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A Numerical and Experimental Study on Dynamics of A Towed Low-Tension Cable

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ABSTRACT

The paper presents a numerical and experimental investigation on dynamic behaviors of a towed low tension cable. In the numerical study, an implicit finite difference algorithm is employed for three-dimensional cable equations. Fluid and geometric non-linearity and bending stiffness are considered and solved by Newton-Raphson iteration. Block tri-diagonal matrix method is applied for the fast calculation of the huge size of matrices. In order to verify the numerical results and to see real physical phenomena, an experiment is carried out for a 6m cable in a deep and long towing tank. The cable is towed in two different ways; one is towed at a constant speed and the other is towed at a constant speed with top end horizontal oscillations. Cable tension and shear forces are measured at the top end. Numerical and experimental results are compared with good agreements in most cases but with some differences in a few cases. The differences are due to drag coefficients caused by vortex shedding. In the numerical modeling, non-uniform element length needs to be employed to cope with the sharp variation of tension and shear forces at near top end.

Key Words: towed cable; low tension; 3-D numerical method; finite non-uniform differences; physical experiment.

INTRODUCTION

Cable problems can be divided into several categories but a primary distinction can be made as a highly tensioned (taut) cable and a low-tension cable. Most researches and applications have been made for taut cables. General introduction to advanced mechanics of a cable is given in a text-book (Irvine, 1981). There have been lots of researches on nonlinear mechanics of taut marine cables. Triantafyllou(1991) described research trends on dynamics of cables in detail.

However, low-tension cables have increasingly used in recent years due to the advent of synthetic cables that are almost neutrally buoyant in water. A low-tension problem means that dynamic tension is of the same order as static tension. This fact makes cable mechanics be particularly complex to tackle. Because of relatively small restoring force of tension, large displacements may occur and thus the effect of geometric non-linearity becomes dominant. Apart from the difficulty, the dynamics of low-tension cables are further complicated by isolated points of zero tension. The most obvious way to eliminate singular behavior for zero tension is to consider bending stiffness in the regions.

Leonard (1972) investigated low-tension slack cables for the first time. After that work, some advanced researches have been made on the dynamics of a low-tension marine cable (Dowing, 1988; Triantafyllou and Triantafyllou, 1991; Howell, 1992; Triantafyllou and Howell, 1994).

The hydrodynamic coefficients should be carefully chosen in a towed low tension cable analysis. Normal drag coefficient has been more thoroughly studied and is known to be dependent on Reynolds number, K-C number and roughness (Sarpkaya and Isaacson, 1981). Nevertheless, there is uncertainty also due to the vortex shedding effect in a certain range of values.

In this study, a numerical program is developed to simulate the dynamic behavior of a towed low tension cable. In order to verify the numerical results and to see real physical phenomena, an experiment is carried out for a 6m cable in a deep and long towed tank. The cable is towed in two different ways; one is towed at a constant speed and the other is towed at a constant speed with top end horizontal oscillations. Cable tension and shear forces are measured at the top end.

NATURAL FREQUENCY

The natural frequency of a free hanging cable needs to be obtained to analyze the response frequency of the towed cable. The natural frequency is dependent on mass, bending stiffness, tension (self-weight) and boundary conditions. In the case of a cable, bending stiffness effect is very small compared to tension effect and can be neglected. A closed-form solution for natural frequency of the model can be obtained by a Bessel Function. In this model case, tension effect is important and thus considered to be variant along its length.

The free vibration equation of the model is as follows;

$$\left[\frac{w_0' + w_2}{g}\right]\frac{\partial^2 y}{\partial t^2} - \frac{\partial}{\partial z}\left[w_1 z \frac{\partial y}{\partial z}\right] = 0 \qquad (1)$$

where, w_0': weight per unit length in water

w_1: weight per unit length in air

w_2: additional weight per unit length due to added mass and internal fluid

g : gravity acceleration

y : horizontal coordinate

z : vertical coordinate

In order to solve Eq. (1), the method of separation of variables is used as follows;

$$y(z,t) = Z(x)Q(t) \tag{2}$$

Then equations for $Q(t)$ are as follow;

$$(1+\beta)\frac{d^2Q}{dt^2} + k^2\beta_1 gQ = 0 \tag{3}$$

where, $\beta_1 = w_1/w_0'$, $\beta_2 = w_2/w_0'$ and k is an arbitrary constant.

Table 1 Cable properties in experiment

Length (m)	6.0
Diameter (m)	0.02
Weight in water (kg/m)	1.597
Weight in air (kg/m)	4.753
Elastic Modulus (N/m^2)	2.33E6

Table 2 Natural periods and frequencies of a free hanging cable

Mode number	Period(s)/Frequency(rad/s)	
	Freely hanging cable	Towed Cable
1st	8.85/0.71	16.58/0.38
2nd	3.89/1.62	7.22/0.87
3rd	2.47/2.54	4.61/1.36
4th	1.81/3.47	3.38/1.86
5th	1.43/4.40	2.76/2.35
6th	1.18/5.33	2.21/2.85
7th	1.00/6.26	1.90/3.34

The natural frequencies of the model, ω_n and the corresponding natural period, T_n can be obtained by solving Eq. (3) as follows (McLachlan, 1955);

$$\omega_n = \frac{2\pi}{T_n} = \frac{1}{2\sqrt{L}} j_{0,n}\sqrt{\frac{g\beta_1}{1+\beta_2}} \tag{4}$$

where, $j_{0,n}$ is the *nth* zero of $J_n(z)$

When the cable is towed in a speed, w_0 is changed due to drag force as follows;

$$w' = w_0' - \cos(\theta)F_{DN} + \sin(\theta)F_{DT} \tag{5}$$

where, F_{DN} : Normal drag force

F_{DT} : Tangential drag force

Using Eq.(4) and data given in Table 1, the first seven natural frequencies of the model are calculated and given in Table 2.

NUMERICAL ANALYSIS

In the numerical analysis, a centered finite difference method is employed. For time integration, implicit scheme is used since it unconditionally stable and more suitable for nonlinear cable dynamics.

Governing equations can be expressed in a matrix form as follows (Park and Jung, 2002);

$$\overrightarrow{JH}' = K\vec{H} + \vec{L} \tag{6}$$

where, $\vec{H} = (T, S_n, S_b, v_t, v_n, v_b, \phi, \theta, \Omega_n, \Omega_b)^T$

The cable is first discretized into n nodes separating by Δs and time is divided into a series of steps of length Δt. Using centered finite differences and evaluating Equation (6) at mid-node $j+1/2$ and at the time $i+1/2$ gives

$$[J_{j+1}^{i+1} + J_j^{i+1}]\frac{H_{j+1}^{i+1} - H_j^{i+1}}{\Delta s} + [J_{j+1}^{i} + J_j^{i}]\frac{H_{j+1}^{i} - H_j^{i}}{\Delta s} =$$
$$[K_{j+1}^{i+1} + K_{j+1}^{i}]\frac{H_{j+1}^{i+1} - H_{j+1}^{i}}{\Delta t} + [K_j^{i+1} + K_j^{i}]\frac{H_j^{i+1} - H_j^{i}}{\Delta t}$$
$$+ L_{j+1}^{i+1} + L_j^{i+1} + L_{j+1}^{i} + L_j^{i} \tag{7}$$

In the first numerical modeling, a uniform element length is employed but extraordinary results are come out. It is found that there is a region where tension and shear forces vary sharply and that fine elements are necessary near the fixed boundary. In the final version of the program, fifty non-uniform elements which increase linearly from top to bottom are employed and satisfactory results are obtained.

EXPERIMENTAL METHOD

Test Set-Up

In the model test, tension and shear force of a towed cable are measured at the top end point. The test model is made from a rubber pipe and its property is given in Table 1. The upper end is fixed to a towing cart and the bottom part is set to be free.

Experiments are carried out in a 7m deep towing tank that is designed and constructed for deep-water researches by Research Institute for Applied Mechanics in Kyushu University. Fig. 1 shows an experiment scene.

Table 3 Experiment list

Exp #	Towing speed (m/s)	Excitation period (sec)	Excitation amp. (m)	Acceleration (m/s^2)
Exp.301	0.4	0	0	0.04g
Exp.302	0.5	0	0	0.04g
Exp.303	0.6	0	0	0.04g
Exp.304	0.6	4	0.2	0.04g
Exp.305	0.6	6	0.2	0.04g
Exp.306	0.6	8	0.2	0.04g

At a starting transient situation, the model cable is accelerated at 0.04g by the cart and after a prescribed speed is reached the cable is towed in two different ways; one is towed at a constant speed and the other is towed at a constant speed with top end horizontal oscillations. The oscillation amplitude is 0.2m and the periods are 4, 6 and 8 seconds. Table 3 shows the experiment cases carried out in this study.

Experiment Results

Fig. 2 shows the time history of tension variation at top point and the corresponding energy spectrum for each constant towing speed. The units of horizontal axes for time history and spectrum are respectively second and hertz. Although a cable is towed at a constant speed, there is tension fluctuation in the time history. The fluctuation is regarded to be caused by vortex shedding. If top end boundary is simply supported, the effect of vortex shedding can be indirectly measured by bending strain at top point as was done by Welch and Pulin (1993). In this experiment, the top end boundary is fixed, so the effect of vortex shedding is obtained by measuring the tension fluctuation.

By looking at the spectrum in Fig. 2, we can find that the frequency of tension fluctuation is nearly equal for the three different towing speeds and about 4 hertz. The tension fluctuation frequency represents vortex shedding frequency. Vortex shedding frequency is normally related to a reduced velocity for this kind of model. Since the reduced velocities of the three cases are different but extraordinarily the vortex shedding frequency is identical. The unordinary phenomena should be studied further in the future. In the case of towing speed of 0.4m/s, there is a beating phenomenon for the tension variation.

Fig. 2 shows that the magnitude of tension fluctuation, i.e. vortex shedding intensity is inversely proportional to the towing speed. As can be seen from Table 4 obtained by numerical analysis, as the towing speed increases, the inclination angle of the towed cable becomes decreases. The phenomenon of vortex shedding intensity being weak at the higher speed is due to the decrease of inclination angle of the towed cable.

Table 4 Inclination angle according to towing speeds

Towing Speed(m/s)	Inclination Angle(°)
0.4	45
0.6	32
0.8	28

Fig. 3 represents the time history of tension variation and energy spectrum for towing speed of 0.6m/s with top end horizontal oscillations. There are two kinds of high and low frequencies in tension variation. The high frequency is originated from vortex shedding as was in Fig. 2. The energy spectrum of high frequency response by vortex shedding is distributed at wider range than that of the previous constant towing speed. Meanwhile the low frequency response comes from oscillating excitation at the top end. As can be seen from Fig. 3(a), the response amplitude of 4 second excitation period is largest of the three cases. The reason is probably due to 4 seconds excitation period being near the 3rd natural period of the towed cable.

COMPARISON BETWEEN NUMERICAL AND EXPERIMENTAL RESULTS

The results between numerical and experimental analyses are compared to verify the algorism of the developed program.

Mean Tension for Constant Towing Speed

It is very important to select accurate hydrodynamic coefficients in a towing cable analysis. As a general approach, a normal coefficient is chosen by considering normal component velocity of the towed cable and using the reference (Sarpkaya and Isaacson, 1981). A tangential coefficient is taken as 1% of the normal coefficient. They are given in Table 5.

Table 5 Drag coefficients for different towing speeds

Towing speed (m/s)	General Coefficients		Adjusted Coefficients	
	C_D	C_T	C_D'	C_T'
0.4	0.93	0.01	1.5	0.017
0.6	0.94	0.01	1.35	0.011
0.8	0.95	0.01	'1.15	0.008

Using the coefficients, mean tension and shear force of numerical and experimental analyses are compared for a constant towing speed (Fig. 4). There exist large differences between two methods. The large discrepancy is caused by inappropriate drag coefficients.

The normal and tangential coefficients are known to be dependent on Reynolds number and a tow angle in the towed cable case. As the tow cart accelerates to a prescribed speed, the hanging cable becomes gradually slanted until it reaches a steady state position. The inclined angle of the cable depends on the normal and tangential velocities. The normal and tangential drag coefficients must be properly adjusted based on the velocity.

Taylor(1951) found that the analogy between a heat transfer and a boundary layer equilibrium could be used for estimation of the C_T

$$C_t = B \tan \alpha^{1/2} Re_t^{-1/2} \quad L > L^* \tag{8}$$

where B a constant, α inclined angle, R_T Reynolds numbers in tangential direction. Taylor suggested a value of $B=1.2$ from heat transfer data, and Relf and Powell (1917) tested for inclined cables in high Reynolds number condition and proposed $B=1.72$

The towed cable is always affected by the vortex shedding to some extent. The easiest way to consider the effect is to increase the value of a drag coefficient. Skop et al. (1977) introduced an approximate equation for a drag coefficient with function of cylinder diameter and the response amplitude by vortex shedding as follows;

$$C_D' = C_{D0}[1 + 1.043(2A_v/D)^{0.65}] \tag{9}$$

where, C_{D0} : general drag coefficienthere,

C_D' : adjusted drag coefficient

A_v : amplitude of cross-flow displacement

It is very difficult to measure the amplitude in cross-flow displacement for a towed cable. In this study, the drag coefficient is thus increased in the ratio of the tension fluctuation amplitude. Table 5 gives ordinary and adjusted drag coefficients for different towing speeds.

As can be seen from Fig. 5, for the adjusted drag coefficients, the results of numerical analysis by the develop program, called KMU Cable are in good agreements with those of the experiment method. There are still small differences in shear forces between two analyses. However the magnitude of shear force is small compared to the tension.

Fig. 6 shows another comparison between two methods with adjusted drag coefficients. The time history of tension is compared for the case where the cable is towed at 0.6m/s speed and then the speed is reduced at 0.04g of negative acceleration. As the towing speed reduces, the tension gradually increases and finally reaches to the immersed weight of the cable. The result of numerical analysis is in a good agreement with that of experiment in the transient condition.

Tension for Constant Towing Speed with Horizontal Oscillation

Fig. 7 shows the comparison of the oscillating tension between numerical and experimental analyses for constant towing speed with horizontal oscillations. The cable velocity is dominated by towing speed rather than oscillating speed and thus the hydrodynamic coefficients is chosen from Table 5 for the case of 0.6m/s. In the case of 4 second oscillation period, the numerical result is in an excellent agreement with that of the experiment. There is slight discrepancy between two methods for other two excitation periods. This is presumably due to cross flow vibrations by vortex shedding at increased Keulegan-Carpenter(KC) number which is proportional to the excitation period. A further study is necessary to correct the error.

CONCLUSION

In this study, a three-dimensional numerical program is developed for the analysis of towed low-tension cables. An implicit finite difference method is employed with non-uniform element length. In order to solve the nonlinear and coupling problems, a Newton-Raphon iteration scheme is used and satisfactory results were obtained. In order to verify the numerical results, an experiment is carried out for a 6m cable in a deep and long towing tank. The cable is towed in two different ways; one is towed at a constant speed and the other is towed at a constant speed with top end horizontal oscillations. Numerical and experimental results of tension and shear forces are compared with good agreements in most cases but with some differences in a few cases. The differences are due to drag coefficients caused by vortex shedding. In the numerical modeling, employing a non-uniform element length gives good results especially for the region where the sharp variation of tension and shear forces exist.

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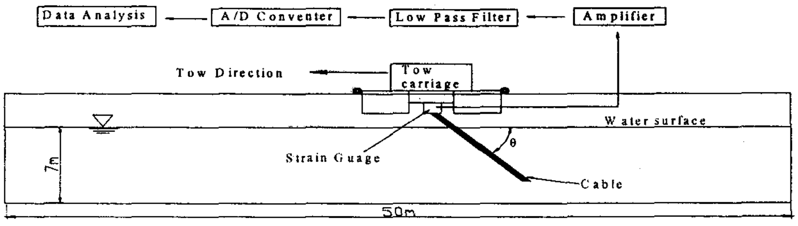
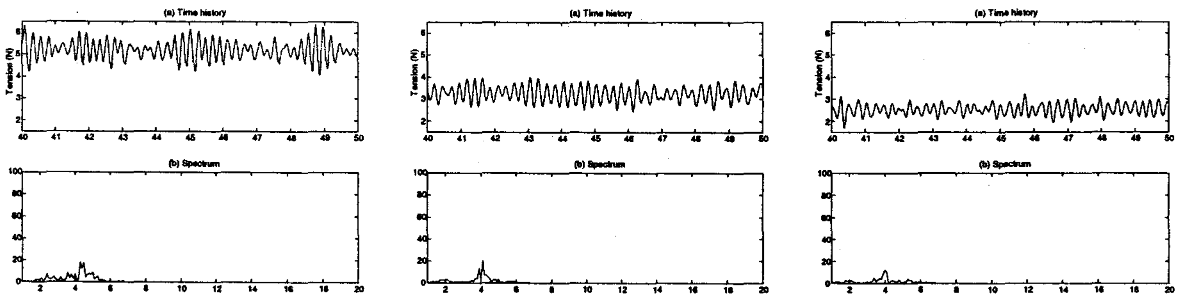
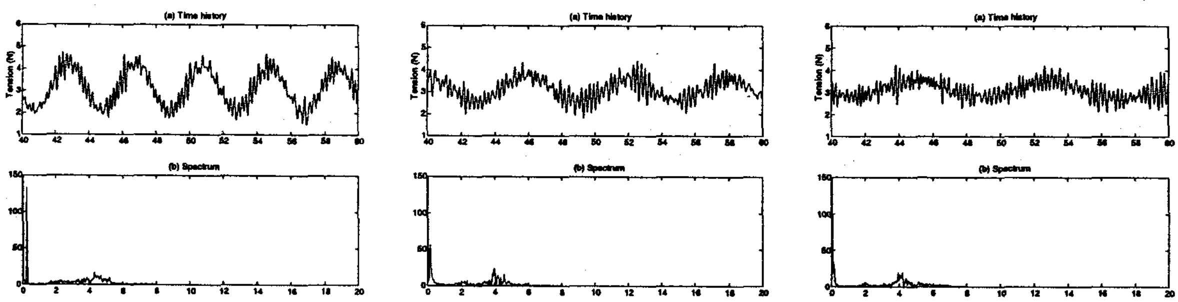


Fig. 1 Side view of tow carriage, tank and experimental apparatus



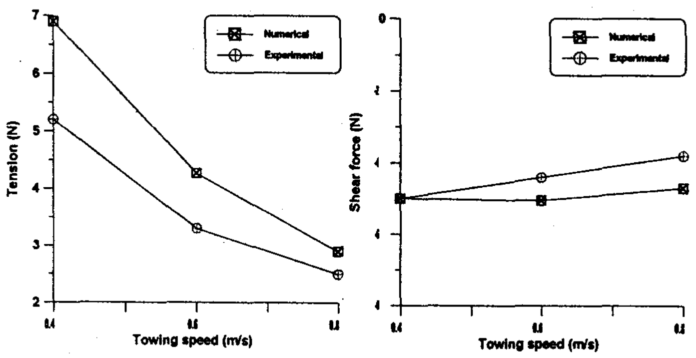
(a) Towing speed = 0.4m/s (b) Towing speed = 0.6m/s (c) Towing speed = 0.8m/s

Fig. 2 Tension and energy spectrum for different towing speeds



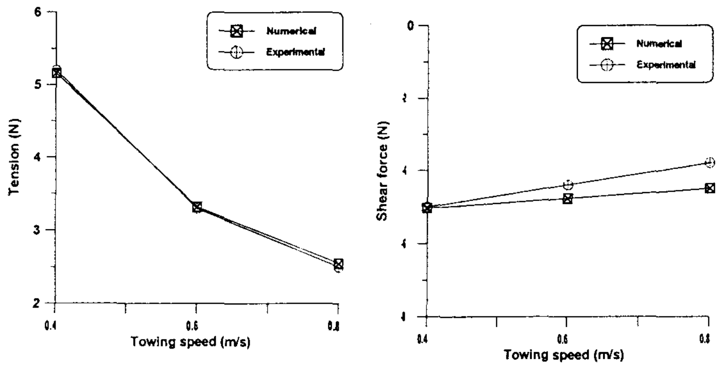
(a) T = 4 (sec) (b) T = 6 (sec) (c) T = 8 (sec)

Fig. 3 Tensions and energy spectrum in towing with excitation



(a) Tension (b) Shear force

Fig. 4 Comparison of the mean tension and shear (Hydrodynamic coefficients is chosen by considering just a normal velocity to the towed cable)



(a) Tension

(b) Shear force

Fig. 5 Comparison of mean tension and shear force between numerical and experiment analysis (Hydrodynamic coefficients is adjusted by considering vortex shedding effect)

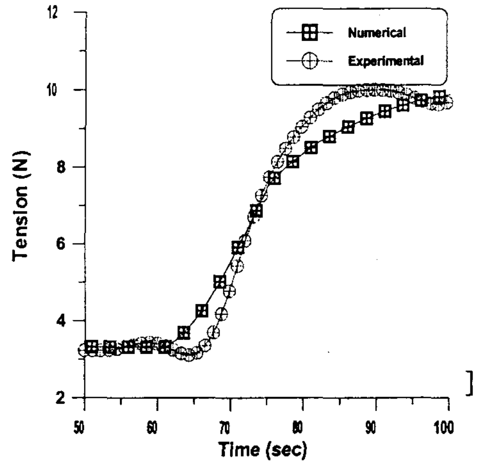
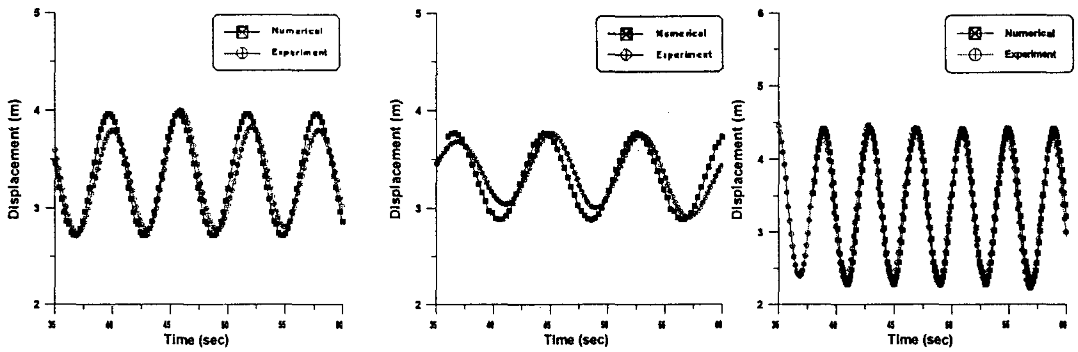


Fig. 6 Comparison of tension variation between numerical and experiment results at starting transient situation for towing speed of 0.6m/s and and deceleration of 0.04g



(b) T = 6 (sec)

(c) T = 8 (sec)

(a) T = 4 (sec)

Fig. 7 Comparison of the oscillating tension between numerical and experiment analysis in towing with horizontal osillation periods