n^{(s)res,sp} = \frac{z_{1n}^{sp} - z_{2n}^{sp}}{z_{2n}^{sp} - F_n} = \frac{S_n^{sp}}{S_n^{(s)sp}} - 1 = \frac{C_n - C_n^{(s)}}{C_n^{(s)}},$$

$$S_n^{(s)res,ss} = \frac{z_{1n}^{ss} - z_{2n}^{ss}}{z_{2n}^{ss} - F_n} = \frac{S_n^{ss}}{S_n^{(s)ss}} - 1 = 2\frac{D_n - D_n^{(s)}}{1 + 2D_n^{(s)}} \quad , \tag{19}$$

From Eq. (19), the relationship among the expansion coefficients, backgrounds and the resonance scattering functions can be expressed as

$$A_n = A_n^{(s)} + \tfrac{1}{2} S_n^{(s)res,pp} + A_n^{(s)} S_n^{(s)res,pp}, \tag{20a}$$

$$B_n = B_n^{(s)} + B_n^{(s)} S_n^{(s)res,ps}, \tag{20b}$$

$$C_n = C_n^{(s)} + C_n^{(s)} S_n^{(s)res,sp}, \tag{20c}$$

$$D_n = D_n^{(s)} + \tfrac{1}{2} S_n^{(s)res,ss} + D_n^{(s)} S_n^{(s)res,ss}. \tag{20d}$$

Note that from Eq. (20) we can easily see that the resonance scattering function always interacts with the scattering function corresponding to the background as a product term. In other words, Eq. (20) clearly shows that the total scattering is not just a simple summation of the backgrounds and resonances as expressed in Eq. (16), but the total scattering always includes a interaction term between the resonance and background.

In order to extract the resonances for *PP* case (*P* wave scattering with *P* wave incidence), $S_n^{pp} - 1$ in Eq. (15a) should be replaced by the corresponding resonance scattering function $S_n^{(s)res,pp}$ by the same rationale in Ref.1. For *PS* case (*S* wave scattering with *P* wave incidence), $S_n^{ps}$ in Eq. (14b) should be replaced by the corresponding resonance scattering function $S_n^{(s)res,ps}$. As such, the resonance information of scatterers can be obtained as follows :

$$f_\infty^{(s)res,pp}(\phi) = \sum_{n=0}^{\infty} f_n^{(s)res,pp}$$
$$= \sqrt{\frac{2}{\pi i x_p}} \sum_{n=0}^{\infty} \varepsilon_n \frac{1}{2} S_n^{(s)res,pp} \cos n\phi$$
$$= \sqrt{\frac{2}{\pi i x_p}} \sum_{n=0}^{\infty} \varepsilon_n \frac{A_n - A_n^{(s)}}{1 + 2A_n^{(s)}} \cos n\phi, \tag{21a}$$

$$f_\infty^{(s)res,ps}(\phi) = \sum_{n=1}^{\infty} f_n^{(s)res,ps}$$
$$= \sqrt{\frac{2}{\pi i x_s}} \sum_{n=1}^{\infty} \varepsilon_n \frac{1}{2} S_n^{(s)res,ps} \sin n\phi$$
$$= \sqrt{\frac{2}{\pi i x_s}} \sum_{n=1}^{\infty} \varepsilon_n \frac{1}{2} \frac{B_n - B_n^{(s)}}{B_n^{(s)}} \sin n\phi, \tag{21b}$$

$$f_\infty^{(s)res,sp}(\phi) = \sum_{n=1}^{\infty} f_n^{(s)res,sp}$$
$$= \sqrt{\frac{2}{\pi i x_p}} \sum_{n=1}^{\infty} \varepsilon_n \frac{1}{2} S_n^{(s)res,sp} \sin n\phi$$
$$= \sqrt{\frac{2}{\pi i x_p}} \sum_{n=1}^{\infty} \varepsilon_n \frac{1}{2} \frac{C_n - C_n^{(s)}}{C_n^{(s)}} \sin n\phi \tag{21c}$$

$$f_\infty^{(s)res,ss}(\phi) = \sum_{n=0}^{\infty} f_n^{(s)res,ss}$$
$$= \sqrt{\frac{2}{\pi i x_s}} \sum_{n=0}^{\infty} \varepsilon_n \frac{1}{2} S_n^{(s)res,ss} \cos n\phi$$
$$= \sqrt{\frac{2}{\pi i x_s}} \sum_{n=0}^{\infty} \varepsilon_n \frac{D_n - D_n^{(s)}}{1 + 2D_n^{(s)}} \cos n\phi, \tag{21d}$$

Using Eq. (21), the resonance information, which are interacting with the background as seen in Eq. (20), can be uncovered. The only difference between Eq. (16) and Eq. (21) is the existence of the denominator, which is equal to the soft background scattering function. This fact is consistent with acoustic wave resonance scattering problem in Ref.1. However, unlike acoustic wave scattering, the background scattering functions in the denominators of Eq. (21) are not unitary except for $n$ = 0 mode due to mode conversion. Therefore, as we predicted in Ref.1, both magnitudes and phases of the resonances of the individual normal modes computed by Eqs. (16) and (21) are generally different for elastic wave resonance scattering, while, for acoustic wave scattering, their magnitudes are always identical.

## 4. NUMERICAL ANALYSIS AND DISCUSSION

We consider the backscattering case ( $\phi = \pi$ ) for *PP* and *SS* cases. For the case of longitudinal wave scattering with longitudinal wave incidence (*PP*), mode conversion does not occur when the normal mode number $n$ = 0. Therefore, in Fig. 3(a) we obtain identical magnitudes by the previous [Eq. (15)] and new [Eq. (21)] methods. This is because the denominator $1 + 2A_n^{(s)}$, which is $S_n^{(s)pp}$, in Eq. (21a) is unitary as can easily be noticed in Nyquist plot. However, their phases are quite different because $S_n^{(s)pp}$ has its own phase shift. While the phases obtained by the previous method show physically unexplainable behavior, the new method generates exact $\pi$ - phase shifts through the resonances and at the anti-resonances. For $n \geq 1$, as shown in Fig. 2 (b), even magnitudes computed by the two methods are different because $S_n^{(s)pp}$ in the denominator in Eq. (21a) is not unitary due to mode conversion, which is clearly shown from the trajectory of $S_n^{(s)pp}$ in the complex plane. (For $n \geq 1$ , the unitarity property applies to the scattering matrix $S_n$ or $S_n^{(s)}$ rather than the individual scattering function $S_n^{\nu\sigma}$ or $S_n^{(s)\nu\sigma}$) In Fig. 2(b), the phase shifts through resonance peaks computed by the new method are not exactly $\pi$ radians especially in the low frequency region. However, the phase shifts converge to $\pi$ radians as the frequency increases, accordingly, mode-converted energy decreases. This is a physically reasonable behavior because the energy leakage due to mode conversion affects the amount of phase shifts as numerically examined in Ref. 3.

$S_n^{pp} - S_n^{(s)pp}$ was mentioned as "background removed" by Solomon *et al* [4], however, it is not considered as a correct statement in this paper. $S_n^{(s)res,pp}$ is actually more reasonable "background removed" information which consists purely of the resonances.

For the case of shear wave scattering with shear wave incidence (*SS*), $f_n^{ss}$ and $f_n^{(s)ss}$ are exactly the same when the normal mode number $n = 0$[5], which implies that the cavity fluid is not excited at all. Therefore, $S_n^{(s)res,ss}$ and the resonances are null. Fig. 8 compares the resonances computed by the new and previous methods for *SS* case, $n = 1,2$, and 3 modes. As *PP* case, both magnitude and phase are differently obtained by the two methods. As *PP* case, the phases computed by the new method show physically explainable behavior compared with the previous method

For a mode converted case such as shear wave scattering with longitudinal wave incidence (*PS*), we basically see the same trend with *PP* or *SS* case in the differences between the isolated resonances by the two methods. The new method generates physically explainable behavior of the phases compared with the previous method. But, the differences in magnitudes of the resonances are larger than *PP* or *SS* case. This is because the magnitude of the denominator, which is $S_n^{(s)ps}$, in Eq. 21(b), is smaller than unity. The detailed discussion on the mode conversion cases will be made in a separate paper.

Due to the differences in the phases and magnitudes of resonances of each partial wave, the total resonance spectra computed by the two methods are quite different. An example for *PP* case is shown in Fig. 4.

Although we analyzed a relatively simple example of elastic wave scattering problems for a convenience in this paper, we can apply the novel formalism developed here to more complex problems such as elastic wave scattering from cylindrical (or spherical) elastic (or fluid) bodies in a similar manner.

## 5. CONCLUDING REMARKS

In this paper, a novel formalism for the isolation of resonances from partial waves for elastic wave scattering is proposed. The concept of the resonance scattering function consisting purely of the resonance information, which was originally developed for acoustic wave scattering, has been extended to elastic wave scattering problems. Plane compressive and shear wave scatterings from an fluid-filled cylinder imbedded in an aluminum matrix are numerically analyzed by utilizing the proposed resonance scattering functions, and the isolated resonances are compared with the results of previous studies. For a non- mode conversion case such as longitudinal wave scattering with longitudinal wave incidence when the normal mode number $n = 0$, both the new and previous methods calculate identical magnitudes. However, the new method generates exact $\pi$ radians phase shifts through resonances and at the anti-resonances while the previous method produces phases which are physically unexplainable. For mode conversion cases such as $n \geq 1$ modes, both magnitudes and phases of the resonances of each partial wave isolated by the two methods are different. While the behavior of the phases obtained by the previous method are not physically explainable, the new method computes the phases which show reasonable behavior. In the low frequency region, where mode-converted energy is large, the phase shifts are close to but not exactly $\pi$ radians. However, as the frequency increases, accordingly, the mode-converted energy becomes smaller, the phase shifts converge to $\pi$ radians. Based on this fact the proposed formalism properly extracts the resonances from scattered waves.

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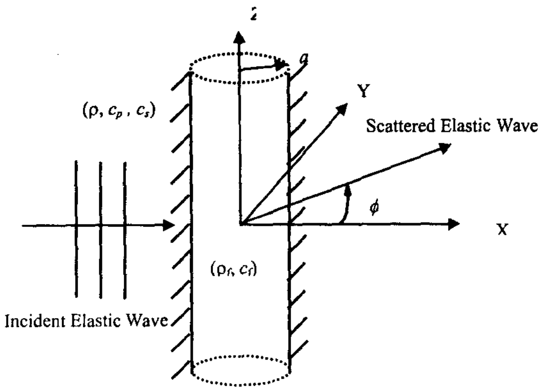



FIG. 1 Geometry of elastic wave scattering from an infinitely long cylindrical fluid-filled cavity

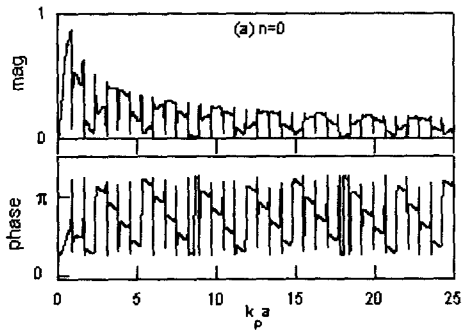
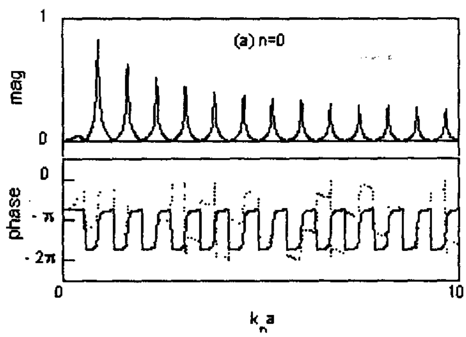



FIG. 2 *n*th partial wave for a water-filled cylindrical cavity imbedded in a aluminum matrix



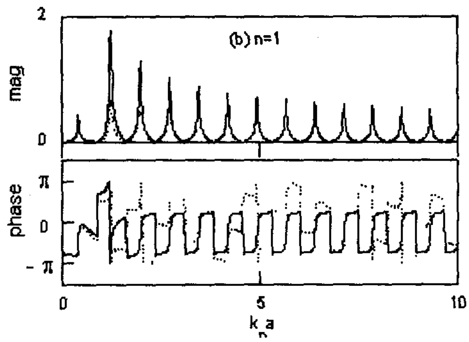




FIG. 3 Comparison of isolated resonances by the new method(solid) and RST(dotted) for *PP* case

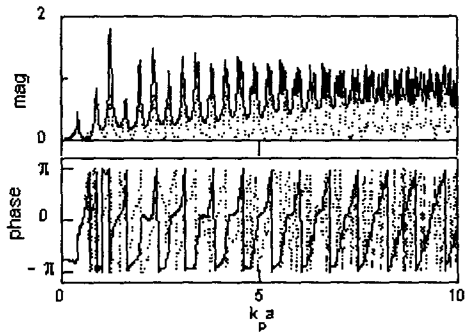



FIG. 4 Total resonance spectra by the new method (solid) and RST (dotted) for *PP* case