대한산업공학회/한국경영과학회 2002 춘계공동학술대회
한국과학기술원(KAIST) 2002 년 5 월 3 일-4 일

SCM 환경의 다단계 재고모형에서 긴급상호대차의 효과에 관한 연구
Analysis of the effect of emergency lateral transsphipment on a multi-echelon inventory model in SCM Environment

Chang Sup Sung[1], Julie Kim[2], Su Jin Jung[3]
[1] Ph.D. IE department, KAIST, Daejeon
[2] Samsung Card, Seoul
[3] Ph.D. Candidate, IE department, KAIST, Daejeon
Tel : 042-869-3154; Fax : 042-869-3110
e-mail : cssung@sorak.kaist.ac.kr

Abstract

This paper deals with a continuous-review two-echelon inventory model with one-for-one replenishment and Poisson demand where transshipments among retailers are allowed. Two classes of inventory systems are considered by the number of distribution centers(DCs) which provide each retailer with inventory items. 1:N class inventory system and M:N class inventory system respectively.

Two-phase model is constructed to find out the optimal inventory positions which minimize supply chain costs. Approximations for customer service levels of the system are evaluated in the first phase, and the optimal inventory positions are found subject to the constraints for service level in the second phase. Simulation tests are performed to assure the effectiveness of the proposed model. The effect of transshipment is evaluated.

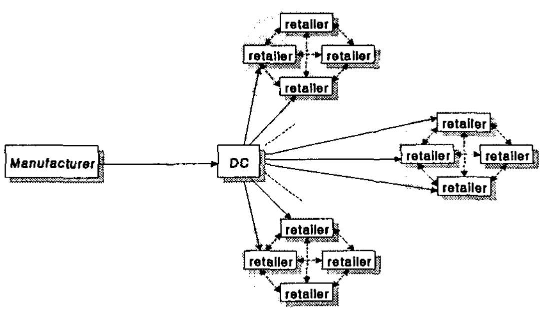
1. Introduction

These days industrial organizations have been forced to reduce operating costs but to improve customer service, which is due to increasing competitive pressures and market globalization. This has directed most firms to make their efforts for effective decisions on their supply chain management. According to recent advances in communications and information technology, it is possible to integrate functional components, geographically distributed facilities, and various decision levels by sharing and using other supply chain members' information. In order to share and take advantage of information among supply chain members, various supply chain strategies have been researched practically and theoretically.

Emergency lateral transshipment is one of the supply chain strategies, referring to Lee[10] where emergency lateral transshipment is defined as "*the shipment of stock from one retailer to another that faces a demand when it is out of stock*". [Figure 1] shows the graphical illustration of a typical multi-echelon inventory system with emergency lateral transshipment, which will be called just "*transshipment*" shortly in the rest of the paper.

When transshipment is allowed, demand uncertainty is pooled across the broad geographical region (Evers[5]). The advantages of this approach include inventory level reduction and customer waiting time reduction for a vast majority of products filled directly from stock (Evers[4]). In this paper, when a stockout case occurs at a retailer, the retailer places an order to another retailer which has on-hand inventory. It is called an "*emergency order*". On the other hand, when a demand arrives at a retailer and the retailer has positive on-hand inventory, the retailer places a "*regular order*" to DC. If stockout occurs at a retailer and the other retailers have no inventory, the retailer also places a regular order to DC (Jovan & Amiya[9]). Emergency orders may result in substantial improvement in service performance of the system, especially when neighboring retailers are at shorter distances than the central DCs. However, benefits from transshipment are not without a cost. The drawbacks include the increased transportation, handling, and administration costs associated with transshipment for any small quantity of redistributed items (Hill[8]). These trade-offs will be discussed in § 5.

This paper deals with a continuous-review two-

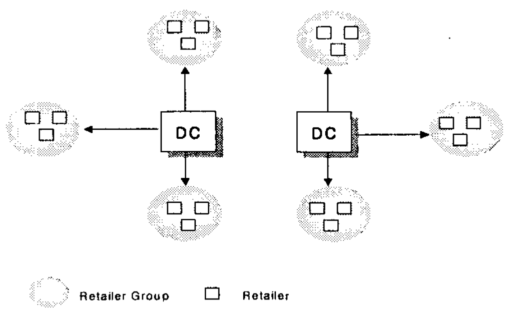


[Figure 1] A typical multi-echelon inventory system with emergency lateral transshipment

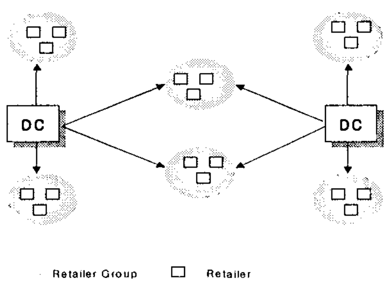
echelon model with one-for-one replenishment and Poisson demand. According to the number of DCs, multi-echelon inventory systems are divided into two classes. The first class is the inventory system where every retailer receives items from only one DC as shown in [Figure 2]. It is called "*1:N class inventory system*" in this paper. The second class is the inventory system where each retailer can receive items from one or more DCs as shown in [Figure 3]. It is called "*M:N class inventory system*". *M:N* class problems include *1:N* class problems. Therefore, only *M:N* class problem needs to be considered.

Two-phase model is constructed to find out the optimal inventory positions which minimize supply chain costs. Approximations for customer service levels of the system are evaluated in the first phase and the resulting approximate values are employed as the input parameters of a minimum total cost model in the second phase. The optimal inventory positions are found subject to the constraints for service level in the second phase. Simulation tests are performed to assure the effectiveness of the proposed model. The effect of transshipment is evaluated.

The organization of this paper is as follows. § 2 reviews related papers. § 3 is devoted to the problem description and formulation. § 4 analyzes the solution properties and proposes an algorithm based on the solution properties. § 5 performs simulation test and discusses the effect of transshipment. § 6 gives some concluding remarks.



[Figure 2] 1:N Class Inventory System



[Figure 3] M:N Class Inventory system

2. Literature review

Studies about multi-echelon inventory systems began as early as 1960 by Clark and Scarf [2]. Since then, many researches have investigated multi-echelon inventory systems. However, theoretic multi-echelon inventory models have not applied practically because of high interaction costs among facilities. In recent years, multi-echelon inventory systems have been implemented practically because interaction costs have been reduced by advanced IT. This trend offers the motivation to pay attention to the multi-echelon inventory system management.

Earlier inventory models that have considered transshipments among stocking locations include Gross[7] and Das[3]. Gross' model[7] is basically a single-echelon model with multiple stocking points. Das' periodic review inventory model[3] consists of only two locations. A classical multi-echelon inventory model with Poisson demand and one-for-one replenishment considered was first analyzed by Sherbrooke[11] who suggested an approximate technique(Metric). Lee[10] has considered a multi-echelon inventory system for repairable items that employs a continuous-review policy similar to the METRIC model of Sherbrooke[11] and Graves[6]. Most assumptions of this paper are based on the literature[10] where approximations for the performance measures and properties for a minimum total cost problem have been developed when transshipment is allowed. However, the demand at a retailer was not described correctly. To correct this defect, Axsäter[1] has put more emphasis on modeling the demand at a retailer correctly. His queueing model[1] is mainly to be employed in this paper. However, only approximations for the performance measure were considered but the total cost issue was not dealt with in the literature. Jovan[9] has extended the concept of transshipment in the inventory systems with one supplier and *n* identical retailers. Generally transshipment occurs when the retailer is out of stock. However, transshipment was made if a demand arrived while on-hand inventory level was lower than a constant *K* in the literature.

As stated above, researches regarding transshipment under various assumptions have been studied. However, the problem environment has been limited to inventory systems with one supplier and several retailers. Thus this paper will extend problem environment to inventory systems with multiple DCs and retailers: *M:N* class inventory problem.

3. Problem Description and Formulation

3.1 Assumptions

① Retailers in the same group have identical demand arrival rates, say λ_j.

② Transshipment is allowed only between retailers in the same group.

③ Low demand-rated items are considered.

④ DCs receive items from a manufacturer in the upper level.

⑤ A demand is backordered either when it is to be filled by emergency order or when all retailers of the group are out of stock.

3.2 Notation

(s) or (s_j) denote the function of s or s_j.

m_j Number of retailers in group j

a_{ij} fraction of j retailer group's demand that i DC serves. ($0 \le a_{ij} \le 1$ and $\sum_i a_{ij} = 1$)

Demand Rate:

λ_j Demand rate at a retailer in group j

$\lambda_j^o = m_j \lambda_j$ Total demand rate of group j

$\lambda_{Di} = \sum_{j=1}^{N} a_{ij} \lambda_j^o$ Demand rate at DC i

Inventory Position:

s_j Inventory position of a retailer in group j

$s_j^o = m_j s_j$ Total inventory position of group j

S_{Di} Inventory position of DC i

Average Backorder Level:

$B_{j(s_j)}$ Average backorder level of a retailer in group j

$B^0_{j(s_j)} = m_j B_{j(s_j)}$ Average total backorder level of group j

$B_{Di(S_{Di})}$ Average backorder level of DC i

$R_{Di} \sim Pois(\lambda_{Di} / \mu_{Di})$ Outstanding order at DC i in steady state

$N_{j(s_j)}$ Average number of transshipments per unit time in group j

Time Parameters:

T_{ij} Transportation time from DC i to a retailer in group j

t_j Transshipment time in group j

$1/\mu_{Di}$ Average lead time from a manufacturer to DC i

$1/\mu_j$ Average lead time from DC to a retailer in group j

Steady-state Probabilities :

π_l^j Steady-state probability that the net inventory at a retailer in group j is l units, with l<0 representing backorders

Π_l^j Steady-state probability that the net inventory of group j is l units, with l<0 representing backorders

Measure for Service Level (at a retailer in group j):

$\alpha_{j(s_j)}$ Proportion of demand met by transshipment

$\beta_{j(s_j)}$ Proportion of demand met by on-hand inventory

$\theta_{j(s_j)}$ Proportion of demand met by DC's inventory due to stock out at all retailers in group j ($\alpha_{j(s_j)} + \beta_{j(s_j)} + \theta_{j(s_j)} = 1$)

$\tau_{j(s_j)}$ Proportion of demand backordered in the inventory system without transshipment allowed

Cost Parameters:

h_j Unit holding cost per time at a retailer in group j

h_{Di} Unit holding cost per time at DC i

b_j Unit backorder cost per time at a retailer in group j

b_{Di} Unit backorder cost per time at DC i

e_j Unit transshipment cost

3.3 Problem Formulation

3.3.1 Phase I: Queueing Model

① Inventory System with transshipment allowed

(1-1)

$$B_{Di(S_{Di})} = \sum_{k=S_{Ci}}^{\infty} (k - S_{Di}) \exp(-\lambda_{Di} / \mu_{Di}) \frac{(\lambda_{Di} / \mu_{Di})^k}{k!}$$

(1-2) $$1/\mu_j = \sum_i a_{ij} \times (T_{ij} + B_{Di(S_{Di})} / \lambda_{Di})$$

(1-3) $$\prod\nolimits_{s_j^0 - k}^{j} = \exp(-\lambda_j^0 / \mu_j) \frac{(\lambda_j^0 / \mu_j)^k}{k!} \quad k = 0,1,2,\Lambda$$

(1-4) $$c_j = \lambda_j (1 - \theta_{j(s_j)}) / \beta_{j(s_j)}$$

(1-5) $d_j = \lambda_j \theta_{j(S_j)} / (1 - \beta_{j(S_j)})$

(1-6) $\pi^j_{S_j} c_j = \pi^j_{S_j - 1} \mu_j$

$$\pi^j_{S_j-k}(c_j + k\mu_j) = \pi^j_{S_j-k+1} c_j + \pi^j_{S_j-k-1}(k+1)\mu_j$$

$$k = 1, 2, \Lambda, S_j - 1$$

$$\pi^j_0 (d_j + S_j \mu_j) = \pi^j_1 c_j + \pi^j_{-1}(S_j + 1)\mu_j$$

$$\pi^j_{S_j-k}(d_j + k\mu_j) = \pi^j_{S_j-k+1} d_j + \pi^j_{S_j-k-1}(k+1)\mu_j$$

$$k = S_j + 1, S_j + 2, \Lambda$$

(1-7) $\pi^j_{S_j-k} = \pi^j_0 \dfrac{S_j! \mu_j^{S_j-k}}{k! c_j^{S_j-k}}$

(1-8) $\pi^j_{S_j-k} = \pi^j_0 \dfrac{S_j! d_j^{k-S_j}}{k! \mu_j^{k-S_j}}$

(1-9) $\dfrac{1}{\pi^j_0} = \sum_{k=0}^{S_j-1} \dfrac{S_j! \mu_j^{S_j-k}}{k! c_j^{S_j-k}} + \sum_{k=S_j}^{\infty} \dfrac{S_j! d_j^{k-S_j}}{k! \mu_j^{k-S_j}}$

(1-10) $\beta_{j(S_j)} = \sum_{l=1}^{S_j} \pi^j_l$

(1-11) $\theta_{j(S_j)} = \sum_{l=0}^{\infty} \Pi^j_{-l}$

(1-12) $\alpha_{j(S_j)} = 1 - \beta_{j(S_j)} - \theta_{j(S_j)}$

From Eq. (1-3), Π^j_l can be obtained with the given S_{Di}, s_j. However, π^j_l is not acquired from one simple equation, but equations from (1-7) to (1-9) provide π^j_l. It is noted that demand rate at a retailer depends on the state of on-hand inventory. Eq. (1-4) represents demand rate at the time when the retailer has positive on-hand inventory. It is composed of external demands and transshipment requests from other retailers. Eq. (1-5) specifies demand rate at the time when the retailer is out of stock.

② Inventory System without transshipment allowed

(2-4) $c_j = \lambda_j$

(2-5) $d_j = \lambda_j$

(2-6) $\pi^j_{S_j-k} = \exp(-\lambda_j / \mu_j) \dfrac{(\lambda_j / \mu_j)^k}{k!}$

(2-7) $\beta_{j(S_j)} = \sum_{l=1}^{S_j} \pi^j_l$

(2-8) $\tau_{j(S_j)} = 1 - \beta_{j(S_j)}$

Equations (2-1), (2-2), (2-3) are equal to the equations (1-1), (1-2), (1-3). The difference caused by transshipment is the demand rate at a retailer. Because demand rate is always identical, the state of on-hand inventory at a retailer is Poisson distributed with mean λ_j / μ_j.

3.3.2 Phase II: Minimum Cost Inventory Model

① Inventory System with transshipment allowed

Min $TC_{(S_{Di}, S_j)} = \{\sum_{i=1}^{M} h_{Di} S_{Di} + \sum_{j=1}^{N} h_j m_j S_j\}$

$$+ \{\sum_{i=1}^{M} b_{Di} B_{Di(S_{Di})} + \sum_{j=1}^{N} b_j m_j B_{j(S_j)}\} + \sum_{j=1}^{N} e_j N_{j(S_j)}$$

$k = 0, 1, \Lambda, S_j - 1$

s.t. $\beta_{j(S_j)} \geq \xi_j$

$1 - \theta_{j(S_j)} \geq \xi'_j$

$k = S_j, S_j + 1, \Lambda$

$\Pr[R_{Di} \leq S_{Di}] \geq \xi_{Di}$

In the objective function, the total cost consists of holding cost, backorder cost and transshipment cost. The lower bound of inventory position is acquired from constraints. The first two constraints are related to inventory position of a retailer and the last constraint is concerned with DC i.

$m_j B_{j(S_j)}$ = backorders met by transshipment + backorders met by the DC

$$= (\alpha_{j(S_j)} m_j \lambda_j) t_j + \sum_{k=m_j S_j}^{\infty} (k - m_j S_j) \Pi^j_{m_j S_j - k},$$

$$N_{j(S_j)} = (\alpha_{j(S_j)} m_j \lambda_j)$$

② Inventory System without transshipment allowed

Min $TC_{(S_{Di}, S_j)} = \{\sum_{i=1}^{M} h_{Di} S_{Di} + \sum_{j=1}^{N} h_j m_j S_j\}$

$$+ \{\sum_{i=1}^{M} b_{Di} B_{Di(S_{Di})} + \sum_{j=1}^{N} b_j m_j B_{j(S_j)}\}$$

s.t. $\beta_{j(S_j)} \geq \xi_j$

$\Pr[R_{Di} \leq S_{Di}] \geq \xi_{Di}$

It should be noticed that $B_{j(S_j)}$ is not the same as $B_{j(S_j)}$ in inventory system with transshipment allowed, while $B_{Di(S_{Di})}$ is identical to $B_{Di(S_{Di})}$ in inventory system with transshipment allowed. $B_{j(S_j)}$ is expressed as follows;

$$B_{j(S_j)} = \sum_{k=S_j}^{\infty}(k-S_j)\pi^j_{S_j-k} = \sum_{k=S_j}^{\infty}(k-S_j)\exp(-\lambda_j/\mu_j)\frac{(\lambda_j/\mu_j)^k}{k!}$$

4. Algorithm

Solution procedures to find the optimal inventory positions S^*_{Di} and S^*_j which minimize the total supply chain cost are proposed based on several solution properties.

① Inventory System with transshipment allowed

<DC *i* Problem>

Min $TC_{(S_{Di})} = h_{Di}S_{Di} + b_{Di}B_{Di(S_{Di})}$

s.t. $\Pr[R_{Di} \le S_{Di}] \ge \xi_{Di}$

<Retailer Group *j* Problem>

Min $TC_{(S_j)}$

$$= h_j m_j S_j + b_j m_j B_{j(S_j)} + e_j N_{j(S_j)}$$

s.t. $\beta_{j(S_j)} \ge \xi_j$

$$1-\theta_{j(S_j)} \ge \xi'_j$$

The original problem can be decomposed into DC problem and retailer problem because the terms of the objective function and constraints depend on either S_{Di} or S_j. In the DC Problem, a lower bound of S^*_{Di}, denoted by LS_{Di}, is easily obtained from a constraint. All feasible S_{Di} should satisfy $S_{Di} \ge LS_{Di}$. It is known that R_{Di} is Poisson distributed with mean λ_{Di}/μ_{Di}. ξ_{Di} is a given constant, say $\xi_{Di} = 0.7$. $A_{(S_{Di})} = B_{Di(S_{Di}+1)} - B_{Di(S_{Di})}$ holds. Then, S^*_{Di} is obtained by Proposition 1.

Proposition 1

i) If $h_{Di} \ge b_{Di}A_{(0)}$, then $S^*_{Di} = LS_{Di}$.

ii) If $h_{Di} < b_{Di}A_{(0)}$ and $S^m_{Di} = \min\{S_{Di} \mid h_{Di} \ge b_{Di}A_{(S_{Di})}\} \ge LS_{Di}$, then $S^*_{Di} = S^m_{Di}$.

iii) If $h_{Di} < b_{Di}A_{(0)}$ and $S^m_{Di} < LS_{Di}$, then $S^*_{Di} = LS_{Di}$.

For the problem of retailer group *j*, it is not easy to find out the optimal S^*_j directly. However, both lower bound and upper bound can be defined. The lower bound of S_j, denoted by LS_j, is obtained from service level constraints. The upper bound, denoted by US_j, is given by Propositions 2 and 3.

$$D_{j(S_j)} = m_j B_{j(S_j)} = \sum_{k=m_jS_j}^{\infty}(k-m_jS_j)\Pi^j_{m_jS_j-k}$$

holds, where $\Pi^{*j}_k = \Pi^j_{m_jS_j-k} = \Pi^j_{m_j(S_j+1)-k}$.

Proposition 2

$D_{j(S_j)}$ monotonically converges to a lower bound.

Proposition 3

$$\lim_{S_j\to\infty}(TC_{j(S_j+1)} - TC_{j(S_j)}) = h_j m_j$$

By Propositions 2 and 3, it is found that US_j is the smallest S_j which satisfies the relations $\beta_{j(S_j)} \cong 1$ and $TC_{j(S_j+1)} - TC_{j(S_j)} = h_j m_j$. Therefore, the total cost should be calculated for the S_j between LS_j and US_j. S^*_j is found among them. Solution procedure based on above propositions is as follows.

Step 0 Assign DC

i) Set *i=1*. Go Step 1.

ii) Set *i=i*+1. Go Step 1.

Step 1 Solve DC *i* Problem

i) Calculate LS_{Di} and set $S_{Di} = LS_{Di}$.

ii) Evaluate $A_{(S_{Di})} = \Pr(k \ge S_{Di}+1)$, where $\Pr(k)$ is a Poisson pdf. with mean λ_{Di}/μ_{Di}.

iii) If $h_{Di} \ge b_{Di}A_{(S_{Di})}$, then $S^*_{Di} = LS_{Di}$ and go to vi). Otherwise, go to iv).

iv) Set $S_{Di} = S_{Di}+1$.

v) If $h_{Di} \ge b_{Di}A_{(S_{Di})}$, then $S^*_{Di} = S_{Di}$ and go to vi). Otherwise, go to iv).

vi) If *i=M*, go to Step 2. Otherwise, go to Step 0-ii).

Step 2 Assign retailer group

i) Set *j=1*. Go to Step 3.

ii) Set *j=j+1*. Go to Step 3.

Step 3 Solve Retailer Group j Problem

i) From the constraints, obtain LS_j.

Set $s_j = LS_j$, $TC_{j(S_j-1)} = 0$.

Give a convergence tolerance parameter $\varepsilon \ge 0$, say the value 0.005.

ii) Evaluate $\alpha_{j(S_j)} = 1 - \beta_{j(S_j)} - \theta_{j(S_j)}$ and $TC_{j(S_j)}$.

iii) If $s_j = LS_j$, then $S_j^* = s_j$ and $TC_{j(s_j)}^* = TC_{j(s_j)}$.

If $s_j > LS_j$ and $TC_{j(s_j)} < TC_{j(s_j)}^*$, then $S_j^* = s_j$ and $TC_{j(s_j)}^* = TC_{j(s_j)}$.

iv) If $\beta_{j(s_j)} \geq 1-\varepsilon$ and $TC_{j(s_j+1)} - TC_{j(s_j)} = h_j m_j$, go to v).

The optimal solution is at S_j^* and the minimum cost is at $TC_{j(s_j)}^*$.

Otherwise, set $s_j = s_j + 1$ and go to ii).

v) If $j=N$, then terminate.
Otherwise, go to Step 2-ii).

② Inventory System without transshipment allowed
<DC i Problem>

Min $TC_{(S_{Di})} = h_{Di} S_{Di} + b_{Di} B_{Di(S_{Di})}$

s.t. $\Pr[R_{Di} \leq S_{Di}] \geq \xi_{Di}$

<Retailer Group j Problem>

Min $TC_{(S_j)} = h_j m_j S_j + b_j m_j B_{j(S_j)}$

s.t. $\beta_{j(S_j)} \geq \xi_j$

The optimal solution of the DC i Problem, denoted by S_{Di}^*, is equal to the optimal solution of the inventory system with transshipment. On the other hand, S_j^* is closely affected by transshipment. In the retailer group j problem, the structure of the objective function is very similar to that of the DC problem. Accordingly, Proposition 4 is characterized to find S_j^*.

Proposition 4

i) If $h_j \geq b_j \Phi_{j(0)}$, then $S_j^* = LS_j$.

ii) If $h_j < b_j \Phi_{j(0)}$ and

$$S_j^m = \min\{S_j \mid h_j \geq b_j \Phi_{j(S_j)}\} \geq LS_j,$$

then $S_j^* = S_j^m$.

iii) If $h_j < b_j \Phi_{j(0)}$ and $S_j^m < LS_j$, then $S_j^* = LS_j$.

Solution procedure is as follows. Procedure from Step 0 to Step 2 is the same as that of inventory system with transshipment allowed

Step 3 <u>*Solve Retailer Group j Problem*</u>

i) From the constraints, obtain LS_j. Set $s_j = LS_j$.

ii) Evaluate $\Phi_{j(S_j)} = | B_{j(S_j+1)} - B_{j(S_j)} |$.

iii) If $h_j \geq b_j \Phi_{j(S_j)}$, then $S_j^* = LS_j$ and go to vi). Otherwise, go to iv).

iv) Set $s_j = s_j + 1$.

v) If $h_j \geq b_j \Phi_{j(S_j)}$, then $S_j^* = S_j$ and go to vi). Otherwise, go to iv).

vi) If $j=N$, then terminate. Otherwise, go to Step2-ii).

6. Computational Results

6.1 Simulation Test

The results are shown in Tables 1 and 2.

λ_j	S	S_j	$\alpha_{j(S_j)}$		$\beta_{j(S_j)}$	
			Approximation	Simulation	Approximation	Simulation
0.06	0	1	0.26	0.27	0.61	0.63
	2	1	0.18	0.16	0.80	0.82
	4	1	0.16	0.14	0.82	0.85
0.08	0	1	*0.25	0.30	0.51	0.51
	2	1	0.22	0.22	0.72	0.73
	4	1	0.20	0.20	0.76	0.76
0.1	2	1	0.25	0.26	0.64	0.65
		2	0.05	0.05	0.95	0.95
	4	1	0.23	0.24	0.70	0.71
		2	0.04	0.03	0.96	0.97
	6	1	.0.23	0.23	0.71	0.72
		2	0.04	0.03	0.96	0.97
0.2	4	2	0.14	0.14	0.84	0.84
		3	0.03	0.03	0.97	0.97
		4	0.01	0.01	1.00	0.99
	8	1	*0.24	0.30	*0.49	0.47
		2	0.11	0.12	0.88	0.87
		3	0.02	0.02	0.98	0.98
0.4	4	3	*0.19	0.21	0.71	0.69
		4	0.09	0.11	0.90	0.88
		5	0.03	0.04	0.97	0.96
	8	3	0.12	0.13	0.86	0.86
		4	0.04	0.04	0.96	0.96
		5	0.01	0.01	0.99	0.99

[Table 1] Computational Results in the M:N inventory system with transshipment allowed.

λ_j	S	S_j	$\beta_{j(S_j)}$	
			Approximation	Simulation
0.06	0	1	0.66	0.68
	2	1	0.82	0.84
	4	1	0.84	0.85
0.08	0	1	0.57	0.59
	2	1	0.75	0.76

	4	1	0.78	0.79
0.1	2	1	0.68	0.70
		2	0.94	0.95
	4	1	0.74	0.75
		2	0.96	0.97
	6	1	0.74	0.75
		2	0.96	0.97
0.2	4	2	0.85	0.84
		3	0.97	0.96
		4	0.99	0.99
	8	1	0.55	0.55
		2	0.88	0.88
		3	0.98	0.97
0.4	4	3	0.72	0.71
		4	0.88	0.88
		5	0.96	0.96
	8	3	0.86	0.86
		4	0.96	0.96
		5	0.99	0.99

[Table 2] Computational Results in the M:N inventory system without transshipment allowed.

In the above tables, (*) means that the approximate values lie outside 95% confidence interval of the simulation results. In [Table 1], 3 out of 24 $\alpha_{j(s_j)}$ instances and 1 out of 24 $\beta_{j(s_j)}$ instances exist outside the confidence interval. However, most of them are not far from the simulation results. In [Table 2], there are nothing outside the confidence interval. These results show that the approximate queueing model is very effective. However, some comments are needed. Most outside instances belong to $\alpha_{j(s_j)}$ and most of them are smaller than the simulation results. It means that the real $\theta_{j(s_j)}$ value may be smaller than its approximation. Most outside instances of $\alpha_{j(s_j)}$ have fairly large values of $\alpha_{j(s_j)}$. Thus, the proposed queueing model works better when the value of $\alpha_{j(s_j)}$ is small.

6.2 Effect of transshipment

Problems with variable transportation time from DCs to retailers, unit backorder cost and unit transshipment cost are considered to explain the effect of transshipment. For the effect investigation, two problem sets are generated. The first set consists of 5 problems with varying transportation time which refers to movement time from a DC to a retailer. It is assumed that transportation time is the same as at all retailers. The second set includes 9 problems with varying unit backorder cost and unit transshipment cost. Unit costs at DC and all retailers are the same for each type of cost parameter. The computational results are given in [Table 3] and [Table 4].

In the tables, the symbol (+) points to the case where the stock level in the system with transshipment allowed is higher than the stock level in the system without transshipment allowed. The symbol (*) indicates the case where the total cost in the system with transshipment allowed are lower than the total cost in the system without transshipment allowed. Unexpectedly, the total cost increases by transshipment in several cases. These observations occur because of two reasons. They are discussed in § 7.

From the [Table 3], transshipment has a strong effect of reducing total cost as transportation time gets longer. It really makes sense. From the [Table 4], total cost significantly decreases as unit transshipment cost gets smaller and unit backorder cost gets larger.

T_j	With transshipment	Without transshipemnt

Backorder cost	Transshipment cost	Transshipment allowed		Transshipment not allowed	
		(s^*, s_j^*)	Total Cost	(s^*, s_j^*)	Total Cost
60	10	+(5,2,1,2)	208.37	(5,1,1,1)	162.90
	20	+(5,2,1,2)	208.94	(5,1,1,1)	162.90
	30	+(5,2,1,2)	209.50	(5,1,1,1)	162.90
100	10	*(6,1,1,1)	175.45	(6,1,1,1)	181.44
	20	*(6,1,1,1)	177.16	(6,1,1,1)	181.44
	30	*(6,1,1,1)	178.86	(6,1,1,1)	181.44
150	10	*(6,1,1,1)	192.32	(6,1,1,1)	202.15
	20	*(6,1,1,1)	194.02	(6,1,1,1)	202.15
	30	*(6,1,1,1)	195.73	(6,1,1,1)	202.15

[Table 4] Varying cost parameters

	(S^*,S_j^*)	TC	(S^*,S_j^*)	TC
2	(1,1)	191.03	(1,1)	190.91
*3	(1,1)	191.36	(1,1)	191.57
*4	(1,1)	191.69	(1,1)	192.40
*5	(2,1)	231.25	(2,1)	238.08
*8	(2,2)	383.93	(2,2)	389.33

[Table 3] Varying transportation time from DC to pooling groups

7. Conclusion

This paper deals with a continuous-review two-echelon inventory model with one-for-one replenishment and Poisson demand considered. The inventory systems are divided into two classes by the number of DCs: *1:N* Class and *M:N* Class. Because *M:N* class problem include *1:N* problem, only *M:N* class was considered. A two-phase model is constructed to find out the optimal inventory position which minimizes supply chain cost. The supply chain cost consists of inventory holding costs and backorder costs. Transshipment costs should also be considered, especially in the case when transshipments are allowed. Approximations for the service levels of the system are evaluated in the first phase via the queueing model. The optimal inventory position is then found subject to the constraints imposed on service level in the second phase via the minimum total cost model.

Simulation tests are performed to assure the effectiveness of the proposed queueing model. From the result of the simulation test, the queueing model was shown to be very effective. The result shows that the effect of transshipment depends on two factors: transportation time from DC to retailer groups and unit backorder cost. As transportation time gets longer, and as unit backorder cost gets higher, the effect of transshipment to reduce total cost gets significant. Therefore, transshipment tends to be more suitable for inventory systems where long transportation time and high unit backorder costs are considered.

However, it should be noticed that the effect of transshipment is not as large as expected. There are two reasons for that. First, low demand items are assumed in this paper. Backorders seldom occur and so backorder costs are very small within total cost. Moreover, any reduction in backorder cost by transshipment is traded off an additional transshipment cost. Therefore, the effect of transshipment needs to be investigated for high demand-rated items. Second, the largest proportion of total cost is holding cost, which is mainly determined by the constraints imposed on service level. Therefore, the behavior of the system performance should be examined for various kinds of constraints.

As a further study, the following may be considered to extend the problem;

① High demand-rated items
② Other order policies including (R, r), (nQ, r), etc.
③ Additional constraints, including backorder time, backorder level, etc.
④ Higher levels of multi-echelon systems

References

[1] Axsäter, S., "Modeling emergency lateral transshipments in inventory systems," *Management Science*, Vol.36, No.1, 1990
[2] Clark, A.J. and Scarf, H., "Optimal policies for a multi-echelon inventory problem," *Management Science*, Vol.6, 1960
[3] Das, C., "Supply and redistribution rules for two-location inventory systems: One Period Analysis," *Management Science*, Vol.21, 1975
[4] Evers, P.T., "Hidden benefits of emergency transshipments," *Journal of Business Logistics*, Vol.18, 1997
[5] Evers, P.T., "Heuristics for assessing emergency transshipments," *European Journal of Operational Research*, Vol.129, 2001
[6] Graves, S.C., "A multi-echelon inventory model for a repairable item with one-for-one replenishment," *Management Science*, Vol.31, 1985
[7] Gross, D., "Centralized inventory control in multi-location supply systems," *Multistage Inventory Models and Techniques*, H.E. Scarf, D.M. Gilford, and M.W. Shelly (Eds,), Stanford University Press, Stanford, CA, 1963
[8] Hill, R.M., "Using inter-branch stock transfers to meet demand during a stockout," *International Journal of Retail and Distribution Management*, Vol.20, 1992
[9] Jovan, G. and Amiya, C., "Sharing and lateral transshipments of inventory in a supply chain with expensive low-demand items," *Management Science*, Vol.47, No.4, 2001
[10] Lee, H.L., "A multi-echelon inventory model for repairable items with emergency lateral transshipments," *Management Science*, Vol.33, No.10, 1987
[11] Sherbrooke, C.C., "METRIC: A multi-echelon technique for recoverable item control," *Oper. Res.*, Vol.16, 1968