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Gain-phase margin specified PI speed control of a PM synchronous motor

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ABSTRACT - Simple tuning formulae are derived to design a PI controller to meet the gain and phase margin specifications. These formulae are suitable for the auto-tuning of a process where the robustness should be guaranteed. The auto-tuned PI controller is examined for the speed regulation of a PM synchronous motor.

1. Introduction

Vector current control technique based on field orientation enabled the electromagnetic torque of AC servomotors to be controlled such as separately excited DC motors. In most cases, the dynamics of current control for the torque generation could be designed much faster than the mechanical dynamics of the motor. Therefore, the electrical dynamics is often eliminated for the design of speed or position controllers. Assuming a constant inertia load, the servomotor under speed control can be modeled as a first order plus dead time process because the open loop response for a stepwise torque input shows an over-damped characteristic. PI controllers are known to be suitable for the processes that have over-damped step responses. For this reason, PI controllers are widely used for the speed control of a motor. Considering model uncertainties of the motor mechanical dynamics, gain and phase margins (GPM) specified PI tuning formulae have special advantages. Since gain and phase margins are defined as a set of nonlinear equations, simple tuning rules are limited to the low order processes and approximations are generally involved. Astrom *et al.*[1] introduced the relay auto-tuner that approximately identified the critical frequency information to move the compensated Nyquist curve to pass through a specified design point. Ho *et al.*[2] used a rough linear approximation to obtain a closed form solution. Though the accuracy is limited, the PI controller is designed to pass through two design points at the Nyquist curve. In this paper, more accurate GPM tuning formulae are obtained by using the optimal approximation theory. With the proposed tuning formulae, the first order plus dead time (FOPDT) processes are easily auto-tuned with accuracy.

2. Robust PI Speed Control

2.1 Gain-phase margin PI tuning formulae

The process transfer function $Gp(s)$ and the controller transfer function $Gc(s)$ are denoted as follows:

$$G_p(S) = \frac{k_m}{\tau_m s + 1} e^{-Ls}, \quad (1)$$

$$G_c(S) = k_p + \frac{k_i}{s}. \quad (2)$$

The gain and phase margin definition equations are rearranged as follows:

$$w_g = k_m k_i \sqrt{\frac{\left(\frac{w_g k_p}{k_i}\right)^2 + 1}{(w_g \tau_m)^2 + 1}}, \quad (3)$$

$$A_m = \frac{w_p}{k_m k_i} \sqrt{\frac{(w_p \tau_m)^2 + 1}{\left(\frac{w_p k_p}{k_i}\right)^2 + 1}}, \quad (4)$$

$$\phi_m = \frac{\pi}{2} + atan\left(\frac{w_g k_p}{k_i}\right) - atan(w_g \tau_m) - w_g L \quad (5)$$

$$\frac{\pi}{2} + atan\left(\frac{w_p k_p}{k_i}\right) - atan(w_p \tau_m) - w_p L = 0 \quad (6)$$

where ω_p and ω_g are the phase crossover frequency and gain crossover frequency, respectively. If the parameters $(\omega_g k_p/k_i)^2$, $(\omega_g \tau_m)^2$, $(\omega_p \tau_m)^2$, $(\omega_p k_p/k_i)^2$, in eqns. 3-4 are determined as sufficiently large values compared to the unity, the square roots can be eliminated for the approximation. Without loss of generality, the minimum value is selected as 10. On the other hand, the arctangent functions in eqns. 5-6 are approximated by the first order polynomials as follows:

$$atanx \cong \begin{cases} 1.4680 - \dfrac{0.7007}{x} & \sqrt{10} > x > 1 \\ 1.5689 - \dfrac{0.9685}{x} & x \geqq \sqrt{10} \end{cases} \tag{7}$$

where x is one of $(\omega_g k_p/k_i)$, $(\omega_g \tau_m)$, $(\omega_p \tau_m)$, $(\omega_p k_p/k_i)$. The coefficients in eqn. 7 are determined by using the Remez exchange algorithm[3] where the maximum approximation error is minimized over the defined ranges. Fig. 1 shows the approximated arctangent function and related error. Thus, eqns. 2-6 can be rewritten as follows:

$$w_g = \frac{k_m k_p}{\tau_m}, \; A_m = \frac{w_p \tau_m}{k_m k_p}, \tag{8}$$

$$\phi_m = \frac{\pi}{2} - \frac{0.96853 k_i}{w_g k_p} + \frac{0.96853}{w_g \tau_m} - w_g L, \tag{9}$$

$$\frac{\pi}{2} - \frac{0.96853 k_i}{w_p k_p} + \frac{0.96853}{w_p \tau_m} - w_p L = 0. \tag{10}$$

Eqns. 8-10 are solved to derive the GPM PI tuning formelae as follows:

$$w_p = A_m \frac{\{\phi_m + 0.5\pi(A_m - 1)\}}{L(A_m^2 - 1)}, \tag{11}$$

$$k_p = \frac{w_p \tau_m}{A_m k_m}, \tag{12}$$

$$k_i = k_p(1.62184 w_p - 1.03249 L w_p^2 + 1/\tau_m). \tag{13}$$

For the given restrictions on approximation, the tuning formulae in eqns. 11-13 are analyzed for the FOPDT processes to obtain the achievable GPM values. In Fig. 2, the selectable boundaries of gain and phase margins are uniquely determined by the normalized dead time L/τ_m.

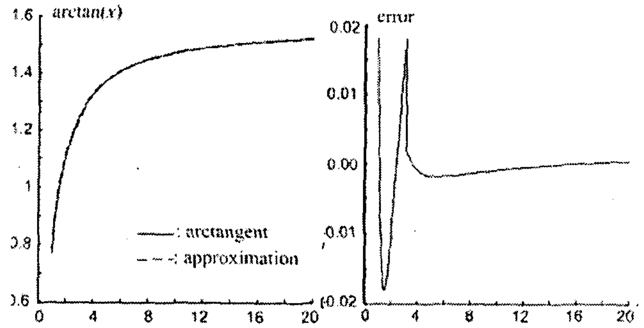


Fig. 1 Minimax optimal approximation of arctangent function.

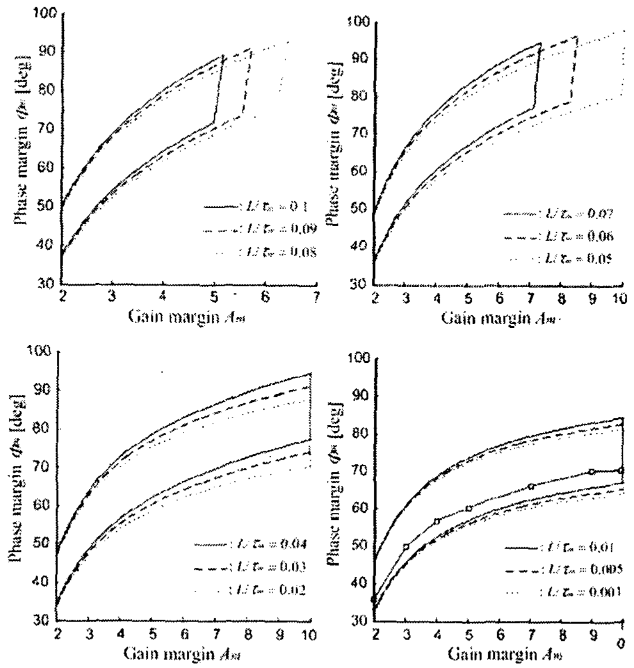


Fig. 2 Achievable GPM boundaries for the FOPDT processes.

2.2 Identification of a motor mechanical model

Process identification is one of the most important procedures for the controller design and tuning. Traditional ways of process identification can be divided into the time domain identification and the frequency domain identification. The relay feedback is a special case of frequency domain identifier that approximately identified the ultimate gain and the phase crossover frequency of a process.

Mechanical transfer function of a motor for the speed output is described in eqn. 1. Identification of the nominal model is not easy because of nonlinear friction terms and backlash. For the process with negligibly small dead time, traditional Ziegler-Nichols step response method does not give an accurate result. Therefore this paper tried to use the input-output relations of a relay feedback for the identification of nominal mechanical parameters. The describing function of a hysteric relay is given as follows:

$$N(a) = \frac{4d}{\pi a}\sqrt{1 + \left(\frac{\epsilon}{a}\right)^2} - j\frac{4d\epsilon}{\pi a^2} \tag{14}$$

where d, ε, and a are the relay amplitude, the hysteric band, and the sinusoidal input amplitude, respectively. Since AC servomotors require vector current control, position information is always available. Thus, the position is redefined as a new output and described as follows:

$$G_{ID} = \frac{G_p(s)}{s} = \frac{k_m}{s(\tau_m s + 1)} e^{-Ls} . \tag{15}$$

When the motor in eqn. 15 is controlled by the hysteric relay in eqn. 14 with unity feedback, a stable oscillation may be induced in the loop with the condition as follows:

$$N(a) G_{ID}(jw_c) = -1, \tag{16}$$

$$G_{ID}(jw_c) = -\frac{1}{N(a)} = -\frac{\pi}{4d}\sqrt{a^2 - \epsilon^2} - j\frac{\pi\epsilon}{4d} . \tag{17}$$

Nyquist plot of $-1/N(a)$ function and $G_{ID}(s)$ is presented in Fig. 3 where the stable oscillation is generated at the operating point of cross point.

From eqns. 15 and 17, the parameters of process model in eqn. 1 can be identified as follows:

$$L = \frac{1}{\omega_c}\left\{\sin^{-1}\left(\frac{\pi a \omega_c}{4k_m d}\right) - \sin^{-1}\left(\frac{\varepsilon}{a}\right)\right\}, \tag{18}$$

$$\tau_m = \frac{1}{\omega_c\sqrt{a^2 - \varepsilon^2}}\left\{\frac{4dk_m}{\pi\omega_c}\cos(\omega_c L) - \varepsilon\right\} . \tag{19}$$

where $a > \varepsilon$. The static gain of an integrating process is undefined. Therefore, the parameter k_m is identified by using a pulse control input u_p which is applied for a finite duration Δt. If the change of the output is measured as y, k_m is represented as follows:

$$k_m = \frac{\Delta y}{u_p \Delta t} . \tag{20}$$

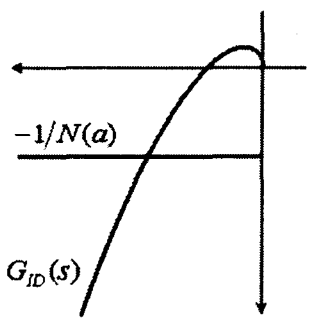


Fig. 3 Nyquist plot of $-1/N(a)$ function and $G_{ID}(s)$

2.3 On-line tuning experiment

The PM synchronous motor used in the experiment is characterized to have a rated torque of 9.8 *N-m*, a rated speed of 4π *rad/sec*, and a rated power of 123 *W*. From the relay test shown in Fig. 4, the process parameters k_m, τ_m, and L are determined as 20.5, 0.3148, and 0.0074, respectively. The normalized dead time L/τ_m is employed in Fig. 2 for the GPM specifications. The rectangular points in Fig.2 show the selected sets of specifications (A_m^*, Φ_m^*). By using eqns. 11-13, the on-line calculations of the control parameters (k_p, k_i) have been done to fill up a look-up table as shown in Table I. In this table, GPM (A_m, Φ_m) are predicted by using the nominal process parameters. Compared to the specified values, the gain margin is designed within 3% error and the phase margin is designed within 6% error.

Fig. 5 shows the transient responses for the different GPM specifications. In this figure, a smaller overshoot is observed for the larger GPM specifications. For the continuous operation, the speed reference is changed from 10 *rad/sec* to 10 *rad/sec* in a period of six seconds.

Fig. 6*a* shows the profile of the load disturbance and compensating torque generated by the motor. In Fig. 6*b*, a good transient response is obtained at (i) by selecting a large GPM specification ($9, 70^0$). During the operations, the maximum overshoot (undershoot) and steady state error are continuously measured. When the disturbances are frequently applied, (A_m^*, ϕ_m^*) are evolved to ($7, 65^0$), ($5, 60^0$), ($3, 50^0$), and ($2, 35^0$) at the time of (ii), (iii), (iv), and (v), respectively. As a result, the disturbances are effectively compensated.

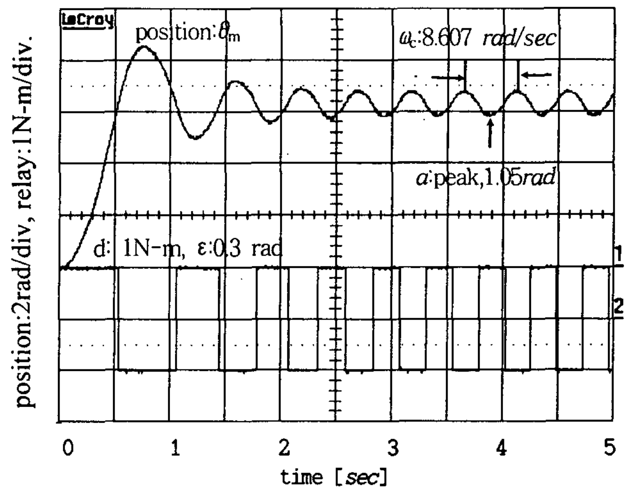


Fig. 4 Relay test for the process identification [experiment].

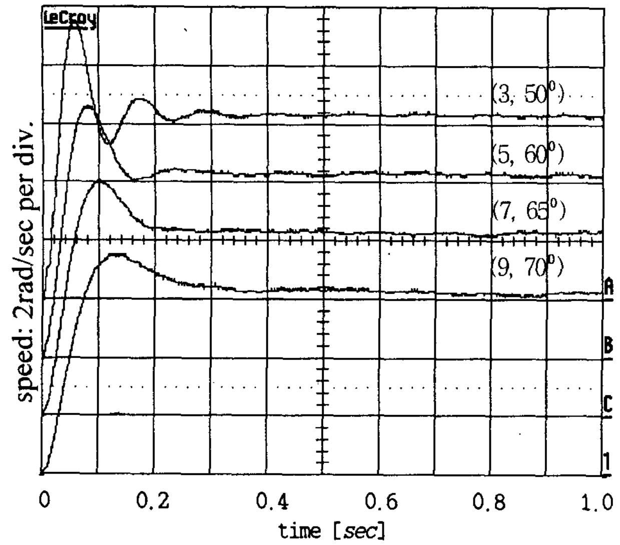
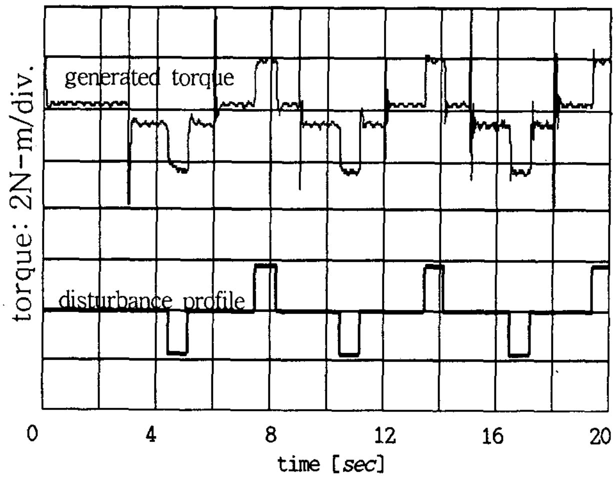
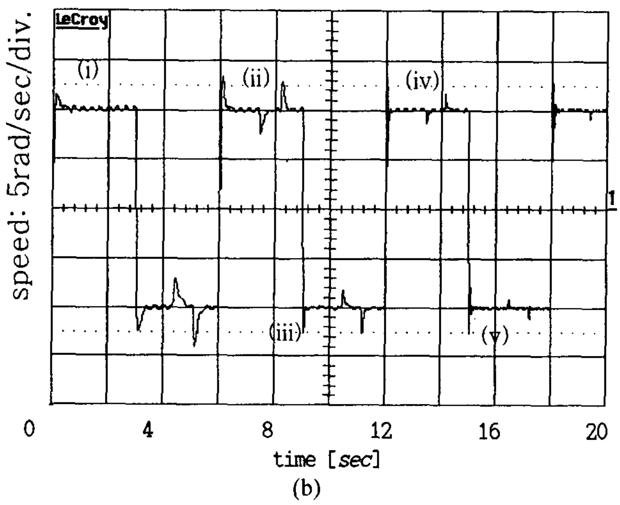


Fig. 5 Transient response of various gain-phase margin specifications (A_m^*, Φ_m^*) [experiment].



(a)



(b)

Fig. 6 Auto-tuning performance for the setpoint changes and frequent load disturbances.

Table 1 Gain look up table for the GPM PI auto-tuner.

GPM spec.		design values		obtained results	
A_m^*	Φ_m^*	k_p	k_i	A_m	Φ_m
2	35^0	1.51	40.52	1.94	33^0
3	50^0	1.04	17.66	2.93	49^0
5	60^0	0.63	7.88	4.90	60^0
7	65^0	0.46	4.48	6.84	65^0
9	70^0	0.32	2.40	8.81	70^0

3. Conclusion

A simple gain and phase margins specified PI tuning formulae is proposed for the FOPDT processes with small dead time. The proposed tuning formulae are expected to be suitable for the auto-tuning speed control of AC servomotors including PM synchronous motors.

References

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