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Takagi-Sugeno 퍼지 시스템의 분리 원리에 관하여

On the Separation Principle of Takagi-Sugeno Fuzzy Systems

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ABSTRACT

In this note, a separation principle of the Takagi-Sugeno (T-S) fuzzy-model-based controller/observer is investigated. The separation principle of T-S fuzzy-model-based controller/observer sharing the premise parts in the fuzzy rule with directly measurable premise variables is well established. In that case, the fact that the augmented observer-based control system has the eigenvalues of the sub-closed-loop control system by the state-feedback controller and the sub-closed-loop observer error system is used to prove the separation principle. This paper studies the separation principle of T-S fuzzy-model-based controller/observer in which the premise variables cannot be directly measurable.

1. Introduction

The Takagi-Sugeno (T-S) fuzzy-model-based control technology can be yet another promising resolution for the output-tracking problem in that: i) it originally aims at controlling plants that are mathematically ill-defined, uncertain, and nonlinear [4]; ii) it bridges the gap between the domain expert's knowledge and the fruitful linear control theories; iii) only simple computation is needful without any complicated adaptive scheme. Plentiful works related to the fuzzy-model-based state-feedback control have been published [4–8, 11, 12]. However, relatively few contributions to the output-feedback control problem seem to be tractable [2, 3, 9, 10].

In practical applications, it is often that all state variables are not fully measurable, while the premise variables are mapped from the state variables. Usually, in the fuzzy-model-based control strategy, the plant rule and the controller rule share the premise parts of the fuzzy rule base. The separation principle of T-S fuzzy-model-based controller/observer sharing the premise parts in the fuzzy rule with directly measurable premise variables is well established [2, 3, 10]. In that case, similarly to the linear controller/observer system case, the fact that the augmented observer-based control system has the eigenvalues of the sub-closed-loop control system by the state-feedback controller and the sub-closed-loop observer error system is used to prove the separation principle.

In practical applications such that all state variables are not fully measurable, while the the premise variables are mapped from the unmeasurable state variables, the controller/observer problem becomes more difficult. Such a problem is studied in [3]. However, the separation principle is not explored. Motivated by the observations, a separation principle of the T-S fuzzy-model-based controller/observer is investigated with partially unmeasurable premise variables.

The following section briefly reviews observer-based T-S fuzzy-model-based control systems. In Section 3, the separation principle of observer-based T-S fuzzy-model-based control systems with measurable premise variables is discussed. Section 4 is devoted to show the main results: the separation principle of observer-based T-S fuzzy-model-based control systems with unmeasurable premise variables. In Section 4 the paper is closed.

2. Observer-Based T-S Fuzzy-Model-Based Control Systems

Many physical systems are very complex in practice and have strong nonlinearities and uncertainties so that rigorous mathematical models can be difficult, if not impossible, to obtain. Fortunately, certain class of nonlinear dynamical systems can be expressed in some forms of a linear mathematical model locally, or as an aggregation of a set of linear mathematical models.

Consider a nonlinear dynamical system of the following form:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input vector; $y(t) \in \mathbb{R}^p$ is the output vector. The vector field, $f : U_x \times U_u \subset \mathbb{R}^n \times \mathbb{R}^m \rightarrow V_x \subset \mathbb{R}^n$ is assumed to be affine in $u(t)$, $f(0,0) = 0$, and piecewise $\mathbf{C}^r, r \geqslant 1$, and the output mapping $h : U_x \subset \mathbb{R}^n \rightarrow V_y \subset \mathbb{R}^p$ is assumed to be $y(0) = 0$.

One way to view a T-S fuzzy system is that it performs nonlinearly interpolated linear mappings $\psi_x(x(t), u(t)) : U_x \times U_u \to V_x$ and $\psi_y(x(t)) : U_x \to V_y$ so as to satisfy

$$\sup_{(x(t),u(t)) \in U_x \times U_u} \|f(x(t), u(t)) - \psi_x(x(t), u(t))\| \leqslant \delta_f$$
$$\sup_{x(t) \in U_x} \|h(x(t)) - \psi_y(x(t))\| \leqslant \delta_h$$

where δ_f and δ_h are arbitrary small positive scalars.

Assume there exist L triplets $v_i = (A_i, B_i, C_i)$ which represent the local dynamic behavior of (1) , such that the matrix polytope

$$\mathcal{P} = \mathbf{Co}\{[A_1, B_1, C_1], \ldots, [A_L, B_L, C_L]\}$$

contains the domain $U_x \times U_u \times U_x$ and the range $V_x \times V_y$, where **Co** denotes a convex hull of the set $V = \{v_1, \ldots, v_L\}$, and $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$, and $C_i \in \mathbb{R}^{p \times n}$. Thus, one can find an adequate mapping at time instant t with δ_f and δ_h of the form:

$$\psi_x(x(t), u(t)) = A(\theta)x(t) + B(\theta)u(t)$$
$$\psi_y(x(t)) = C(\theta)x(t)$$

where $A(\theta)$ ranges over a matrix polytope

$$A(\theta) \in \mathbf{Co}\{A_1, \ldots, A_L\}$$

and $B(\theta) \in \mathbf{Co}\{B_1, \ldots, B_L\}$, $C(\theta) \in \mathbf{Co}\{C_1, \ldots, C_L\}$ with $\sum_{i=1}^{L} \theta_i = 1, \theta_i \in \mathbb{R}_{[0,1]}, i \in \mathcal{I}_L = \{1, 2, \ldots, L\}$. The key feature of the T-S fuzzy inference system is to determine the coefficients θ_i by virtue of the qualitative knowledge available from domain experts that is quantified by 'IF-THEN' rule base. More precisely, the ith rule of the T-S fuzzy system is formulated in the following form:

R^i : IF $z_1(t)$ is about Γ_1^i and $\cdots$ and $z_n(t)$ is about Γ_n^i

$$\text{THEN} \begin{array}{l} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{array} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input vector; R^i denotes the ith fuzzy inference rule; $z_h(t)$ is the premise variable, $\Gamma_h^i, i \in \mathcal{I}_L, h \in \mathcal{I}_N$ is the fuzzy set of the hth premise variable in the ith fuzzy inference rule. Using the center-average defuzzification, product inference, and singleton fuzzifier, the global dynamics of this T-S fuzzy system (2) is described by

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{L} \theta_i(z(t))(A_i x(t) + B_i u(t)) \\ y(t) &= \sum_{i=1}^{L} \theta_i(z(t)) C_i x(t) \end{aligned} \tag{3}$$

in which $\omega_i(z(t)) = \prod_{h=1}^{N} \Gamma_h^i(z_h(t))$, $\theta_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{L} \omega_i(z(t))}$ and $\Gamma_h^i(z_h(t))$ is the membership value of the hth premise variable $z_h(t)$ in Γ_h^i.

In control problems, the full system state vector is often unknown and only some functions of state variables, called system outputs, can be measured. One way to estimate the full state vector is to build an observer, for example, taking the following form:

R^i : IF $z_1(t)$ is Γ_1^i and $\cdots$ and $z_n(t)$ is Γ_n^i,

$$\text{THEN} \begin{array}{l} \dot{\widehat{x}}(t) = A_i \widehat{x}(t) + B_i u(t) + L_i(y(t) - \widehat{y}(t)) \\ \widehat{y}(t) = C_i \widehat{x}(t) \end{array}$$

The defuzzified output of the observer rules is represented by

$$\begin{aligned} \dot{\widehat{x}}(t) &= \sum_{i=1}^{L} \theta_i(z(t)) \left(A_i \widehat{x}(t) + B_i u(t) + L_i(y(t) - \widehat{y}(t))\right) \\ \widehat{y}(t) &= \sum_{i=1}^{L} \theta_i(z(t)) C_i \widehat{x}(t) \end{aligned} \tag{4}$$

The controller rule is of the following form:

R^i : IF $z_1(t)$ is Γ_1^i and $\cdots$ and $z_n(t)$ is Γ_n^i,
THEN $u(t) = K^i \widehat{x}(t)$

The defuzzified output of the controller rules is given by

$$u(t) = \sum_{i=1}^{L} \theta_i(z(t)) K^i \widehat{x}(t) \tag{5}$$

3. Observer-Based T-S Fuzzy-Model-Based Control: Measurable Premise Variable

In this section, we assume that there exists a mapping from y to z. Let the estimation error $e(t) = x(t) - \widehat{x}(t)$, then we obtain the augmented continuous-time closed-loop T-S fuzzy system is

$$\dot{\chi}(t) = \sum_{i=1}^{L} \sum_{j=1}^{L} \theta_i(z(t)) \theta_j(z(t)) \Phi_{ij} \chi(t) \tag{6}$$

where

$$\Phi_{ij} = \begin{bmatrix} A_i + B_i K_j & -B_i K_j \\ 0 & A_i - L_i C_j \end{bmatrix}$$

for the pair $(i, j) \in \mathcal{I}_L \times \mathcal{I}_L$, where $\chi(t) = \left[x(t)^T, e(t)^T\right]^T$.

Theorem 1 *Suppose* (3) *is globally exponentially observable and globally exponentially controllable with* $u(t) = \sum_{i=1}^{L} \theta_i(z(t)) K_i x(t)$. *Then the observer-based output-feedback fuzzy-model-based control* (5) *with* (4) *globally exponentially stabilizes* (3) *at the zero equilibrium.*

proof: From the controllability we can assign all eigenvalues of $A_i - B_iK_j$ such that the values of their real part is negative. From the observability we can make $A_i - L_iC_j$ to be Hurwitz. Since the eigenvalues of Φ_{ij} are those of $A_i - B_iK_j$ and $A_i - L_iC_j$, the augmented closed-loop system (6) is exponentially stable. ■

Remark 1 *From the fact that the eigenvalues of* Φ_{ij} *are those of* $A_i - B_iK_j$ *and* $A_i - L_iC_j$*, it is easily seen that the fuzzy-model-based controller* (5) *and the fuzzy-model-based observer* (4) *can be designed separately.*

4. Observer-Based T-S Fuzzy-Model-Based Control: Unmeasurable Premise Variable

In this section, we study the case that there exists some unmeasurable premise variable: they should be obtained from the estimated state. Then we have the following plant, observer, and controller dynamics.

$$\dot{x}(t) = \sum_{i=1}^{L}\theta_i(z(t))(A_ix(t) + B_iu(t))$$
$$y(t) = \sum_{i=1}^{L}\theta_i(z(t))C_ix(t) \tag{7}$$

$$\dot{\widehat{x}}(t) = \sum_{i=1}^{L}\theta_i(\widehat{z}(t))(A_i\widehat{x}(t) + B_iu(t) + L_i(y(t) - \widehat{y}(t)))$$
$$\widehat{y}(t) = \sum_{i=1}^{L}\theta_i(\widehat{z}(t))C_i\widehat{x}(t) \tag{8}$$

$$u(t) = \sum_{i=1}^{L}\theta_i(\widehat{z}(t))K^i\widehat{x}(t) \tag{9}$$

Remark 2 *It is noted that the firing strength is function of* $\widehat{z}(t)$ *mapped from* $\widehat{x}(t)$*, not from* $x(t)$.

The closed-loop system is constructed as follows:

$$\begin{aligned}\dot{\chi}(t) &= \sum_{i=1}^{L}\sum_{j=1}^{L}\theta_i(z(t))\theta_j(\widehat{z}(t))\theta_h(\widehat{z}(t)) \\ &\quad\times\begin{bmatrix} A_i + B_iK_h & -B_iK_h \\ \begin{pmatrix} A_i - A_j \\ +(B_i - B_j)K_h \\ +L_j(C_h - C_i)\end{pmatrix} & \begin{pmatrix} A_j - L_jC_h \\ -(B_i - B_j)K_h\end{pmatrix}\end{bmatrix}\chi(t) \\ &= \sum_{i=1}^{L}\sum_{j=1}^{L}\theta_i(z(t))\theta_j(\widehat{z}(t))\theta_h(\widehat{z}(t)) \\ &\quad\times\begin{bmatrix} A_i + B_iK_h & -B_iK_h \\ 0 & A_j - L_jC_h\end{bmatrix}\chi(t) \\ &\quad+\begin{bmatrix} 0 & 0 \\ \begin{pmatrix} A_i - A_j \\ +(B_i - B_j)K_h \\ +L_j(C_h - C_i)\end{pmatrix} & -(B_i - B_j)K_h\end{bmatrix}\chi(t)\end{aligned} \tag{10}$$

Theorem 2 *Suppose* (3) *is exponentially observable and exponentially stabilizable with* $u(t) = \sum_{i=1}^{L}\theta_i(z(t))K_ix(t)$. *Further assume that their decay rates are sufficiently fast. Then the observer-based output-feedback fuzzy-model-based control* (9) *with* (8) *exponentially stabilizes* (3) *at the zero equilibrium.*

proof: Since the closed-loop system (3) fed by $u(t) = \sum_{i=1}^{L}\theta_i(z(t))K_ix(t)$ back is exponentially stable, it follows from the converse Lyapunov theorem that there exists a Lyapunov function $V_1(x(t))$ satisfying the conditions

$$\kappa_1\|x\| \leqslant V_1(x(t)) \leqslant \kappa_2\|x\|$$
$$\left\|\frac{\partial V_1(x(t))}{\partial x}\right\| \leqslant \kappa_3\|x\|$$
$$\frac{\partial V_1}{\partial x}\left(\sum_{i=1}^{L}\sum_{h=1}^{L}\theta_i(z(t))\theta_h(\widehat{z}(t))(A_i + B_iK_j)x(t)\right) \leqslant -\kappa_4\|x\|^2$$

On the other hand, from the observability assumption we know matrix C_i can be arbitrary assigned such that $A_i - L_iC_j$ is Hurwitz for all $(i,j) \in \mathcal{I}_L \times \mathcal{I}_L$. Therefore, there exist a Lyapunov function $V_2(e(t))$ such that

$$\varsigma_1\|e\| \leqslant V_2(e(t)) \leqslant \varsigma_2\|e\|$$
$$\left\|\frac{\partial V_2(e(t))}{\partial x}\right\| \leqslant \varsigma_3\|e\|$$
$$\frac{\partial V_2}{\partial e}\left(\sum_{j=1}^{L}\sum_{h=1}^{L}\theta_j(\widehat{z}(t))\theta_h(\widehat{z}(t))(A_j + L_jC_h)x(t)\right) \leqslant -\varsigma_4\|e\|^2$$

To prove $\chi(t) = [0]_{2n\times 1}$ is a stable equilibrium point of (10), we choose a Lyapunov function in the form $V(x(t), e(t)) = \lambda V_1(x(t)) + V_2(e(t))$, where $\lambda \in \mathbb{R}_{(0,\infty)}$ is a constant to be determined [1]. The time derivative of $V(x(t), e(t))$ along the trajectory of (10) is computed by

$$\begin{aligned}\dot{V}(x,e)\Big|_{(10)} &= \lambda\frac{\partial V_1}{\partial x}\frac{dx}{dt} + \frac{\partial V_2}{\partial e}\frac{de}{dt}\Big|_{(10)} \\ &\leqslant -\lambda\kappa_4\|x\|^2 \\ &\quad+\lambda\kappa_3\left\|\sum_{i=1}^{L}\sum_{h=1}^{L}\theta_i(z(t))\theta_h(\widehat{z}(t))B_iK_h\right\|\|x\|\|e\| \\ &\quad-\varsigma_4\|e\|^2 \\ &\quad+\varsigma_3\Bigg\|\sum_{i=1}^{L}\sum_{j=1}^{L}\sum_{h=1}^{L}\theta_i(z(t))\theta_j(\widehat{z}(t))\theta_h(\widehat{z}(t)) \\ &\quad\times(A_i - A_j + (B_i - B_j)K_h + L_j(C_h - C_i))\Bigg\| \\ &\quad\times\|x\|\|e\| \\ &\quad+\varsigma_3\Bigg\|\sum_{i=1}^{L}\sum_{j=1}^{L}\sum_{h=1}^{L}\theta_i(z(t))\theta_j(\widehat{z}(t))\theta_h(\widehat{z}(t)) \\ &\quad\times((B_i - B_j)K_h)\Bigg\|\|e\|^2\end{aligned} \tag{11}$$

Let

$$\nu_1 = \sup \left\| \sum_{i=1}^{L} \sum_{h=1}^{L} \theta_i(z(t))\theta_h(\hat{z}(t))B_iK_h \right\|$$

$$\nu_2 = \sup \left\| \sum_{i=1}^{L} \sum_{j=1}^{L} \sum_{h=1}^{L} \theta_i(z(t))\theta_j(\hat{z}(t))\theta_h(\hat{z}(t)) \times (A_i - A_j + (B_i - B_j)K_h + L_j(C_h - C_i)) \right\|$$

$$\nu_3 = \sup \left\| \sum_{i=1}^{L} \sum_{j=1}^{L} \sum_{h=1}^{L} \theta_i(z(t))\theta_j(\hat{z}(t))\theta_h(\hat{z}(t))((B_i - B_j)K_h) \right\|$$

Then (11) becomes

$$\dot{V}(x(t), e(t))\Big|_{(10)} \leqslant -\lambda\kappa_4 \|x\|^2 + (\lambda\kappa_3\nu_1 + \varsigma_3\nu_2)\|x\|\|e\| + (\varsigma_3\nu_3 - \varsigma_4)\|e\|^2 \quad (12)$$

Completing the square in the two variables $\|x\|$ and $\|e\|$, (12) is negative definite if and only if

$$\frac{(\lambda\nu_1 + \varsigma_3\nu_2)^2}{4\lambda\kappa_4} + \varsigma_3\nu_3 - \varsigma_4 < 0 \quad (13)$$

Completing the square in λ again and by the Sylbester's criterion, there exists some λ guaranteeing (13) if and only if

$$\begin{aligned}(\nu_1\nu_2\varsigma_3 + 2(\varsigma_4 - \varsigma_3\nu_3)\kappa_4)^2 - \nu_1^2\nu_2\varsigma_3 &= 4((\varsigma_4 - \varsigma_3\nu_3)\kappa_4^2) \\ &+ 4\nu_1\nu_2\varsigma_3(\varsigma_4 - \varsigma_3\nu_3)\kappa_4 \\ &+ (\nu_1\nu_2\varsigma_3)^2 - \nu_1^2\nu_2\varsigma_3 \\ &> 0\end{aligned}$$

which is equivalent to

$$4(\varsigma_4 - \varsigma_3\nu_3)\kappa_4 - \nu_1 > 0 \quad (14)$$

which completes the proof. ■

Remark 3 *Condition* (14) *shows that sufficiently fast convergent observer provides the separation principle of the fuzzy-model-based controller/observer.*

5. Conclusions

This note has discussed a separation principle of the T-S fuzzy-model-based controller/observer is investigated. The separation principle of T-S fuzzy-model-based controller/observer with unmeasurable premise variable has been presented. The future research effort will be devoted to the relaxation of the separating controller/observer design condition.

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