Adaptive Robust Output Tracking for Nonlinear MIMO Systems

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Abstract

The robust output tracking control problem of general nonlinear MIMO systems is discussed. The robustness against parameter uncertainties is considered. In this paper, we proposed the robust output tracking control scheme for a class of MIMO nonlinear dynamical systems using output feedback linearization method. The presented control scheme is based on the VSS. We assume that the nonlinear dynamical system is minimum phase, the relative degree of the system is $r_1+r_2+\cdots+r_m \le n$ and zero dynamics is stable. It is shown that the outputs of the closed-loop system asymptotically track given output trajectories despite the uncertainties while maintaining the boundedness of all signals inside the loop. And we verified that the proposed control scheme is then applied to the control of a two degree of freedom (DOF) robotic manipulator with payload.

I. Introduction

A general class of nonlinear systems is considered in this paper. Towards a robust output tracking control of this system, an input-output linearization is performed, first. Then a SMC is applied on the input-output linearized system. The relation between input-output linearization and SMC method is discussed. It is shown that much broader class of nonlinear dynamics can be treated with this strategy, compared to the earlier applications of other investigators on the systems in canonical forms. For many physical systems, it is difficult to develop accurate mathematical descriptions for them. Thus, there are inevitable uncertainties in their constructed models. These uncertainties may be due to unknown or imperfectly known parameter values of the controlled systems and their changing environment as well as to the unpredicted disturbances, such as measurement noises. Therefore, the design of a robust controller that deals with uncertainties of a system is an important subject for the design of a good and efficient control system.

So far, to deal with uncertain nonlinear systems, three main approaches have been proposed: 1) adaptive control, 2) Lyapunov-based control, 3) variable structure control(VSC). The first one is applied to systems mainly with parametrized uncertainties and the other to then allow nonparametrized (unstructured) uncertainties. Specifically, adaptive control [1], [2] uses the linear in parameter assumption to formulate error equations which relate measurable signals to parameter errors whereby a parameter update law can be formed. The objective of either stabilization or tracking is thus achieved during the adaptation process. The Lyapunov-based approach [3] relies on an explicit construction of a Lyapunov function based on which a state feedback control is synthesized using the bounds on the uncertainties. Variable structure control[4], related to the previous one, exploits the variable structure concept[4]. It first defines the sliding surface in the error state space and forms a switching state feedback control also using the same information on the bounds. The high-speed switching forces the error state to slide along the surface until it converges and then the tracking is attained.

In this paper, a robust output tracking control scheme for a class of nonlinear dynamical system is proposed by using output-feedback linearization method proposed by Isidori et al.. The presented control scheme is based on the VSS concept proposed by Utkin and Itkis. In this control scheme, we assume that the nonlinear dynamical system is minimum phase, i.e., the relative degree of the system is $r_1+\cdots+r_m \le n$ and the zero dynamics is stable. It is also shown that the global asymptotically stability is guaranted under the proposed control scheme. The paper is organized as follows. In section II, mathematical tools is presented and discussed output feedback linearization. In section III, VSS controller is presented and its stability analysis is shown in the Lyapunov sense. In section IV, the feasibility of the proposed control scheme is verified through a computer simulation.

II. Preliminary

The purpose of this section is to show how multi-input multi-output nonlinear systems can be locally given, by means of a suitable change of coordinates in the state space, a normal form of special interest, on which several important properties can be elucidated. The point of departure of the whole analysis is the notation of relative degree of the system, which is formally described in the following way. We consider the multi-input multi-output nonlinear system

$$\dot{x}(t) = f(x(t)) + \sum_{i=1}^{m} g_i(x(t))\, u_i(t)$$

$$y(t)=h_i(x(t)),\quad i=1,\cdots,m \tag{2.1}$$

It is well known in the literature that certain structural conditions the output equations can be decoupled and linearized by a nonlinear state feedback law of the form

$$u=\alpha(x)+\beta(x)v \tag{2.2}$$

which results in the linear output equations

$$y_i^{r_i}=v_i,\quad i=1,\cdots,m \tag{2.3}$$

where $y_i^{r_i}$ denotes the r_i-th derivative of output y_i, $i=1,\cdots,m$. To explore the conditions, we need the following the definition. The nonlinear system (2.1) is said to have relative degree $r_1+r_2+\cdots+r_m\le n$ at a point x^0 if

(i) $L_{g_j}L_f^k h_i(x)=0$ for all $1\le j\le m$, for all, $k< r_i-1$, for all, $1\le i\le m$ and for all x in a neighborhood U of x^0.

(ii) the $m\times m$

$$A(x)=\begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1(x) & \cdots & L_{g_m}L_f^{r_1-1}h_1(x) \\ L_{g_1}L_f^{r_2-1}h_2(x) & \cdots & L_{g_m}L_f^{r_2-1}h_2(x) \\ \vdots & \cdots & \vdots \\ L_{g_1}L_f^{r_m-1}h_m(x) & \cdots & L_{g_m}L_f^{r_m-1}h_m(x) \end{bmatrix}$$

is nonsingular at $x=x^0$. Where $x(\cdot):R_+\to R^n$ is the state, $u(\cdot),y(\cdot):R_+\to R^m$ are system inputs and outputs, respectively, $f(\cdot),g_i(\cdot):R^n\to R^n, i=1,\cdots,m$, are sufficiently smooth vector fields, $h_i(\cdot):R^n\to R, i=1,\cdots,m$, are sufficiently smooth scalar functions. Associated with these relative degrees r_i, $i=1,\cdots,m$, it can be easily verified that

$$\begin{bmatrix} y_1^{r_1} \\ \vdots \\ y_m^{r_m}\end{bmatrix}=\begin{bmatrix} L_f^{r_1}h_1(x) \\ \vdots \\ L_f^{r_m}h_m(x)\end{bmatrix}+\begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1(x) & \cdots & L_{g_m}L_f^{r_1-1}h_1(x) \\ L_{g_1}L_f^{r_2-1}h_2(x) & \cdots & L_{g_m}L_f^{r_2-1}h_2(x) \\ \vdots & \cdots & \vdots \\ L_{g_1}L_f^{r_m-1}h_m(x) & \cdots & L_{g_m}L_f^{r_m-1}h_m(x) \end{bmatrix}\begin{bmatrix} u_1 \\ \vdots \\ u_m\end{bmatrix}$$

This mathematical property of relative degree can be summarized in the following theorem.

Theorem 1. [5] *The nonlinear system (2.1), with $f(x)$ and $g_i(x)$ being smooth vector fields, is input-state linearizable if, and only if, there exists a region Ω such that the following conditions hold :*

(i) the vector fields

$$\{g_1, ad_f g_1,\cdots, ad_f^{r_1-1}g_1,\cdots,g_m,\cdots, ad_f^{r_m-1}g_m\}$$

are linearly independent in Ω.

(ii) the distribution

$$\triangle(x)=span\{g_1, ad_f g_1,\cdots, ad_f^{r_1-2}g_1,\cdots,g_m,\cdots, ad_f^{r_m-2}\}$$

is involutive in Ω.

Where Ω is an open and connected subset of R^n which includes the origin.

Also, $ad_f g_i=[f,g_i]$, $ad_f^2 g_i=[f,[f,g_i]]$, $i=1,\cdots,m$ and $[f,g_i]$ is the Lie bracket of f and g_i.

Theorem 2. [5]
A nontrivial real-valued function $h(x)=[h_1(x),\cdots,h_m(x)]^T$ exists whose differential is a annihilator of the distribution $\triangle(x)$ defined in theorem 1, i.e.,

$$dh(x)\neq 0 \text{ and } dh(x)\cdot\triangle(x)=0$$

if and only if system (2.1) satisfies the theorem 1.

We can see that theorem 2. is equivalent to the fact that the function $h(x)$, when interpreted as the output of the system (1), produces a relative degree r_i. From the above theorem 1. and 2., we can construct the diffeomorphism $\Phi(x)=[h_1(x),\cdots,L_f^{r_1-1}h_1(x),\cdots,h_m,\cdots,L_f^{r_m-1}h_m(x),\psi_{r_i+1}(x),\cdots,\psi_n(x)]^T, i=1,\cdots,m$ which has the property $\Phi(0)=0$, and transforms the system (1) to the following cannonical form

$$\begin{aligned} \dot z_{11}&=z_{12} \\ &\vdots \\ \dot z_{1r_1-1}&=z_{1r_1} \\ \dot z_{1r_1}&=a_1(z,\Psi_1)+b_1(z,\Psi_1)u \\ &\vdots \\ \dot z_{m1}&=z_{m2} \\ &\vdots \\ \dot z_{mr_m}&=a_m(z,\Psi_m)+b_m(z,\Psi_m)u \end{aligned}$$

and

$$\dot z_{ir_i}=a_i(z,\Psi_i)+b_i((z,\Psi_i)u \tag{2.4}$$

$$\dot\Psi=\begin{bmatrix}\dot\psi_{r_i+1} \\ \vdots \\ \dot\psi_n\end{bmatrix}=\begin{bmatrix} w_{r_i+1}(z,\Psi) \\ \vdots \\ w_n(z,\Psi)\end{bmatrix},\ i=1,\cdots,m$$

where $\dot\Psi=w(0,\Psi)$ is zero dynamics and in this paper we assume that it is stable(minimum phase system) and

$$\begin{aligned} &p=r_1+r_2+\cdots+r_m\le n \\ &z=[z_{i1},\cdots,z_{ir_i}]^T, i=1,\cdots,m \\ &\Psi_i=[\psi_{r_i+1}(x),\cdots,\psi_n(x)]^T, i=1,\cdots,m \\ &z_{1i}=h_i(x)=L_f^{j-1}h_i(x),\ i=1,\cdots,r_j,\ j=1,\cdots,m \\ &a_i(z,\Psi)=L_f^{r_i}h_i(x)|_{x=\Phi^{-1}(z)},\ i=1,\cdots,m \\ &b_i(z,\Psi)=L_{g_j}L_f^{r_i-1}h_i(x)|_{x=\Phi^{-1}(z)},\ i,j=1,\cdots,m. \end{aligned}$$

III. VSS Robust Controller Design

In this section, we propose a control law which guarantees that closed-loop system has the uniformly ultimate bounded stability with a tolerable tracking error. Because the states of zero dynamics are not accessible, we consider these states as bounded disturbances under the assumption that zero dynamics is stable and also we have no priori knowledge concerning the magnitude of these disturbances. Now let $y_d=[y_{d1},\cdots,y_{dm}]^T$ be the desired output trajectories where $y_{di}^{(j)}, j=1,\cdots,\nu_i, i=1,\cdots,m$ are bounded signals. The following assumptions are needed for the development of a controller.

Assumption 1 Assume that zero dynamics is stable (or nonlinear minimum phase). $a_i(z,\Psi)$ and $b_i(z,\Psi)$ can be approximated as follows

$$a_i(z,\Psi)\cong a_i(z,\Psi_o)+\frac{\partial a_i}{\partial\Psi}(z,\Psi_o)\delta\Psi$$
$$b_i(z,\Psi)\cong b_i(z,\Psi_o)+\frac{\partial b_i}{\partial\Psi}(z,\Psi_o)\delta\Psi.$$

Assumption 2 There exists some positive constant vector $\rho_{i\nu}, \rho_{i+1\nu}$ such that

$$\|\frac{\partial a_i}{\partial\Psi}(z,\Psi_o)\delta\Psi\|\le\rho_{i\nu}^T\mu_{i\nu}(t,z)$$
$$\|\frac{\partial b_i}{\partial\Psi}(z,\Psi_o)\delta\Psi\|\le\rho_{i+1\nu}^T\mu_{i\nu}(t,z)$$

where $\rho_{i\nu}^T=(\rho_{i1}\ \rho_{i2}\ \rho_{i3})$, $\rho_{i+1\nu}^T=(\rho_{i+11}\ \rho_{i+12}\ \rho_{i+13})$ are unknown parameter vectors and

$$\mu_{i\nu}(t,z)=(1\ \|z\|\ \|z\|^2)^T.$$

From the above assumptions, the time derivative of z_{ir}, $i=1,\cdots,m$ can be rewritten as

$$\dot z_{ir_i}=a_i(z,\Psi_o)+b_i(z,\Psi_o)u+\eta_i(z,u,t) \qquad (3.1)$$

where

$$\eta_i(z,u,t)=\frac{\partial a_i}{\partial\Psi}(z,\Psi_o)\delta\Psi+\frac{\partial b_i}{\partial\Psi}(z,\Psi_o)\delta\Psi u.$$

Because control input $u(t)$ must be bounded, the norm of $\eta_i(z,u,t)$ can satisfy the following inequality.

$$\|\eta_i(z,u,t)\|\le\sigma_{i\nu}^T\varphi_{i\nu}(t,z)$$

where $\sigma_{i\nu}^T=(\gamma_{i1}\ \gamma_{i2}\ \gamma_{i3})$ is unknown parameter vector and $\varphi_{i\nu}(t,z)$ can be any positive vector function. In this paper, we set $\varphi_{i\nu}(t,z)$ to be the same as $\mu_{i\nu}(t,z)$ as follows

$$\varphi_{i\nu}(t,z)=(1\ \|z\|\ \|z\|^2)^T,\ i=1,\cdots,m.$$

Throughout this paper, the norm $\|\cdot\|$ is assumed to be the Euclidean vector norm. Now we utilize the VSS concept to derive a control law. First let us define a *sliding surface* vector $S\in R^m$ as follows

$$S=\begin{bmatrix}(y_1^{r_1-1}-y_{d1}^{r_1-1})+\sum_{i=1}^{r_1-1}a_{1i}(y_1^{r_1-i-1}-y_{d1}^{r_1-i-1})\\ \vdots\\ (y_m^{r_m-1}-y_{dm}^{r_m-1})+\sum_{i=1}^{r_m-1}a_{mi}(y_m^{r_m-i-1}-y_{dm}^{r_m-i-1})\end{bmatrix} \qquad (3.2)$$

where $a_{ij}, j=1,\cdots,r_{i-1}, i=1,\cdots,m,$ are appropriate constants to be specified. The sliding surface is then defined as

$$\begin{bmatrix}s_1\\ \vdots\\ s_m\end{bmatrix}=\begin{bmatrix}e_1^{(r_1-1)}+\sum_{i=1}^{(r_1-1)}a_{1i}e_1^{(r_1-i-1)}\\ \vdots\\ e_m^{(r_m-1)}+\sum_{i=1}^{(r_m-1)}a_{mi}e_m^{(r_m-i-1)}\end{bmatrix} \qquad (3.3)$$

where $a_{ij}, j=1,\cdots,r_{i-1}, i=1,\cdots,m,$ are chosen so that the following polynomials:

$$p_i(s)=s^{(r_i-1)}+a_{i1}s^{(r_i-2)}+\cdots+a_{i(r_i-1)},\ i=1,\cdots,m \qquad (3.4)$$

are Hurwitz. Thus, when the state trajectories reach the sliding surface $S=0$ and stay on it

$$e_i^{(r_i-1)}+a_{i1}e^{(r_i-2)}+\cdots+a_{i(r_i-1)}e_i=0,\ i=1,\cdots,m \qquad (3.5)$$

which implies that $e_i^{(j)}$ tends to zero as t tends to infinity $j=1,\cdots,(r_i-1), i=1,\cdots,m,$. This obviously includes the control objective $e_j=y-y_d\to 0$ as $t\to\infty$.

Now, we consider a following VSS-like type control law

$$u=u_{eq}+u_\Delta, \qquad (3.6)$$

where u_{eq} is the equivalent control input of the nominal system, u_Δ is the control input overcoming the uncertainties (or disturbance which represent the term concerning zero dynamics). We derive u_{eq} from the fact that the derivative of $1/2s_i^2$ along the trajectory of the closed- loop system should be equal to zero and u_Δ is found such that

$$s_i[\dot s_i]<-\beta\|s_i\|,\ \beta>0. \qquad (3.7)$$

This inequality implies that the trajectory reaches the sliding surface in a finite time and stays on the sliding surface thereafter. Now we discuss how to derive the u_{eq} and u_Δ which satisfy the above conditions. The time derivative $\dot s$ can be expressed as

$$\dot s_i=\sum_{j=1}^{r_i-1}a_{ij}e_{ij+1}-y_{di}^{r_i}+a_i(z,\Psi_o)+b_i(z,\Psi_o)u+\eta_i(z,u,t)$$
$$,i=1,\cdots,m \qquad (3.8)$$

If we choose u_{eq} as follows

$$u_{eq}=\frac{1}{b_i(z,\Psi_o)}\left\{\sum_{i=1}^{m}y_i^{r_i}-\dot s_i-a_i(z,\Psi_o)\right\} \qquad (3.9)$$

then

$$\dot s(z_{i1}\cdots z_{ir_i})=b_i(z,\Psi_o)u_\Delta+\eta_i(z,u,t),\ i=1,\cdots,m. \qquad (3.10)$$

Now u_Δ is chosen by

$$u_\Delta = -\frac{1}{b_i(z, \Psi_o)}\{ \hat{\sigma}_\nu^T \varphi_\nu(t, z) + k_i\} \cdot sgn(s_i) \quad (3.11)$$

where $\hat{\sigma}_{i\nu}$ is estimate of $\sigma_{i\nu}$ and k_i is a switching feedback gain yet to be determined and

$$sgn(S) = [sgn(s_1), \cdots, sgn(s_m)]^T$$

with

$$sgn(s_i) = \begin{cases} 1 & s_i > 0 \\ 0 & s_i = 0 \\ -1 & s_i < 0 \end{cases} \quad i = 1, \cdots, m$$

then $\sum_{i=1}^{m} y_i^{r_i} - \dot{s}_i$ can be shown that

$$\sum_{i=1}^{m} y_i^{r_i} - \dot{s}_i = [y_{d1}^{r_1}, \cdots, y_{dm}^{r_m}]^T - \left\{ \sum_{i=1}^{r_1-1} a_{1i} e_1^{r_1-1}, \cdots, \sum_{i=1}^{r_m-1} a_{mi} e_1^{r_m-1} \right\}$$

which is an implementable signal. We can summarized the controller structure as follows

$$u = u_{eq} + \frac{1}{b_i(z, \Psi_o)}\{-k_i - \hat{\sigma}_\nu^T \varphi_\nu(t, z)\} sgn(s_i). \quad (3.12)$$

Now the objective of control is to drive the parameter update law which guarantee that $z(t)$ converge to zero vector as time goes to infinity. Therefore, we suggest the parameter update law as follows

$$\dot{\hat{\gamma}}_1 = s_i \cdot sgn(s_i)$$
$$\dot{\hat{\gamma}}_2 = s_i \|z\| sgn(s_i)$$
$$\dot{\hat{\gamma}}_3 = s_i \|z\|^2 sgn(s_i) \quad (3.13)$$

where

$$\tilde{\gamma}_1 = \hat{\gamma}_1 - \gamma_1, \quad \tilde{\gamma}_2 = \hat{\gamma}_2 - \gamma_2, \quad \tilde{\gamma}_3 = \hat{\gamma}_3 - \gamma_3.$$
$$\gamma_1 = \gamma_{i1}, \gamma_2 = \gamma_{i2}, \gamma_3 = \gamma_{i3}, i = 1, \cdots, m.$$

The stability of the proposed control law is analyzed by the following theorem.

Theorem 3. Under the assumption [1]-[2], the uncertain dynamical system (3.1) with a robust control law (3.12) and parameter update law (3.13), is globally uniformly ultimately bounded.

Proof: The proof is based on the Lyapunov-like function

$$V = \frac{1}{2}(\tilde{\gamma}_1^2 + \tilde{\gamma}_2^2 + \tilde{\gamma}_3^2 + s_i^2). \quad (3.14)$$

Taking the time derivative of V along the trajectory of (3.10) yields

$$\dot{V} \le \tilde{\gamma}_1 \dot{\hat{\gamma}}_1 + \tilde{\gamma}_2 \dot{\hat{\gamma}}_2 + \tilde{\gamma}_3 \dot{\hat{\gamma}}_3 + s_i b_i(z, \Psi_o) u_\Delta + |s_i| \sigma_{i\nu}^T \varphi_{i\nu}(t, z)$$
$$\le \tilde{\gamma}_1 \dot{\hat{\gamma}}_1 + \tilde{\gamma}_2 \dot{\hat{\gamma}}_2 + \tilde{\gamma}_3 \dot{\hat{\gamma}}_3 - s_i(\hat{\sigma}_{i\nu}^T \varphi_{i\nu}(t, z) + k_i)$$
$$\cdot sgn(s) + |s_i| \sigma_{i\nu}^T \varphi_{i\nu}(t, z)$$
$$\le \tilde{\gamma}_1 \dot{\hat{\gamma}}_1 + \tilde{\gamma}_2 \dot{\hat{\gamma}}_2 + \tilde{\gamma}_3 \dot{\hat{\gamma}}_3 - s_i \tilde{\sigma}_{i\nu}^T \varphi_{i\nu}(t, z) sgn(s_i)$$
$$- s_i k_i sgn(s_i).$$

From (3.14), V can be expressed as

$$\dot{V} \le -k_i s_i sgn(s_i) < 0.$$

Therefore $s_i \to 0$ and $e \to 0$ as $t \to \infty$.

Ⅳ. Computer Simulation

In this section, computer simulations are conducted to verify the feasibility and effectiveness of the proposed robust output tracking control scheme. The following fourth-order nonlinear dynamical system is used for a nonlinear plant. A two DOF robotic manipulator studied by[6] is used here to illustrate the efficiency of the variable structure control law proposed. The end effector of the manipulator shown in Fig.1 can be extended or extracted from and rotated about a vertical axis. To obtain numerical results, the desired trajectories $y_d = [y_{d1}, y_{d2}]^T = [r(\cdot), \theta(\cdot)]^T$ are C^2 function pair given by $r(\cdot) = 2.0 + 0.93\cos(0.2t)$, $\theta(\cdot) = 1$.

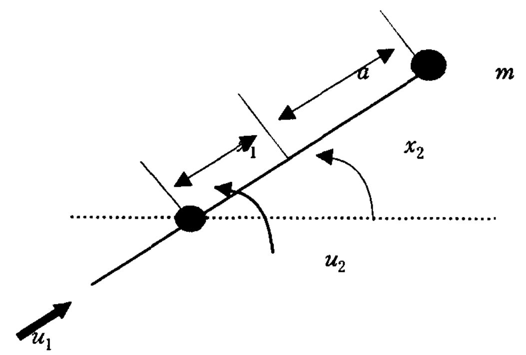

Fig.1. Two DOF manipulator for horizantal tracking

Let (x_1, x_2) denote the position of the center of mass m_c of the motional link in polar coordinate, and then, the equations of motion are given by

$$\dot{x} = \begin{bmatrix} x_3 \\ x_4 \\ \frac{(m_c x_1 + m(a + x_1))x_4^2}{M} \\ -\frac{2(m_c x_1 + m(a + x_1))x_3 x_4}{R(x)} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{R(x)} \end{bmatrix} u_2.$$

$$y_1 = h_1(x) = x_1, \quad y_2 = h_2(x) = x_2 \quad (4.1)$$

where

$M = m_c + m$, $R(x) = J_1 + J_2 + m_c x_1^2$

m_c, m : mass of the motional link and the payload,

J_1, J_2 : moments of inertia of the motional link with respect to the vertical axis through c and o,

u_1, u_2 : a linear force and a torque required to execute the motion,

a : distance from c to the center of payload.

From the feedback linearization method, we can derive the following equations

$$\begin{aligned}
y_1 &= h_1(x) = x_1 = z_{11} \\
\dot{y}_1 &= x_3 = z_{12} \\
\ddot{y}_1 &= \frac{(m_c x_1 + m(a+x_1))x_4^2}{M} + \frac{1}{M} u_1 = z_{13} \\
y_2 &= h_2(x) = x_2 = z_{21} \\
\dot{y}_2 &= x_4 = z_{22} \\
\ddot{y}_2 &= -\frac{2(m_c x_1 + m(a+x_1))x_3 x_4}{R(x)} + \frac{1}{R(x)} u_2 = z_{23}
\end{aligned}$$

Using these new coordinates, we obtain the following equations

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= \frac{(m_c z_1 + m(a+z_1))z_4^2}{M} + \frac{1}{M} u_1 \\
\dot{z}_3 &= z_4 \\
\dot{z}_4 &= -\frac{2(m_c z_1 + m(a+z_1))z_2 z_4}{R(z)} + \frac{1}{R(z)} u_2
\end{aligned} \qquad (4.2)$$

where $R(z) = J_1 + J_2 + m_c z_1^2 + m(a+z_1)^2$. To derive a robust output tracking controller based on VSS concepts, it can be easily verified that $L_{g_i} L_f^2 h_i(x) \neq 0,\ i, j = 1, 2$ and thus relative degree of the system is $r_1 = r_2 = 2$. We select sliding surface vector $S \in R^2$ as follows

$$\begin{aligned}
s_1 &= 5(y_1 - y_{d1}) + (z_2 - \dot{y}_{d1}), \\
s_2 &= 5(y_2 - y_{d2}) + (z_4 - \dot{y}_{d2}).
\end{aligned} \qquad (4.3)$$

Then the equivalent control input can be expressed as follows

$$\begin{aligned}
u_{eq1} &= -5Mz_2 + M(\ddot{y}_{d1} - 5(z_2 - \dot{y}_{d1})) \\
u_{eq2} &= -5(J_1 + J_2)z_4 + (J_1 + J_2)(\ddot{y}_{d2} - 5(z_4 - \dot{y}_{d2}))
\end{aligned} \qquad (4.4)$$

We choose $u_{\triangle}$ as follows

$$\begin{aligned}
u_{\triangle 1} &= -M\{\hat{\sigma}_{1\nu}^T \varphi_{1\nu}(t, z) + k_1\} \cdot sgn(s_1) \\
u_{\triangle 2} &= -R(z)\{\hat{\sigma}_{2\nu}^T \varphi_{2\nu}(t, z) + k_2\} \cdot sgn(s_2).
\end{aligned} \qquad (4.5)$$

and we set k_1, k_2 to be 1. Therefore the total control input u_1, u_2 and parameter update laws can be expressed as

$$\begin{aligned}
u_1 &= -5Mz_2 + M(\ddot{y}_{d1} - 5(z_2 - \dot{y}_{d1})) \\
&\quad - M\{\hat{\sigma}_{1\nu}^T \varphi_{1\nu}(t, z) + k_1\} \cdot sgn(s_1) \\
u_2 &= -5(J_1 + J_2)z_4 + (J_1 + J_2)(\ddot{y}_{d2} - 5(z_4 - \dot{y}_{d2})) \\
&\quad - R(z)\{\hat{\sigma}_{2\nu}^T \varphi_{2\nu}(t, z) + k_2\} \cdot sgn(s_2)
\end{aligned}$$

$$\begin{aligned}
\dot{\gamma}_{i1} &= s_i \cdot sgn(s_i) \\
\dot{\gamma}_{i2} &= s_i \|z\| sgn(s_i) \\
\dot{\gamma}_{i3} &= s_i \|z\|^2 sgn(s_i),\ i = 1, 2.
\end{aligned} \qquad (4.6)$$

In Fig.2 and Fig.3, the payload mass m is set equal to $100Kg$ and the simulation shows that the tracking errors approach zero asymptotically. Fig.4 and Fig.5 is shown the robust output tracking controller and the corresponding estimate parameter updation $(\gamma_{11}, \gamma_{12}, \gamma_{13})$, $(\gamma_{21}, \gamma_{22}, \gamma_{23})$ are shown in Fig.6 and Fig.7. From the results, we can see that the proposed robust output tracking control scheme is very effective.

Table.1. Physical parameters of two DOF Manipulator

parameters	values	units
m	100	kg
m_c	100	kg
J_1	100	kg・m^2
J_2	100	kg・m^2
a	1.0	m

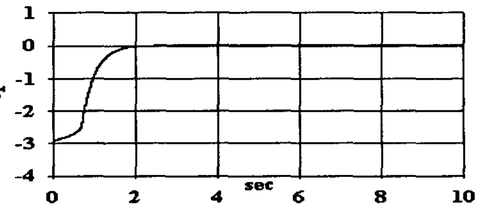

Fig.2. Output Tracking Error 1 (e_1)

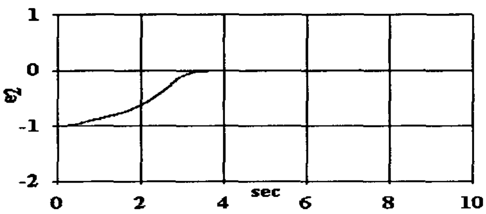

Fig.3. Output Tracking Error 2 (e_2)

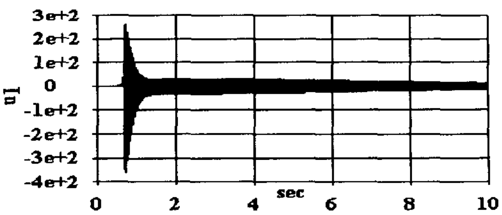

Fig.4. Robust Output Tracking Controller (u_1)

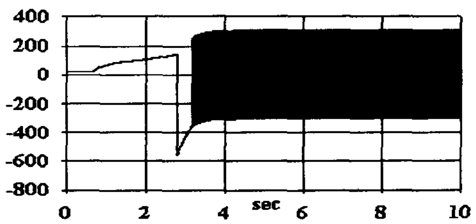

Fig.5. Robust Output Tracking Controller (u_2)

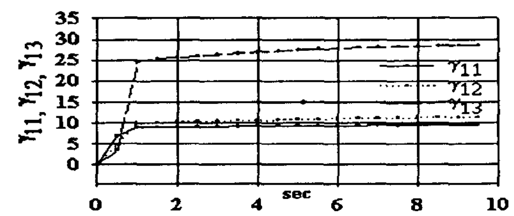

Fig.6. Parameter Update Law (u_1)

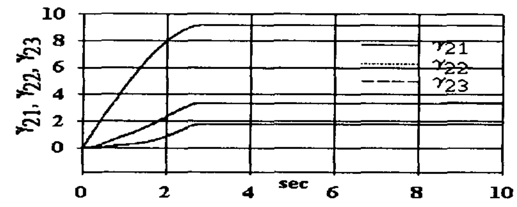

Fig7. Parameter Update Law (u_2)

V. Conclusions

In this paper, a globally stable output tracking control of a class of nonlinear systems which is robust to uncertainties was proposed. Starting from any initial conditions of the system states, the outputs of the closed-loop system will asymptotically track the desired trajectories. Furthermore, all signals inside the loop are shown to be bounded for all time. To illustrate the efficiency of the controller, the approach was then applied to the case of a two DOF cylinderical type manipulator where numerical simulation results are also provided. From the results, we can see that the proposed robust output tracking control scheme is very effective.

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