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A Modified Weighted Least Squares Approach to Range Estimation Problem

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Abstract

A practical recursive weighted least square(WLS) solution is proposed to solve the passive ranging problem. Apart from the previous works based on the extended Kalman filter(EKF), to ensure the convergency at long-range, the proposed scheme makes use of line-of-sight(LOS) rate instead of bearing information. The influence of LOS rate measurement errors is investigated and it is asserted that the WLS estimates contain bias and scale factor errors. Together with simple compensation algorithm, the estimation errors of proposed filter can be reduced dramatically.

1 Introduction

The objective of passive ranging is to estimate the relative range between the missile and the moving target using bearing information. Therefore, the passive ranging problem is regarded as one of representative nonlinear filtering problems. During the last decades, there are many attempts to solve the passive ranging problem based on the EKF and its variants [1]. It is very well known that there exist bias errors in range estimates caused by a correlation between the EKF gain and its innovations sequence [2]. To come up with this drawback, a design of the unbiased EKF was concerned. Another problem is that a conventional angle based passive ranging filter requires long-distance movement proportional to the initial range r_0 to ensure a reasonable accuracy of range information in angle measurements as shown in the figure 1. It causes significant difficulty for the application of long-range passive ranging problem [3].

Depart from the conventional passive ranging filter, a novel modified WLS range filter exploits the fact that the relative velocity perpendicular to LOS vector can be expressed by the multiplication of range and noisy LOS rate measurements. By using the LOS rate measurements as well as LOS angle, the sufficient observability can be easily obtained with moderate missile acceleration. Thus it guarantees fast convergency at long-range. To describe relative kinematics with no approximations, the proposed filter adopts the linear measurement equation. This implies the proposed filter might be more confident in practical applications than the previous methods based on the approximate nonlinear filter. It is able to effectively eliminate the bias errors from range estimates with simple modification. And it can reduce the computational burden compared to the conventional EKF recursions.

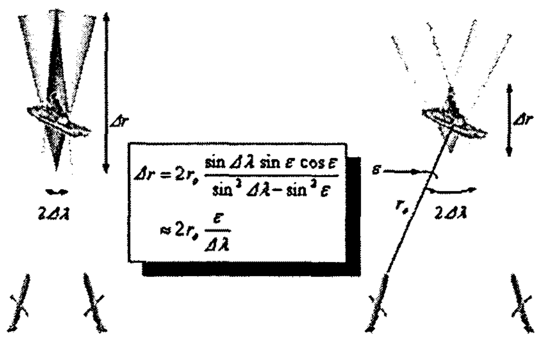

Figure 1: Observability in Passive Ranging Problem

2 Weighted Least Square Estimation

The range estimation problem in horizontal plane against nearly stationary ship target is formulated for simplicity. Let r_k be the relative range between missile and target should be estimated from LOS rate measurement $\tilde{\omega}_k$ and relative velocity measurement $\tilde{v}_k$ perpendicular to LOS vector. The engagement geometry is depicted in figure 2. The LOS rate measurement consists of the true LOS rate ω_k corrupted by additive noise $\delta\omega_k$, which is assumed to be zero-mean Gaussian with covariance Q_k. Also, if the closing velocity v_k^c is given, the range propagation

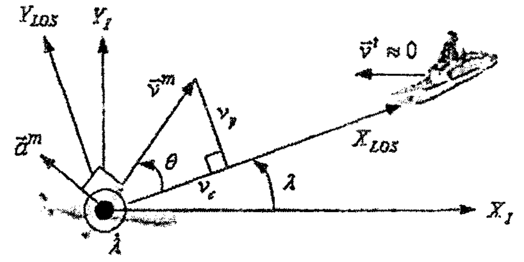

Figure 2: Engagement Geometry

equation can be written by

$$r_{k+1} = r_k + \Delta t \; v_k^c \tag{1}$$

where Δt is the sampling time defined by $\Delta t = t_{k+1} - t_k$. To define the least square problem, first we consider the general recurrent relations for basic angular kinematics.

$$\begin{aligned} v_{k-j}^p &= \omega_{k-j} r_{k-j} = \omega_{k-j}\left(r_k - \Delta t \sum_{i=1}^{j} v_{k-i}^p\right) \quad (2) \\ &\equiv \omega_{k-j}\left(r_k - b_{k-j}\right) \end{aligned}$$

Therefore, the optimal range estimates $\hat{r}_k$ in WLS sense is the minimizing solution to the weighted quadratic cost function J_k defined by

$$J_k\left(r, \tilde{\omega}, \tilde{v}, \tilde{b}\right) = \frac{1}{2}\sum_{j=0}^{k} q^j \left(\tilde{\omega}_{k-j} r_k - \tilde{\omega}_{k-j}\tilde{b}_{k-j} - v_{k-j}^p\right)^2 \tag{3}$$

where q is the forgetting factor.

The sufficient condition for optimal range estimates, $\hat{r}_k$, in WLS setting is readily obtained.

$$\frac{\partial J_k}{\partial r_k} = \sum_{j=0}^{k} q^j \left(\tilde{\omega}_{k-j}\hat{r}_k - \tilde{\omega}_{k-j}\tilde{b}_{k-j} - v_{k-j}^p\right)\tilde{\omega}_{k-j} = 0 \tag{4}$$

Rearranging the above equation results the WLS solution to the range estimation problem.

$$\hat{r}_k = \frac{1}{\Delta_k}\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\left(\tilde{\omega}_{k-j}\tilde{b}_{k-j} + \tilde{v}_{k-j}^p\right), \tag{5}$$

where,

$$\Delta_k \equiv \sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}^2.$$

The correlations between LOS rate measurement error and relative velocity measurement error cause the bias errors of WLS estimates similar to the case of EKF.

3 Property of WLS Estimation Error

To get an insight into the nature of the WLS estimation errors, the valuable analysis are done using the definitions

$$\tilde{\omega}_k = \omega_k + \delta\omega_k, \quad \tilde{b}_{k,j} = b_{k,j} + \delta b_{k,j}, \quad \tilde{v}_k^p = v_k^p + \delta v_k^p. \tag{6}$$

It is also assumed that the measurement errors listed above are mutually uncorrelated zero mean white noises. Substituting eqs. (2) and (6) for numerator in eq. (5), one gets

$$\begin{aligned} &\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\left(\tilde{\omega}_{k-j}\tilde{b}_{k-j} - \tilde{v}_{k-j}^p\right) \\ &= \left[\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}^2\right] r_k - \left[\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\delta\omega_{k-j}\right] r_k \quad (7) \\ &+ \sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\left(\omega_{k-j}\delta b_{k,j} + \delta\omega_{k-j}\tilde{b}_{k,j} + \delta v_{k-j}^p\right). \end{aligned}$$

Consequently, from eqs. (5) and (8) , it is obvious that the WLS range estimate contains scale factor errors α_k and bias errors β_k.

$$\hat{r}_k = (1 - \alpha_k) r_k + \beta_k \tag{8}$$

where α_k and β_k have been defined by

$$\begin{aligned} \alpha_k &= \frac{1}{\Delta_k}\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\delta\omega_{k-j}, \\ \beta_k &= \frac{1}{\Delta_k}\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\left(\omega_{k-j}\delta b_{k,j} + \delta\omega_{k-j}\tilde{b}_{k,j} + \delta v_{k-j}\right). \end{aligned}$$

Now, we consider a simple error correction algorithm in WLS estimates. Maybe, it is a possible correction method to use the statistics of α_k and β_k. Replacing the α_k and β_k with the expectations $E[\alpha_k]$ and $E[\beta_k]$, we have the modified WLS estimates

$$\hat{r}_k' = \frac{\hat{r}_k - E\left[\beta_k\right]}{1 - E\left[\alpha_k\right]} \tag{9}$$

However, we can not find the $E[\alpha_k]$ and $E[\beta_k]$ by the online processing for its nonlinear statistics. In this reason, we suggest a practical alternative idea which uses not the expectations of α_k and β_k but the expectations of their numerators.

$$\hat{\alpha}_k = \frac{1}{\Delta_k} E\left[\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\delta\omega_{k-j}\right] = \frac{1}{\Delta_k}\sum_{j=0}^{k} q^j Q_{k-j} \tag{10}$$

$$\begin{aligned} \hat{\beta}_k &= \frac{1}{\Delta_k} E\left[\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\omega_{k-j}\delta b_{k,j} + \delta\omega_{k-j}\tilde{b}_{k,j} + \delta v_{k-j}^p\right] \\ &= \frac{1}{\Delta_k}\sum_{j=0}^{k} q^j Q_{k-j}\tilde{b}_{k-j} \end{aligned}$$

Then, the modifications of WLS estimates is completed with the same manner used in (9).

$$\hat{r}_k^* = \frac{\hat{r}_k - \hat{\beta}_k}{1 - \hat{\alpha}_k} \tag{11}$$

4 Recursive Form of Modified WLS Algorithm

To implement the modified WLS range filter, some tedious works are necessary. We should find the recursive forms of the parameters in (5) and (11). To do this, first consider the recursion of Δ_k.

$$\Delta_{k+1} = q \cdot \Delta_k + \tilde{\omega}_{k+1}^2 \tag{12}$$

Since $\tilde{b}_{k+1,0} = 0$ and $\tilde{b}_{k+1,j+1} = \tilde{b}_{k,j} + \Delta t \cdot \tilde{v}_k^c$, we can find the following recursions.

$$\begin{aligned} & \Delta_{k+1}\ \hat{r}_{k+1} \qquad (13) \\ = & \sum_{j=0}^{k+1} q^j \left(\tilde{\omega}_{k+1-j}\ \tilde{v}_{k+1-j}^p + \tilde{\omega}_{k+1-j}^2\ \tilde{b}_{k+1,j} \right) \\ = & \ q \sum_{j=0}^{k} q^j \left(\tilde{\omega}_{k-j}\ \tilde{v}_{k-j}^p + \tilde{\omega}_{k-j}^2 \tilde{b}_{k+1,j+1} \right) + \tilde{\omega}_{k+1} \tilde{v}_{k+1} \\ = & \ q \sum_{j=0}^{k} q^j \left(\tilde{\omega}_{k-j} \tilde{v}_{k-j}^p + \tilde{\omega}_{k-j}^2 \tilde{b}_{k,j} \right) + \tilde{\omega}_{k+1} \tilde{v}_{k+1}^p + q \Delta t \tilde{v}_k^c\ \Delta_k \\ = & \ q\ \Delta_k\ (\hat{r}_k + \Delta t \cdot \tilde{v}_k^c) + \tilde{\omega}_{k+1} \tilde{v}_{k+1}^p \end{aligned}$$

By the similar way,

$$\Delta_{k+1}\ \hat{\alpha}_{k+1} = q\ \Delta_k\ \hat{\alpha}_k + Q_{k+1}, \tag{14}$$

$$\Delta_{k+1}\ \hat{\beta}_{k+1} = q\ \Delta_k\ \hat{\beta}_k + q\ \Delta t\ v_k^c\ \hat{\alpha}_k. \tag{15}$$

Using the above results (12)-(15), the recursive modified WLS range filter can be obtained. It should be noted that the above recurrent equations are require only first order scalar manipulations.

5 Simulation Results

To verify the performance of the proposed filter and reveal the fast convergency, a typical anti-ship missile engagement scenario in XY plane is considered. The simulation condition is summarized in table 1. For comparison, we design an EKF with LOS and LOS rate measurements because an EKF with bearing only measurement is diverge under the same simulation condition. As shown in the figure 3, the range estimation performance of proposed WLS filter is superior to the EKF at long-range. Also, the filter convergency come up to our expectations. The initial fluctuation of the modified WLS estimate is due to the deficiency of the measurements corresponding to the forgetting factor q. Once, sufficient measurements are collected, this fluctuation is diminished.

6 Conclusion

In this paper, a practical range estimator was proposed. It is shown that the filter convergency problem at long-range is easily solved by using the LOS rate information and the proposed modified WLS algorithm.

Table 1: Simulation Condition

Item	Assumption
Target	initial pos. : $\vec{p}_{to} = [0.0,\ 0.0]$ Km
Missile	initial pos. : $\vec{p}_{mo} = [-10.0,\ 0.1]$ Km missile vel. : $v_m = 300.0$m/s initial heading : $\psi_0 = 0.0°$
Guidance	$a_m = -3.0\ v_k^c\ \omega_k + 10.0 \sin(0.315 \cdot r_k / v_k^c)$
Errors	LOS, LOS rate error std : $0.1°, 0.1°/\text{sec}$
EKF	$\hat{p}_0 = -[49.9, 0.58]$Km $\hat{v}_0 = [300.0, 0.0]$m/s $Q = 0.01^2$, $R = diag(0.1°^2, 0.1°^2)$
WLS	initial range : $\hat{r}_0 = 50.0$Km $Q = (0.1°)^2$ $q = 0.999$

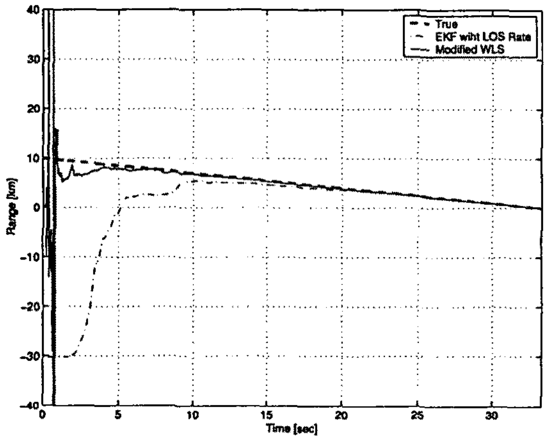

Figure 3: Range Estimates

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