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WDM 환형 망에서 경로 설정 및 파장 할당 문제의 새로운 모형
A New Model of a Routing and Wavelength Assignment Problem on WDM Ring Networks

강동한 · 강장하 · 박성수
대전광역시 유성구 구성동 373-1 한국과학기술원 산업공학과

Abstract

We consider a routing and wavelength assignment (RWA) problem on wavelength division multiplexing (WDM) ring networks, which is to maximize the established connections between nodes, given a set of usable wavelengths. We propose two new mathematical formulations of it and efficient algorithms based on branch-and-price method. Computational experiments on random instances show that one of the proposed formulations yields optimal solutions in much shorter time on the average than the previous formulation due to Lee (1998).

1. Introduction

The optical transport network based on the optical path (or lightpath) concept, which employs wavelength division multiplexing (WDM) technologies and the wavelength routing technologies, is the key approach to realize large bandwidth networks (Lee *et al.*, 2000, Nagatsu *et al.*, 1999). We focus on WDM ring networks (Lee *et al.*, 2000) in this paper.

Routing and Wavelength (RWA) problem in WDM networks is to construct optical paths between pairs of nodes and to assign a wavelength to each optical path with a certain objective. The problem has been a key issue for years in WDM network design (Lee *et al.*, 2000, Wuttisittikulkij *et al.*, 2000). There are two versions of RWA problem commonly appear in the literature (Lee, 1998 and the references therein). The first is to get the maximum number of connections that can be established, given a set of usable wavelengths for a network is given. Another problem is to establish all connections between nodes while minimizing the number of used wavelengths. Hereafter, we denote the problems by (RWA1) and (RWA2), respectively.

Recently, Lee *et al.* (2000) solved (RWA2) successfully in ring networks using a branch-and-price algorithm (Barnhart *et al.*, 1998). (RWA1) with the formulation due to Lee (1998) can be solved with the same approach. Though LP relaxation of the formulation provides tight upper bounds on the optimal values, there are wide variations in implementation time depending on the number of usable wavelengths in a ring. In this research, we suggest two new formulations of (RWA1), and compare the performance of the branch-and-price algorithms for all three formulations.

In the wavelength routing, no two optical paths can be assigned the same wavelength in a link (Lee *et al.*, 2000). We assume that wavelength conversion is not allowed; hence, one wavelength is assigned to each path for a connection.

2. Models of RWA problem

2.1 Traditional formulations

Lee (1998) presented formulations of (RWA1) and (RWA2) based on decomposition and column generation, using the notion of independent routing configuration.

Let K be the set of node pairs such that for each $k \in K$, the number of required connections d_k is given. We denote the number of usable wavelengths (we also call this the ring capacity) by b. A routing configuration is a set of paths given for some node pairs in K. Note that, there are two kinds of paths between two nodes of a node pair since the physical network is a cycle. Routing configuration c can be represented as a nonnegative integer vector $r_c = (r_{c1}, r_{c2}, \ldots, r_{c|K|})$, where r_{ck} represents the number of paths appearing in routing configuration c which are used for the connections of node pair $k \in K$. Routing configuration c is called *independent* if all the optical paths in c can be established using only one wavelength. For an independent routing configuration c, r_c is $2e_k$ or composed of 0s or 1s due to the constraint that no two optical paths are assigned the same wavelength in a link, where e_k is kth unit vector of size $|K|$.

Let C be a set of all independent routing configurations. Decision variables $z_c, c \in C$ are introduced to represent the number of routing configuration c in the final solution. Then, the mathematical formulations for the two problems can be stated as follows (Lee, 1998).

(RWA1)

$$\max \sum_{c \in C} \sum_{k \in K} r_{ck} z_c$$

s.t.

$$\sum_{c \in C} r_{ck} z_c \le d_k, \quad k \in K$$

$$\sum_{c \in C} z_c \le b$$

$$z_c \in Z_+, \quad c \in C$$

(RWA2)

$$\min \sum_{c \in C} z_c$$

s.t.

$$\sum_{c \in C} r_{ck} z_c \ge d_k, \quad k \in K \qquad (1)$$

$$z_c \in Z_+, \quad c \in C$$

For details of the formulations, please refer to Lee (1998).

(RWA2) on WDM ring networks was proved to be NP-hard (Erlebach and Jansen, 1997). (RWA1) can be shown to be NP-hard by polynomial reduction from (RWA2) based on the next property.

Property 1. (RWA1) has a solution of value $\sum_{k \in K} d_k$ if and only if (RWA2) has a solution of value less than or equal to b.

In the branch-and-price algorithm for (RWA2) (Lee *et al.*, 2000), the subproblem (column generation problem) can be solved in polynomial time using the algorithm for obtaining maximum weight independent set (MWIS) on interval graphs (Hsiao *et al.*, 1992) as a subroutine. They applied the branching rule based on variable dichotomy with a procedure of finding nth best solution in case that an existing column is generated again after branching. Two kinds of primal heuristics were implemented to get feasible solutions. The first, denoted by PH1, is a greedy-style heuristic to get a set of independent routing configurations by implementing MWIS algorithms iteratively, until all connection requirements are covered. The final solution is added to the formulation as initial columns. The second, denoted by PH2, is applied at each node of the branch-and-bound tree. First, they determine that $\lfloor \overline{z_c} \rfloor$ configurations are always selected for $c \in C$, and then they apply PH1 to the rest of connection requirements, where $\overline{z}$ is an LP optimal solution at the node. As a result, they found optimal or near-optimal solutions in a reasonable time.

Note that (RWA1) can also be solved using branch and price. We can solve the subproblems in polynomial time using MWIS algorithm. We can also apply PH1 and PH2 to find feasible solutions. We can adopt the same branching rule as that used in the algorithm for (RWA2).

2.2 New formulations of (RWA1)

We note that there is still difficulty in solving (RWA1). Depending on the value of b, it takes several hundreds seconds to solve the problem (it will be shown in section 3). In order to tackle the problem, we consider two alternative optimization problems, (RWA1-1) and RWA1-2), that provide an optimal solution to (RWA1). They are closely related and the second one is induced by the first. Both problems can be solved using branch and price as (RWA1) or (RWA2). With the latter one, we could obtain optimal solutions to (RWA1) for all test problems by solving LP relaxation and applying branch-and-bound algorithm. In other words, we may solve the problems with only the columns generated at the root node of the branch-and-bound tree. This method considerably outperforms the branch-and-price algorithm for (RWA1) in terms of computing time.

We introduce decision variables u_k, $k \in K$ representing the amount of unsatisfied connections of node pair k. Then the first problem can be formulated as follows.

(RWA1-1)

$$\min \sum_{k \in K} u_k \qquad (2)$$

s.t.

$$\sum_{c \in C} r_{ck} z_c + u_k \geq d_k, \quad k \in K \tag{3}$$

$$\sum_{c \in C} z_c \leq b$$

$$z_c \in Z_+, \; c \in C, \quad u_k \in R_+, \; k \in K$$

The objective (2) is to minimize the sum of unsatisfied connections.

We show that the formulation of (RWA1-1) is valid in the following proposition.

Proposition 1. (RWA1) has an optimal solution of value α_1 if and only if (RWA1-1) has an optimal solution of value $\sum_{d \in K} d_k - \alpha_1$. Furthermore, the optimal solution to (RWA1) can be transformed into that to (RWA1-1) in $O(B|K|)$ time, and vice versa, where $B = \min(b, \sum_k d_k)$.

Now, we consider another problem referred to as (RWA1-2). The formulation of it has the same constraints as those of (RWA1-1), but the objective is replaced by

$$\min \quad g(u,z) = \sum_{k \in K} M u_k - \sum_{c \in C'} \sum_{k \in K} r_{ck} z_c,$$

where $C' \subset C$ and M is a sufficiently large number. Note that $0 \leq \sum_{c \in C'} \sum_{k \in K} r_{ck} z_c \leq B|K|$. If we are given two feasible solutions (u^1, z^1) and (u^2, z^2) to (RWA1-1) such that $\sum_{k \in K} u_k^1 = \sum_{k \in K} u_k^2$, then $|g(u^2, z^2) - g(u^1, z^1)| \leq B|K|$.

Proposition 2. An optimal solution to (RWA1-2) is also optimal for (RWA1-1) when $M > B|K|$.

Suppose we have a feasible solution to (RWA1-1) or (RWA1-2). Let f_u, g_u be the objective values of (RWA1-1) and (RWA1-2), respectively, provided by the solution. Then, we can write $g_u = M f_u - \alpha$, where $0 \leq \alpha \leq B|K|$.

2.3 Pruning test in implementing branch and price

Since the objective value of (RWA1) or (RWA1-1) is integer, we can prune the branch-and-bound tree at a node if difference between LP value and a value of a feasible solution is less than 1. For (RWA1-2), the threshold value is determined based on the next proposition.

Proposition 3. Let g_l and g_u be objective values of LP optimal solution and a known feasible solution to (RWA1-2), respectively. If $g_u - g_l < M(1-\varepsilon)$, where $M > 2B|K|$ and $B|K|/M \leq \varepsilon < 1$, we can prune the tree at the current node.

2.4 Determining C' in solving (RWA1-2)

The aim of introducing additional term $-\sum_{c \in C'} \sum_{k \in K} r_{ck} z_c$ in the objective function of (RWA1-2) formulation is mainly to enlarge the left-hand sides of constraints (3) enough to accomplish the optimality of (RWA1) early (possibly, at the root node of the branch-and-bount tree). As will be seen in the next section, the upper bound provided by the LP relaxation of (RWA1) is very close to the optimal value, but the lower bound provided by the branch-and-bound method run at the root node is apart from it a little depending on the values of b. Introducing the term helps reduce the gap between lower and upper bounds. If $|C'|$ is too small, we may not expect the effect much. On the other hand, if C' is set to be close to C, the effect becomes significant but more columns than necessary would be generated. One possible way is to set C' as the columns due to PH1. With the C', we could terminate the algorithm at the root node for all test problems in a short time.

3. Test results

To compare performance of the branch-and-price algorithms for the three problems, we consider 25 test problems with $|V| = 13$ and $|K| = 78$. The connection requirement of each node pair varies randomly from 1 to 5. We generate five sets of connection requirements, and for each set, we consider five ring capacities (b): $\lceil 0.7 w_0 \rceil, w_0 - 1, w_0, w_0 + 1, \lfloor 1.3 w_0 \rfloor$, where w_0 is the optimal value of (RWA2) with the instance of the set of connection requirements. Note that when $b \geq w_0$, an optimal solution to (RWA1) satisfies $\sum_c r_{ck} z_c = d_k, k \in K$ (see Property 1). The tests were run on Pentium III PC with speed of 866MHz.

Table 1, Table 2 and Table 3 show the results of branch-and-price algorithms for (RWA1), (RWA1-1), and (RWA1-2), respectively. Columns 'IP' and 'LP' refer to the optimal values of the problem and LP relaxation, respectively. Entries of the column

Table 1. Results of branch-and-price algorithm for (RWA1)

No.	*b*	IP	LP	N0	B&B nodes	columns	time(s)
1	51	200	200	199	253	315	290.8
	71	240	240	237	4692	364	4163.5
	72	241	241	238	214	297	213.3
	73	241	241	240	2	257	8.5
	93	241	241	241	0	208	2.4
2	47	187	187.5	186	213	304	210.9
	65	225	225	223	19	290	35.1
	66	226	226	224	89	311	114.3
	67	226	226	225	36	286	57.0
	85	226	226	226	0	203	2.0
3	46	170	170	168	23	253	22.4
	64	206	206	204	73	256	51.0
	65	208	208	206	8	252	10.7
	66	208	208	208	0	204	2.1
	84	208	208	208	0	203	2.0
4	48	198	198	195	77	274	92.6
	67	236	236	234	10	261	17.2
	68	238	238	237	26	272	36.1
	69	238	238	237	3	240	8.9
	88	238	238	238	0	203	2.2
5	49	185	185	182	37	277	38.7
	69	225	225	223	148	290	122.2
	70	226	226	223	2	267	10.6
	71	226	226	224	13	254	14.8
	91	226	226	226	0	203	2.1
							221.3 (Avg.)

'N0' are obtained by implementing branch and bound with the columns generated at the root node of the tree. The objective values of (RWA1-1) and (RWA1-2) were converted into the corresponding ones of (RWA1) in Table 2 and Table 3. Note that there are no columns 'LP' in the tables, since the corresponding LP values are unknown. We used callable library of ILOG CPLEX 7.0 as LP and MIP solvers.

As shown in Table 1, the problems can be solved in a short time in the case of $b=w_0+1$ or $\lfloor 1.3w_0 \rfloor$. In the remaining cases, it takes generally much time to obtain optimal solutions. The results of Table 2 also show this tendency, but the average time decreases significantly. In the algorithm for (RWA1), without PH2, only one problem yielded an optimal solution with the branch-and-bound node limit of 10000; there are numerous nodes with the same LP value though the symmetric structure resulting from the indexing of wavelengths has been destroyed by adopting the model of (RWA1) (Lee, 1998, Lee et al., 2000), and this causes too many nodes to be explored.

The values of M, ε are set to 10^6 and 0.1, respectively in solving (RWA1-2), which comply with the possible ranges of M and ε given in

Table 2. Results of branch-and-price algorithm for (RWA1-1)

No.	*b*	IP	N0	B&B nodes	columns	time(s)
1	51	200	199	309	285	318.3
	71	240	239	191	260	152.3
	72	241	241	0	234	5.5
	73	241	241	0	221	3.2
	93	241	241	0	208	2.3
2	47	187	187	0	228	5.9
	65	225	224	146	276	114.9
	66	226	225	57	264	45.1
	67	226	224	22	247	21.8
	85	226	226	0	203	2.0
3	46	170	169	13	218	10.3
	64	206	206	0	223	3.9
	65	208	207	7	230	7.5
	66	208	208	0	203	2.1
	84	208	208	0	203	2.0
4	48	198	197	10	229	14.7
	67	236	235	59	243	52.5
	68	238	237	5	232	12.3
	69	238	237	4	221	6.3
	88	238	238	0	203	2.2
5	49	185	184	5	232	9.8
	69	225	224	35	242	32.1
	70	226	224	19	235	19.6
	71	226	226	0	219	3.7
	91	226	226	0	203	2.0
						34.1(Avg.)

Proposition 3. (RWA1-2) requires more time than (RWA1) or (RWA1-1) does when $b=\lfloor 1.3w_0 \rfloor$, but it gives the shortest time on the average. By adopting the model of (RWA1-2), all test problems were solved at the root node of the tree. The average elapsed times for the three problems (RWA1), (RWA1-1) and (RWA1-2) were 221.3, 34.1 and 7.7, respectively.

4. Conclusions

We suggested and tested a new formulation of a RWA problem on WDM all-optical networks that gives optimal solutions in a short time. Though we have tested the formulation in ring networks, it can also apply to RWA problem on mesh networks.

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Table 3. Results of branch-and-price algorithm for (RWA1-2)

No.	*b*	IP	NO	B&B nodes	columns	time(s)
1	51	200	200	0	276	10.4
	71	240	240	0	285	10.0
	72	241	241	0	275	10.1
	73	241	241	0	254	6.4
	93	241	241	0	233	4.3
2	47	187	187	0	276	23.3
	65	225	225	0	307	11.6
	66	226	226	0	293	10.6
	67	226	226	0	286	9.9
	85	226	226	0	242	4.9
3	46	170	170	0	246	6.5
	64	206	206	0	259	6.7
	65	208	208	0	250	6.1
	66	208	208	0	221	3.3
	84	208	208	0	228	3.8
4	48	198	198	0	261	8.1
	67	236	236	0	277	8.6
	68	238	238	0	285	9.6
	69	238	238	0	223	3.8
	88	238	238	0	213	3.1
5	49	185	185	0	266	8.4
	69	225	225	0	273	8.5
	70	226	226	0	257	6.4
	71	226	226	0	244	5.4
	91	226	226	0	224	3.7
						7.7(Avg.)