

MPLS 기반 IP 망에서 열생성 기법을 이용한 경로 설정 해법

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Column Generation Approach to the Constraint Based Explicit Routing Problem in MPLS Based IP Networks

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Abstract

We consider the constraint based explicit routing problem in MPLS based IP Network. In this problem, we are given a set of traffic demands and a network with different link capacities. The problem is to assign the demand commodities to the paths in the network while minimizing the maximum link load ratio.

We formulate this problem as an integer programming problem and propose an efficient column generation technique. To strengthen the formulation, we consider some valid inequalities. We also incorporate the column generation technique with variable fixing scheme. Computational results show that the algorithm gives high quality solutions in a short execution time.

1. Introduction

The Internet of today is evolving rapidly. The importance of internet for the communications has been recognized by a great number of users, and Internet is considered as one of the drivers for our new global international society.

The growth in the number of users of the Internet and their bandwidth requirements makes traffic congestions on the Internet service providers' (ISPs) networks. To meet the growing demand for bandwidth and guaranteed Quality of Service (QoS), ISPs need higher performance switching and routing technology [1].

Multiprotocol label switching (MPLS) is one such technology. MPLS offers simple mechanisms for packet-oriented traffic engineering and multiservice functionality with the added benefit of greater scalability. MPLS emerged from the Internet Engineering Task Force (IETF)'s effort to standardize a number of proprietary multilayer switching solutions that were initially proposed in the mid-1990s [11]. MPLS uses short, fixed length, locally significant labels in the packet header and packets are forwarded by network nodes via label swapping similar to layer 2 switching. MPLS is intended to work over any data link layer technology including ATM, frame relay, PPP and Ethernet. A router that supports the MPLS protocol is called a Label Switching Router (LSR). In MPLS, packet flows are mapped to Forwarding Equivalence Classes (FEC) which are mapped to traffic trunks, which in turn are mapped to Label Switched Paths (LSP) [3].

We now consider constraint based routing which is one of the most important innovations enabled by MPLS. The constraint based routing is a step beyond conventional IP routing because, in addition to minimizing some administrative metrics, it also selects paths that satisfy one or more constraints. Typical constraints include the available bandwidth along the path and administrative constraints.

Support for constraint based routing requires explicit routings (or source routing) capability. To provide this capability, we use MPLS based

explicit routing capability. The reason for using MPLS are twofold. First of all, MPLS allows decoupling of the information used for forwarding (a label) from the information carried in the IP header. Second, mapping between an FEC and an LSP is completely confined to the Label Edge Router (LER), where LER is the LSR at the head end of the LSP [1].

This explicit routing feature of MPLS may overcome many shortcomings associated with current Interior Gateway Protocol(IGP) routing schemes. A prime problem of current IGP routing schemes is that some links on the shortest path between certain ingress-egress pairs may get congested while links on possible alternative paths remain free.

Therefore routing schemes that can make better use of network infrastructure are needed. In the MPLS network, explicit routing with Resource Reservation Protocol (RSVP)-extensions allows the network to be able to control the path from ingress node to egress node so as to optimize utilization of network resources and enhance performance [6].

In an operational MPLS network, the routing computations implemented in edge routers automate LSP setups. However, edge routers have only local information about the network resources. These local routing decisions made in LERs may result in a degraded global network performance in the long run. One solution for this is to perform periodical (not too frequent, e.g. daily or weekly) global re-optimization of LSPs with a centralized off-line network optimization tool [5].

In this study, we consider such a global optimization algorithm to solve an explicit routing problem in MPLS based IP network. The problem we try to solve is to place bandwidth guaranteed explicit routes between edge nodes over a physical topology such that all the traffic demands are fulfilled. We also consider the hop constraint for each explicit route to ensure delay factor of QoS. The optimization objective we propose is to minimize the maximum of link load ratio. This optimization objective ensures that the traffic may be moved away from congested hot spots to less utilized parts of the network, and the distribution of traffic is balanced as much as possible across the network. Minimizing the maximum of link load ratio also leaves more space for future traffic growth. Therefore, the growth in traffic in the future are more likely to be accommodated, and can be accepted without requiring the re-arrangement of connections [13].

To obtain these QoS routings and to solve traffic engineering problems, there has been several researches. For examples, Wang and Crowcroft [12] and Guerin and Orda [4] have proposed a QoS routing algorithm, but in the greedy style. Xiang *et al.* [14] have proposed the scheme of QoS routing based on a genetic algorithm.

Recently, Wang and Wang [13] have proposed 4 heuristic algorithms for the explicit routing in the MPLS based IP network. But they have not considered any QoS constraints. Also, Girish *et al.* [3] have formulated the optimization problem for constraint based routing to determine the optimal placement of a set of LSPs in a network, but without any solution approach.

In this study, we propose a column generation approach to the explicit routing problem for the traffic engineering in the MPLS based IP network. Our approach is different from the above researches in the sense that we consider link utilization as optimization objective and provide high quality solution using the column generation approach.

The rest of this paper is organized as follows. In section 2, we formulate the problem. In section 3, we propose some valid inequalities used in the preprocessing. In section 4, we present the column generation procedure. Section 5 describes the overview of our algorithm. Computational results are shown in section 6. Finally, concluding remarks are given in section 7.

2. Formulation of the Problem

In this section, we present the mathematical formulation of constraint based routing problem(CRP). In this problem, we assume that the backbone network consists of a set of nodes (LSRs) connected by directional links with fixed capacities. Thereby, the network which we consider is a directed graph. We also assume that the average bandwidth demand from one edge node to another is known. This demand is measured by ISPs, or in the case of VPNs, specified by customers for the logical connection. Note that an explicit route must start from one edge node (ingress LSR) and terminate at another edge node (egress LSR) in MPLS networks. Following are notations and decision variables to be used for modeling.

Notations

- N : the set of nodes in the network
 E : the set of links in the network (which are defined as directed arcs)
 K : the set of traffic demands between a pair of edge nodes.
 d_k : effective or equivalent bandwidth of a traffic demand k .
 c_{ij} : bandwidth or available bandwidth of a link $(i, j) \in E$
 h_k : maximum allowed number of LSR hops through the network for a LSP $k \in K$
 s_k : ingress LSR (source node) of a traffic demand $k \in K$
 t_k : egress LSR (destination node) of a traffic demand $k \in K$
 $P(k)$: the set of (s_k, t_k) paths of traffic demand k such that the number of hops of path p is less than h_k
 E_p : the set of links of which path p consists
 α : decision variable that shows link load ratio

Then, the problem can be formulated as follows.

(MP)

$$\min \alpha \quad (1)$$

s.t.

$$\sum_{k \in K} d_k \sum_{\{p | p \in P(k), (i, j) \in E_p\}} y_p^k \leq c_{ij} \alpha, \text{ for } (i, j) \in E \quad (2)$$

$$\sum_{p \in P(k)} y_p^k = 1, \text{ for } k \in K \quad (3)$$

$$y_p^k \in \{0, 1\} \text{ for } p \in P(k), k \in K.$$

In this formulation, the variable $y_p^k = 1$ if and only if traffic demand k is assigned to path p , otherwise 0. The objective (1) says the variable to be minimized is the maximum of link load ratio. Constraints (2) ensure that the link capacities are not exceeded. Constraints (3) imply that there must exist a path for each traffic demand.

Note that this CRP is NP-hard. It can be shown by reducing the k disjoint route problem which is NP hard [2] to this problem. For the details of this procedure, refer to Wang and Wang [13].

In our study, we use the efficient column generation technique to solve MP. The column generation is a pricing scheme for solving large-scale linear programs. The column generation technique has usually been used to solve the problems whose natural formulations

have exponential number of variables.

3. Preprocessing

In this section, we propose an efficient procedure which is used to strengthen the initial formulation. First, we consider the result of W. Yang [15], who has considered following proposition and incorporated it with his algorithm to increase the efficiency of the heuristic algorithm.

Proposition 1([15]). Following inequality is valid

$$\alpha \geq \max_{k \in K} d_k / \max_{(i, j) \in E} c_{ij} \quad (4)$$

Now, we consider another valid inequality. First, we define a subset $\delta(i)$ of E , which is defined as links that are adjacent to node $i \in N$. By considering the capacities of links that are adjacent to the source node or destination node of traffic demand k , we can obtain following results.

Proposition 2. Following inequality is valid

$$\alpha \geq \max_{k \in K} \{d_k / \min\{\widehat{c}_{s_k}, \widehat{c}_{t_k}\}\} \quad (5)$$

where $\widehat{c}_{s_k} = \max_{(i, j) \in \delta(s(k))} c_{ij}$, and

$$\widehat{c}_{t_k} = \max_{(i, j) \in \delta(t(k))} c_{ij}.$$

Proof. Note that traffic demand k originates from s_k and terminates at t_k . Since the demand k paths through one link among $\delta(s(k))$ and one link among $\delta(t(k))$, α should satisfy that (i) $\alpha \geq d_k / \widehat{c}_{s_k}$ and (ii) $\alpha \geq d_k / \widehat{c}_{t_k}$. Therefore we have $\alpha \geq d_k / \min\{\widehat{c}_{s_k}, \widehat{c}_{t_k}\}$. Note that we are given a set of traffic demands. Therefore α should satisfy this inequality for each traffic demand $k \in K$. Then we obtain the validity of the inequality (5). \square

Proposition 3. The valid inequality (5) gives tighter bound than that of the valid inequality (4)

Proof. Let $\bar{k} = \arg \max_{k \in K} d_k$. For the \bar{k} , we define $\widehat{c}_{s_{\bar{k}}} = \max_{(i, j) \in \delta(s(\bar{k}))} c_{ij}$ and $\widehat{c}_{t_{\bar{k}}} = \max_{(i, j) \in \delta(t(\bar{k}))} c_{ij}$. Now we consider two cases.
 (i) If $\widehat{c}_{s_{\bar{k}}}$ and $\widehat{c}_{t_{\bar{k}}}$ are not same, then $\max_{(i, j) \in E} c_{ij} > \widehat{c}_{t_{\bar{k}}}$ where $\widehat{c}_{\bar{k}} = \min\{\widehat{c}_{s_{\bar{k}}}, \widehat{c}_{t_{\bar{k}}}\}$,

$\widehat{c}t_{\bar{k}}$. Therefore, $\max_{k \in K} d_k / \max_{(i,j) \in EC} ij < d_{\bar{k}} / \widehat{c}_{\bar{k}} \leq \max_{k \in K} \{d_k / \min\{\widehat{c}s_k, \widehat{c}t_k\}\}$.

(ii) If $\widehat{c}s_{\bar{k}}$ and $\widehat{c}t_{\bar{k}}$ are same, then $\max_{(i,j) \in EC} ij \geq \widehat{c}_{\bar{k}}$. Therefore, we obtain $\max_{k \in K} d_k / \max_{(i,j) \in EC} ij \leq d_{\bar{k}} / \widehat{c}_{\bar{k}} \leq \max_{k \in K} \{d_k / \min\{\widehat{c}s_k, \widehat{c}t_k\}\}$. Therefore we complete the proof. \square

4. Column Generation Problem

In this section, we give an explanation of column generation problem and the algorithm to solve the problem. To represent the column generation procedure for the LP relaxation of MP, let MPL be the linear programming relaxation of MP. We need to have a feasible basis for MPL to use the column generation method. If it is difficult to find an initial feasible solution, we introduce artificial variables with big cost coefficient. We will mention how to find an initial feasible solution to MPL in section 5.

Given a feasible basis to MPL, we need to generate columns to enter the basis. We assume that a subset $P'(k) \subset P(k)$ of path set for each $k \in K$ is given. Replacing $P(k)$ by $P'(k)$ for all $k \in K$ in MPL yields the restricted linear programming MPL', whose solutions are suboptimal to MPL. Let β_{ij} be the dual variable associated to the constraints (2) for each link $(i,j) \in E$. Let θ_k be the dual variable associated to the constraints (3). For the given $k \in K$, the constraints in the dual of MPL' are,

$$\begin{aligned} \theta_k + \sum_{(i,j) \in E_p} d_k \beta_{ij} &\geq 0, & p \in P'(k), \\ \beta_{ij} &\geq 0, & (i,j) \in E. \end{aligned}$$

Let $(\bar{\beta}, \bar{\theta})$ be an optimal solution to the dual of MPL'. Then it is also optimal to the dual of MPL if

$$\bar{\theta}_k + \sum_{(i,j) \in E_p} d_k \bar{\beta}_{ij} \geq 0, \quad p \in P(k) \setminus P'(k), k \in K$$

Therefore, we may write the optimality condition for MPL:

$$\min \left\{ \sum_{(i,j) \in E_p} (d_k \bar{\beta}_{ij}) \mid p \in P(k) \right\} \geq -\bar{\theta}_k \quad (6)$$

Using (6), we can derive the formulation of the column generation problem for MP. For the given $k \in K$, the column generation problem associated to traffic demand k can be formulated as follows.

(SP(k))

$$\min \sum_{(i,j) \in E_p} (d_k \bar{\beta}_{ij})$$

s.t.

$$\sum_{(i,j) \in E} x_{ij} \leq h_k, \quad (7)$$

$$\sum_{j \in N \setminus \{(i,j) \in E\}} x_{ij} - \sum_{j \in N \setminus \{(i,j) \in E\}} x_{ji} = 0, \text{ for } i \neq s_k, t_k \quad (8)$$

$$\sum_{j \in N \setminus \{(i,j) \in E\}} x_{ij} - \sum_{j \in N \setminus \{(i,j) \in E\}} x_{ji} = 1, \text{ for } i = s_k \quad (9)$$

$$x_{ij}^k \in \{0, 1\}, \quad \text{for } (i,j) \in E$$

In this formulation, the variable $x_{ij} = 1$ if and only if traffic demand k is routed on link $(i,j) \in E$, 0 otherwise. Constraints (7) imply that the number of LSR hops of the given traffic demand should be less than the bound h_k . Constraints (8) and (9) imply that there must exist a path for the given traffic demand.

Note that SP(k) is the problem that finds the shortest path from s to t with hop constraint, where s is the source and t is the destination of traffic demand $k \in K$, respectively. Since the link weights are nonnegative, SP(k) can be solved efficiently by Bellman-Ford's algorithm [7]. If the resulting length of the shortest path is less than $-\bar{\theta}_k$, the path can be added to the current formulation. Otherwise, no column is generated with respect to traffic demand k .

5. The Algorithm

5.1 Overview

In this section, we give a brief and overall explanation of our algorithm. First, we construct the initial formulation of LP using artificial variables with big cost coefficient and initial feasible solutions.

After preprocessing and solving the initial LP, we decide whether the present solution is dual feasible or not. If it is not, new columns are generated and added to LP. We repeat this procedure until no more columns are added. If the present solution is dual feasible, i.e., no more column need to be added to LP, the final formulation of LP relaxation of MP is obtained.

Then, we check if the solution obtained by

solving the last LP is integral. If we have obtained an integral solution, we have obtained an optimal solution of MP. Otherwise, we have to initiate the variable fixing procedure to find an integral solution. The procedure incorporates the column generation within the variable fixing scheme. The procedure yields an integral solution of MP.

5.2 Initial Feasible Solution

To use the column generation procedure, we need to have an initial feasible solution to the LP relaxation of MP. We can obtain an initial feasible solution by finding paths between the ingress routers and the egress routers, respectively, for all traffic demands. Note that an initial feasible solution, if found, can also serve as an incumbent solution in the variable fixing procedure.

To find an initial feasible solution, we used following approach. We first try to find a path between the ingress router and the egress router for each traffic demand by using the Dijkstra's shortest path algorithm. If we can get a path that also satisfies the hop restriction, there is no problem. Otherwise, we apply the Bellman-Ford method [7] to find the path that satisfy the hop restriction. We repeat this procedure for all traffic demand $k \in K$.

Note that the computational complexity of Bellman-Ford method is $O(N^2h)$, where h is the number of maximum hops between source and destination pair. This algorithm is much more time consuming than Dijkstra's algorithm whose complexity is of $O(N^2)$.

5.3 Primal Heuristic

In this section, we present simple heuristic algorithm to find an incumbent solution. The incumbent solution can be obtained by the fixing of decision variables during the variable fixing phase, or by the primal heuristic algorithm from the start of the variable fixing phase. In our algorithm, we developed greedy style heuristic algorithm as follows to be incorporated at the start of the variable fixing phase.

Algorithm LHA (LP-based Heuristic Algorithm)

Step 0 : Initialization. Set $l_{ij} = 0$ for all $(i, j) \in E$.

Step 1 : Solve LP. Solve LP relaxation of MP using the column generation algorithm. Let y^* be the obtained optimal solution.

Step 2 : Path assignment. For all $k \in K$, Find \bar{p} , where $y_{\bar{p}}^{k*} = \max\{y_p^{k*} | p \in P'(k), k \in K\}$. Set $l_{ij} = l_{ij} + d_{\bar{p}}$ for all $(i, j) \in \bar{p}$.

Step 3 : Maximum load ratio computation. Find maximum load ratio \bar{l}_{ij} , where $\bar{l}_{ij} = \max\{l_{ij} | (i, j) \in E\}$

Note that \bar{l}_{ij} is an upper bound on the objective value of MP. The number of iterations needed by the algorithm is $|K|$. Now, in the next section, we present variable fixing phase as follows.

5.4 Variable Fixing Phase

When we solve the LP relaxation of MP by the column generation, we may get the fractional solution. In order to get an integral solution, we need to consider branching strategies. Basically branch-and-price procedure can be considered in order to obtain an optimal solution. But, it generates exponential number of branching nodes. Therefore we can not guarantee the termination of the algorithm. To overcome these drawbacks in this study, we use the following variable fixing strategy which is based on one directional branch scheme.

When we get a fractional optimal solution of the LP relaxation of MP, we first select a variable with maximum value among path variables y^* that satisfy the hop restriction. After the variable selection, we fix that variables's value to one. It means that the lower bound of that variable is changed to one. After that, we solve the LP relaxation problem of MP and then again solve the subproblem to generated columns until no more columns are generated. After completing this procedure, if the LP relaxation solution of MP is integral, we are done with an integral feasible solution. Otherwise we repeat this procedure until we get an integral solution. In this way, we finally find an integral feasible solution.

6. Computational Results

In this section, we present the performance of

the proposed algorithm. We first mention the characteristics of the generated problems. Then, we give the computational results of our algorithm.

6.1 Problem Characteristics

We tested the proposed algorithm on some randomly generated problems. We first generate two networks. The underlying networks are randomly generated from a discrete uniform 50×50 Euclidian plane.

For the given network topology, we construct 8 test classes with the number of demands varying from 300 to 1000 in 100 increment. We generate 10 test problems for each class. For each test problem, the link capacities are generated randomly among {800, 900, 1000, 1100, 1200}. The size of each individual demand is generated by a random variable with a uniform integer distribution in [1, 10]. The source and destination pairs are selected also randomly among all edge nodes.

Table 1 summarize the characteristics of the randomly generated data. The column #Dem shows the number of demands. The column "Tot Demand" is the average of the total bandwidth requirement of all demands in the network for 10 test problems. The column "Tot Capacity" is the average of the total capacity of all links in the network for 10 test problems.

Table 1. Characteristics of the test sets

(N, E)	No.	#Dem	Tot Demand	Tot Capacity
(36, 92)	1	300	1643	93010
	2	400	2219	92170
	3	500	2738	92310
	4	600	3286	91760
	5	700	3874	91320
	6	800	4401	92650
	7	900	4928	92740
	8	1000	5520	92340
(45, 150)	1	300	1636	150770
	2	400	2191	149920
	3	500	2751	149780
	4	600	3301	150640
	5	700	3876	150040
	6	800	4427	150530
	7	900	4914	149420
	8	1000	5481	149820

For an LP solver and constraints addition routine, we use the CPLEX 7.0 callable mixed integer library. All tests are performed on a Pentium PC (866MHz).

6.2 Computational Results

The computational results on the 2 networks are summarized in tables 2-3.

In those tables, the column #Dem shows the number of traffic demands. The column #HOP shows the maximum allowed number of LSR hops through the network for a traffic demand. Let MH be the minimum number of hops by which the demand can be transported. For each demand, the hop restrictions T1, T2, T3 are given by $\lceil 1.3MH \rceil$, 2MH, and 3MH respectively. The column #COL shows the number of the columns that include the columns generated and the initial columns. The column FLP shows the objective value obtained by solving the final linear programming relaxation of MP before the variable fixing procedure and the column IP denotes the value of the best integer solution. Then, Gap(%) is defined as follows:

$$\text{Gap}(\%) = (\text{IP} - \text{FLP}) / \text{IP} \times 100.$$

Finally, the column Time refers to the execution time in second needed to solve the problem until variable fixing procedure. Each row shows the average value of 10 problem instances.

Usually, the number of columns generated is relatively large. Note that the number of columns generated increases as the number of demands grows. The Gap is relatively small and almost within 4 percent. It can also be observed that the Gap does not degrade as the number of demands grows.

Note that the maximum link load ratio increases when more demands are added to the network. As we have expected, the tables show that the result of the problems with loose hop restriction show better load balancing than that of the problems with tight hop restriction. But, the tables show that the Gap of the problems with loose hop restriction is not always smaller than the Gap of the problems with tight hop restriction.

The execution times needed to solve the problem of sized, $|K|$ up to 1000, do not exceed 3 minutes. The average execution time is within 90 seconds.

Table 2. Computational results for Network 1
($|N| = 36$, $|E| = 92$)

#Dem	HOP	#COL	FLP	IP	Gap(%)	Time
300	T1	434.2	0.148	0.153	3.558	10.38
300	T2	469.6	0.144	0.149	3.293	13.71
300	T3	477.1	0.144	0.148	3.008	15.60
400	T1	545.6	0.210	0.213	1.725	17.78
400	T2	576.3	0.207	0.212	2.611	22.69
400	T3	581.4	0.207	0.212	2.533	25.74
500	T1	746.3	0.238	0.243	2.380	29.80
500	T2	799.2	0.235	0.240	2.157	39.02
500	T3	815.1	0.235	0.240	2.018	44.96
600	T1	852.8	0.299	0.303	1.442	40.72
600	T2	903.8	0.297	0.302	1.564	52.08
600	T3	922.3	0.297	0.302	1.566	58.05
700	T1	1047.4	0.328	0.333	1.530	57.77
700	T2	1108.8	0.325	0.331	1.911	73.66
700	T3	1137.8	0.325	0.330	1.643	86.18
800	T1	1125.1	0.389	0.394	1.197	71.63
800	T2	1200.7	0.383	0.387	1.194	96.13
800	T3	1224.6	0.383	0.388	1.264	114.43
900	T1	1322.4	0.417	0.422	1.335	94.34
900	T2	1415.6	0.414	0.420	1.453	121.10
900	T3	1441.0	0.414	0.420	1.413	138.89
1000	T1	1420.6	0.492	0.497	1.148	113.81
1000	T2	1501.4	0.488	0.493	1.109	147.00
1000	T3	1525.0	0.488	0.493	1.059	171.97
	T1	936.8	0.315	0.320	1.789	54.53
Avg	T2	996.9	0.312	0.317	1.912	70.67
	T3	1015.5	0.312	0.317	1.813	81.98

Table 3. Computational results for Network 2
($|N| = 45$, $|E| = 150$)

#Dem	HOP	#COL	FLP	IP	Gap(%)	Time
300	T1	325.7	0.122	0.125	2.159	8.72
300	T2	327.3	0.122	0.125	2.325	11.78
300	T3	326.7	0.122	0.125	2.325	13.82
400	T1	422.1	0.177	0.179	1.148	10.30
400	T2	423.5	0.177	0.179	1.307	13.99
400	T3	423.5	0.177	0.179	1.307	16.43
500	T1	530.0	0.201	0.203	1.046	20.72
500	T2	530.3	0.201	0.203	0.984	28.29
500	T3	530.7	0.201	0.203	0.959	33.40
600	T1	647.1	0.242	0.244	1.034	30.62
600	T2	647.4	0.242	0.244	0.842	41.55
600	T3	647.8	0.242	0.244	0.976	48.65
700	T1	745.2	0.285	0.287	0.856	36.58
700	T2	746.1	0.285	0.287	0.844	49.61
700	T3	746.8	0.285	0.287	0.803	58.01
800	T1	852.6	0.308	0.311	0.937	53.64
800	T2	855.2	0.308	0.311	0.833	73.09
800	T3	855.7	0.308	0.310	0.779	85.91
900	T1	965.3	0.365	0.368	0.753	84.43
900	T2	964.8	0.365	0.369	0.890	115.24
900	T3	965.3	0.365	0.369	0.890	135.58
1000	T1	1042.5	0.411	0.413	0.517	71.88
1000	T2	1042.8	0.411	0.414	0.573	111.85
1000	T3	1043.0	0.411	0.414	0.573	131.87
	T1	691.3	0.264	0.266	1.056	39.61
Avg	T2	692.2	0.264	0.267	1.075	55.68
	T3	692.4	0.264	0.266	1.077	65.46

7. Conclusions

In this paper, we proposed integer programming models for the constraint based explicit routing problem in MPLS based IP networks. To solve this problem, we applied column generation approach and the variable fixing procedure. To strength the LP relaxation of the problem, we considered some valid inequalities in the preprocessing procedure. A primal heuristic algorithm is applied to find an incumbent solution

Computational results show that the proposed algorithm can provide high quality solutions in short execution time.

The proposed algorithm can be applied to solve several kinds of routing problems that consider not only hop constraints but also the other QoS attributes such as packet loss or jitter.

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