연속시간 TS 퍼지 시스템의 새로운 이산화

김 도완*, 박 진배*, 정 근호**, 주 영훈**
* 연세대학교 전기전자공학과 ** 군산대학교 전자정보공학부

A New Discretization of Continuous-time TS Fuzzy System

Do Wan Kim*, Jin Bae Park*, Keun Ho Jeong**, and Young Hoon Joo**
* Dep. of Electrical and Electronic Eng., Yonsei University
** School of Electronic and Information Eng., Kunsan National University

Abstract - In this paper, a novel and efficient discretization method of a continuous-time Takagi-Sugeno (TS) fuzzy system is proposed. Because of the highly complex and nonlinear interaction among the subsystems, it is very difficult to develop the discretized version of continuous-time TS fuzzy system. More precisely, to obtain the suitable discretized version, it is necessary to solve two main problems: to discretize global state equations of the continuous-time TS fuzzy system and to hold the polytopic structure after the discretization. This paper will show the solution to above two problems. Main key idea is to transforme the sampling period into the intersampling period. Finally, To show the feasibility and the validity of the proposed method, a computer simulation is provided.

1. Introduction

There exist two digital design approaches for state-feedback digital control systems. The main stream in design approaches to a suitable digital controller is first to discretize the continuous plant, and then to determine a digital controller for the discretized plant, which is called the direct digital design approach [3]. Another efficient approach, which is called digital redesign, is a digital controller design procedure, where an analog controller is first designed and then converted to an equivalent digital controller in the sense of state matching [1,4].

We should notice that, at both design approaches, discretization procedure plays an important and basic part. There are several methods for discretizing linear time-invariant (LTI) continuous-time system. Unfortunately, these discretization methods cannot be directly applied to the discretization of the continuous-time TS fuzzy system since defuzzified output of the TS fuzzy system is not LTI but implicitly time-varying [2].

In this paper, a new discretization method is proposed for the continuous-time TS fuzzy system. To obtain a suitable discretized version, there are two critical points to be considered. First, the discretized version must be obtained by discretizing the global state equation of the continuous-time TS fuzzy system. Second, the discretized version must holds the polytopic structure to be systematically analyzable. To accomplish the above two objectives, we presents a new approximation method based on the inter-sampling time conversion method. Consequently, our discretized version yields the low discretization error and holds polytopic structure. The main contribution of this paper is to originally provide the global discretized version of continuous-time TS fuzzy system. Moreover, our global discretized version is directly applicable for design of the fuzzy-model-based digital controller.

2. Problem Description

Consider the continuous-time TS fuzzy system

$$\dot{x}(t)=\sum_{i=1}^{q}\theta_i(z(t))\left(A_i x(t)+B_i u(t)\right) \tag{1}$$

If the state transition matrix $\Phi(t,t_0)$ satisfies $\Phi(t,t_0)=\Phi(t,t_1)\Phi(t_1,t_0)$ with initial condition $\Phi(t_0,t_0)=I$ and $\frac{\partial}{\partial t}\Phi(t,t_o)=\sum_{i=1}^{q}\theta_i(z(t))A_i\Phi(t,t_0)$, the general solution of (1) excited by the initial state $x(t_0)$ and the input $u(t)$ is given by

$$x(t)=\Phi(t,t_0)x(t_0)+\int_{t_0}^{t}\Phi(t,\tau)\left(\sum_{i=1}^{q}\theta_i(z(\tau))B_i\right)u(\tau)d\tau \tag{2}$$

If an input $u(t)$ is generated by a digital computer followed by a digital-to-analog converter, then $u(t)$ will be piecewise constant, *i.e.*, $u(t)=u(kT)$ for $t\in[kT,kT+T)$. Besides, assume that the firing strength $\theta_i(z(t))$ for $t\in[kT,kT+T)$ is $\theta_i(z(kT))$. For this input and $t\in[kT,kT+T)$, (2) is exactly evaluated by

$$\begin{aligned}x(kT+T)&=\Phi(kT+T,kT)x(kT)\\&\quad+\int_{kT}^{kT+T}\Phi(kT+T,\tau)\left(\sum_{i=1}^{q}\theta_i(z(\tau))B_i\right)u(\tau)d\tau\\&=\exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right)x(kT)\\&\quad+\int_{kT}^{kT+T}\exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i(kT+T-\tau)\right)\\&\quad\times\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right)u(kT)d\tau\\&=\exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right)x(kT)\\&\quad+\left(\exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right)-I\right)\\&\quad\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\right)^{-1}\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right)u(kT)\\&\doteq A_d(kT)x(kT)+B_d(kT)u(kT)\end{aligned} \tag{3}$$

This is a discretized version of (1). Note that there is no approximation error involved in this derivation and yields the exact solution of (1) at $t=kT$ if the input and the firing strength is piecewise constant.

However, (3) has too strong nonlinearities to practically be applied in the problem of the controller design such as intelligent digital redesign. In the other words, it is impossible to systematically analyze (3) since it is not represented in the polytopic structure unlike the TS fuzzy system.

To resolve this problem, Li *et al.* attempt to the conversion (3) to the polytopic structure by the following approximation [2].

$$A_d(kT)\approx\sum_{i=1}^{q}\theta_i(z(kT))\exp(A_iT)$$

$$B_d(kT)\approx\sum_{i=1}^{q}\theta_i(z(kT))(\exp(A_iT)-I)A_i^{-1}B_i$$

Unfortunately, this discretization approach may lead to the big undesired approximation error for the long sampling time T, and also it is only the equivalent result of the defuzzified output of the descretized version of the local state-space equation describing linear time invariant (LTI) system for each TS fuzzy model, *i.e., local discretization*, due to coarse approximation.

Therefore, in this paper, we deal with extremely critical issue, which has not so far been made at providing the satisfactory solution. That briefly is formulated by the following Problem.

Problem 1 (Global discretization of the continuous time TS fuzzy system) *The pointwise dynamical behavior of the continuous-time TS fuzzy system are sufficiently satisfied with the following design objectives:*

(i) It must be obtained to discretize the global state equation of it as accurately as possible.

(ii) It must maintain the polytopic structure to be systematically analyzable.

3. Main results

In this section, we presents new dicstretization version of the continuous time TS fuzzy system through Theorem 1. It is designed to definitely satisfy two objectives of Problem 1. Notice that our key idea is to transform the sampling time period T into inter-sampling time period T/n, where $n>1$ and n is constant.

We begin with the following Corollary, which is necessary to prove Theorem 1.

Corollary 1

$$\exp\Big(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\Big) \approx \sum_{i1=1}^{q}\times\cdots\times\sum_{in=1}^{q}\theta_{i1}(z(kT))\times\cdots\times\theta_{in}(z(kT)) \times\exp\Big(A_{i1}\frac{T}{n}\Big)\times\cdots\times\exp\Big(\frac{A_{in}}{T}n\Big) \quad (4)$$

Proof: It holds since

$$\begin{aligned}\exp\Big(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\Big) &= \exp\Big(\sum_{i=1}^{q}\theta_i(z(kT))A_i\frac{T}{n}\Big)^n \\ &\approx \Big(\sum_{i=1}^{q}\theta_i(z(kT))\exp\Big(A_i\frac{T}{n}\Big)\Big)^n \\ &= \sum_{i1=1}^{q}\times\cdots\times\sum_{in=1}^{q}\theta_{i1}(z(kT))\times\cdots\times\theta_{in}(z(kT)) \times\exp\Big(A_{i1}\frac{T}{n}\Big)\times\cdots\times\exp\Big(A_{in}\frac{T}{n}\Big)\end{aligned} \quad (5)$$

■

The following main theorem gives the rigorous mathematical approximation to accomplish two objectives of Problem 1.

Theorem 1 *The continuous-time TS fuzzy system*

$$\dot{x}(t)=\sum_{i=1}^{q}\theta_i(z(t))\,(A_ix(t)+B_iu(t)) \quad (6)$$

can be converted to the following well approximated pointwise dynamical behavior:

$$x(kT+T) = \sum_{i1=1}^{q}\times\cdots\times\sum_{in=1}^{q}\theta_{i1}(z(kT))\times\cdots\times\theta_{in}(z(kT)) \times(G_{i1,\cdots,in}x(kT)+H_{i1,\cdots,in}u(kT)) \quad (7)$$

where,

$$G_{i1,\cdots,in} = \exp\Big(A_{i1}\frac{T}{n}\Big)\times\cdots\times\exp\Big(A_{in}\frac{T}{n}\Big)$$

$$\begin{aligned}H_{i1,\cdots,in} &= \frac{\big(\exp\big(A_{i1}\frac{T}{n}\big)-I\big)A_{i1}^{-1}B_{i1}+\cdots+\big(\exp\big(A_{in}\frac{T}{n}\big)-I\big)A_{in}^{-1}B_{in}}{n} \\ &+\frac{1}{n-1}\Big(\exp\Big(A_{i1}\frac{T}{n}\Big)\Big(\exp\Big(A_{i2}\frac{T}{n}\Big)-I\Big)A_{i2}^{-1}B_{i2} \\ &+\cdots+\exp\Big(A_{i1}\frac{T}{n}\Big)\Big(\exp\Big(A_{in}\frac{T}{n}\Big)-I\Big)A_{in}^{-1}B_{in}\Big) \\ &+\cdots+\exp\Big(A_{i1}\frac{T}{n}\Big)\times\cdots\times\exp\Big(A_{i(n-1)}\frac{T}{n}\Big)\Big(\exp\Big(A_{in}\frac{T}{n}\Big)-I\Big) \\ &\times A_{in}^{-1}B_{in}\end{aligned}$$

Proof: The general global approach is already illustrated in Section 2.2. Start from the kernel of question, namely approximation of (3). Using Corollary 1, $A_d(kT)$ and $B_d(kT)$ of (3) are finely approximated as follows: $A_d(kT)$ is directly approximated from Corollary 1.

$$\begin{aligned}A_d(kT) &= \exp\Big(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\Big) \\ &\approx \sum_{i1=1}^{q}\times\cdots\times\sum_{in=1}^{q}\theta_{i1}(z(kT)) \times\cdots\times\theta_{in}(z(kT))\exp\Big(A_{i1}\frac{T}{n}\Big)\times\cdots\times\exp\Big(A_{in}\frac{T}{n}\Big) \\ &\doteq \sum_{i1=1}^{q}\times\cdots\times\sum_{in=1}^{q}\theta_{i1}(z(kT))\times\cdots\times\theta_{in}(z(kT))\,G_{i1,\cdots,in}\end{aligned} \quad (8)$$

and

$$\begin{aligned}B_d(kT) &= \Big(\exp\Big(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\Big)-I\Big)\Big(\sum_{i=1}^{q}\theta_i(z(kT))A_i\Big)^{-1} \times\Big(\sum_{i=1}^{q}\theta_i(z(kT))B_i\Big) \\ &\approx\Big(\Big(\sum_{i=1}^{q}\theta_i(z(kT))\exp\Big(A_i\frac{T}{n}\Big)\Big)^n-I\Big) \times\Big(\sum_{i=1}^{q}\theta_i(z(kT))A_i\Big)^{-1}\Big(\sum_{i=1}^{q}\theta_i(z(kT))B_i\Big) \\ &= \frac{\Big(\sum_{i=1}^{q}\theta_i(z(kT))\exp\Big(A_i\frac{T}{n}\Big)\Big)^n-I}{\sum_{i=1}^{q}\theta_i(z(kT))\exp\Big(A_i\frac{T}{n}\Big)-I} \times\Big(\sum_{i=1}^{q}\theta_i(z(kT))\exp\Big(A_i\frac{T}{n}\Big)-I\Big) \times\Big(\sum_{i=1}^{q}\theta_i(z(kT))A_i\Big)^{-1}\Big(\sum_{i=1}^{q}\theta_i(z(kT))B_i\Big) \\ &\approx \frac{\Big(\Big(\sum_{i=1}^{q}\theta_i(z(kT))\exp\Big(\frac{A_i}{T}n\Big)\Big)^n-I\Big)}{\sum_{i=1}^{q}\theta_i(z(kT))\exp\Big(A_i\frac{T}{n}\Big)-I} \times\sum_{i=1}^{q}\theta_i(z(kT))\Big(\exp\Big(A_i\frac{T}{n}\Big)-I\Big)A_i^{-1}B_i \\ &= \Big(I+\sum_{i=1}^{q}\theta_i(z(kT))\exp\Big(A_i\frac{T}{n}\Big) +\cdots+\Big(\sum_{i=1}^{q}\theta_i(z(kT))\exp\Big(A_i\frac{T}{n}\Big)\Big)^{n-1}\Big) \times\sum_{i=1}^{q}\theta_i(z(kT))\Big(\exp\Big(A_i\frac{T}{n}\Big)-I\Big)A_i^{-1}B_i \\ &= \sum_{i1=1}^{q}\times\cdots\times\sum_{in=1}^{q}\theta_{i1}(z(kT))\times\cdots\times\theta_{in}(z(kT)) \\ &\times\Big(\frac{\big(\exp\big(A_{i1}\frac{T}{n}\big)-I\big)A_{i1}^{-1}B_{i1}+\cdots+\big(\exp\big(A_{in}\frac{T}{n}\big)-I\big)A_{in}^{-1}B_{in}}{n} \\ &+\frac{1}{n-1}\Big(\exp\Big(A_{i1}\frac{T}{n}\Big)\Big(\exp\Big(A_{i2}\frac{T}{n}\Big)-I\Big)A_{i2}^{-1}B_{i2} \\ &+\cdots+\exp\Big(A_{i1}\frac{T}{n}\Big)\Big(\exp\Big(A_{in}\frac{T}{n}\Big)-I\Big)A_{in}^{-1}B_{in}\Big) \\ &+\cdots+\exp\Big(A_{i1}\frac{T}{n}\Big)\times\cdots\times\exp\Big(A_{i(n-1)}\frac{T}{n}\Big) \times\Big(\exp\Big(A_{in}\frac{T}{n}\Big)-I\Big)A_{in}^{-1}B_{in}\Big) \\ &\doteq \sum_{i1=1}^{q}\times\cdots\times\sum_{in=1}^{q}\theta_{i1}(z(kT))\times\cdots\times\theta_{in}(z(kT))\,H_{i1,\cdots,in}\end{aligned} \quad (9)$$

■

Remark 1 *Note that the proposed pointwise behavior (7) yields not only the high accurate result as n approaches to infinity but also the polytopic structure.*

Remark 2 *In proposed discretization method. two approximations are performed as follows:*

$$\exp\Big(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\Big)\approx\Big(\sum_{i=1}^{q}\theta_i(z(kT))\exp\Big(A_i\frac{T}{n}\Big)\Big)^n \quad (10)$$

$$\left(\sum_{i=1}^{q}\theta_i(z(kT))\exp\left(A_i\frac{T}{n}\right)-I\right)\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\right)^{-1}$$
$$\times\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right) \qquad (11)$$
$$\approx\sum_{i=1}^{q}\theta_i(z(kT))\left(\exp\left(A_i\frac{T}{n}\right)-I\right)A_i^{-1}B_i$$

In [2], the approximations are also performed at above two cases, and the approximation results are exactly equal to (10) and (11) when $n=1$. Therefore, our discretized version (7) yields smaller approximation error than it of [2], since the approximation error of (10) and (11) clearly decreases as n approaches to infinity.

Remark 3 *The proposed discretized version (7) of the continuous-time TS fuzzy system reduces to the local discretized version of it for $i1 = i2 = \cdots = in$. Therefore, the local discretization method is special case of the proposed discretization method.*

4. Computer Simulation

An application example on discretizing a controlled inverted pendulum system [5] is given in this section. The TS fuzzy system of inverted pendulum system [5] is given by

R^1 : IF $x_1(t)$ is about Γ_1^1,

THEN $\dot{x}(t) = A_1x(t)+B_1u(t)$

R^2 : IF $x_1(t)$ is about Γ_1^2,

THEN $\dot{x}(t) = A_2x(t)+B_2u(t)$ (12)

where

$$A_1=\begin{bmatrix}0 & 1\\ \frac{g}{4l/3-aml} & 0\end{bmatrix}, A_2=\begin{bmatrix}0 & 1\\ \frac{2g}{\pi(4l/3-aml\cos(88^\circ)^2)} & 0\end{bmatrix}$$

$$B_1=\begin{bmatrix}0\\ -\frac{a}{4l/3-aml}\end{bmatrix}, B_2=\begin{bmatrix}0\\ -\frac{a\cos(88^\circ)}{4l/3-aml\cos(88^\circ)^2}\end{bmatrix}$$

From the standard Lyapunov stability criterion for the continuous-time TS fuzzy system, the gain matrixes for the analog fuzzy-model-based controller is obtained as follows:

$F1=[-1172.7\ -334.5]$, $F2=[-2670.2\ -0.782.9]$

As shown Figure 1 and 2, the proposed discretization method yields the discretization error in comparison with [2].

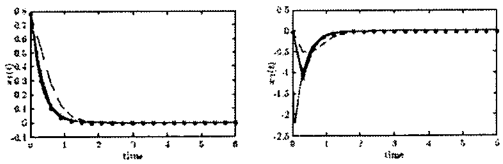

Figure 1: Responses of the controlled inverted pendulum at T=0.3 (solid line: continuous-time, dashed line: discretized version by [2], solid line and bullet point: discretized version by (3), and solid line and circle point: discretized version by the proposed method).

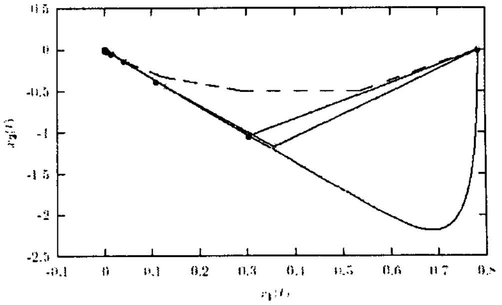

Figure 2: Trajectories of the controlled inverted pendulum at T=0.3 (solid line: continuous-time, dashed line: discretized version by [2], solid line and bullet point: discretized version by (3), and solid line and circle point: discretized version by the proposed method).

5. Conclusions

We formulate and solve the discretization problem for the highly nonlinear fuzzy system. Two main issue is to discrete the global state of the continuous time TS fuzzy system and to hold the polytopic structure after the discretization procedure. With aid of the proposed intersampling time conversion method, these problems can be efficiently solved. Consequently, the obtained discretized version of the continuous-time TS fuzzy system yields low discretization error and holds the polytopic structure. For a given simulation, the result shows that the proposed discretization method yields the low discretization error in comparison with the conventional discretization method, that is, local discretization method. It indicates the great potential for reliable application of design of the fuzzy-model-based digital controller.

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