듀얼레이트 샘플링을 이용한 퍼지 모델 기반 디지털 제어기
Fuzzy Model-Based Digital Controller Using Dual-Rate Sampling

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ABSTRACT

This paper proposes a novel and efficient intelligent digital redesign technique for a Takagi-Sugeno (TS) fuzzy system. The term of intelligent digital redesign involves converting an existing analog fuzzy-model-based controller into an equivalent digital counterpart in the sense of state matching. In this paper, we suggest the discretization method based on the dual-rate sampling approximation is first proposed, and then attempt to globally match the states of the overall closed-loop TS fuzzy system with the pre-designed analog fuzzy-model-based controller and those with the digitally redesigned fuzzy-model-based controller. To show the feasibility and the effectiveness of the proposed method, a computer simulation is provided.

Keywords: Fuzzy-model-based control, intelligent digital redesign, linear matrix inequality.

1. Introduction

One of the digital design approaches, the so-called digital redesign is a controller design procedure for a hybrid control system, where an analog controller is first designed and then converted to an equivalent digital controller. Historically, digital redesign was first studied in detail by Kuo [5]. He proposed a discrete-state matching method and applied it to a simplified one-axis sky-lab satellite system. Recently, Shieh and his colleagues [1] have thoroughly investigated the digital redesign for a class of linear systems. Joo and his colleagues [2–4] have researched the intelligent digital redesign using the T-S fuzzy-model-based controller for complex nonlinear system.

Unlike the digital redesign for a class of linear systems, an exact approach to the intelligent digital redesign may be impossible due to nonlinear behavior among the T-S fuzzy rules. In specific, under influences of this nonlinear behavior, the discretization as well as state matching gives rise to error .

This paper aims at developing a new global discretization methodology for the T-S fuzzy systems. Our key idea is to utilize the dual-rate sampling schemes. The overall dynamics of the T-S fuzzy system are discretized for the slow-rate sampling period, and then the approximation for the polytopic structure is performed in the fast-rate sampling scheme. To match the state of continuous-data system and digital system, this paper formulates the convex minimization problem and presents derivation of some sufficient conditions in terms of the LMIs for it.

2. Preliminaries

Let us consider the continuous-data T-S fuzzy model

R_i :IF $z_1(t)$ is about Γ_{i1} and $\cdots$ and $z_p(t)$ is about Γ_{ip}

$$\text{THEN } \frac{d}{dt}x_c(t) = A_i x_c(t) + B_i u_c(t) \tag{1}$$

where $z_j(t)$, $j \in \mathcal{I}_p = \{1, 2, \ldots, p\}$, is the jth premise variable, and Γ_{ij}, $(i,j) \in \mathcal{I}_q (= \{1, 2, \ldots, q\}) \times \mathcal{I}_p$, is the fuzzy set of jth premise variable in the ith fuzzy rule. Using the center-average defuzzification, product inference, and singleton fuzzifier, the overall dynamics of (1) is described by

$$\frac{d}{dt}x_c(t) = \sum_{i=1}^{q} \theta_i(z(t))(A_i x_c(t) + B_i u_c(t)) \tag{2}$$

where $w_i(z(t)) = \prod_{j=1}^{p} \Gamma_{ij}(z_j(t))$, $\theta_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{q} w_i(z(t))}$, and $\Gamma_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in Γ_{ij}. We use the following fuzzy-model-based controller.

R_i :IF $z_1(t)$ is about Γ_{i1} and $\cdots$ and $z_p(t)$ is about Γ_{ip}

$$\text{THEN } u_c(t) = K_c^i x_c(t) \tag{3}$$

Using the center-average defuzzification, product inference, and singleton fuzzifier, the overall control law is given by

$$u_c(t) = \sum_{i=1}^{q} \theta_i(z(t)) K_c^i x_c(t) \tag{4}$$

Substituting (4) into (2) yields the state equations of the closed-loop system:

$$\frac{d}{dt}x_c(t) = \sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(t))\theta_j(z(t))(A_i + B_iK_c^j)x_c(t) \quad (5)$$

The digital T-S fuzzy system is denoted by

$$\frac{d}{dt}x_d(t) = \sum_{i=1}^{q}\theta_i(z(t))(A_ix_d(t) + B_iu_d(kT)) \quad (6)$$

for $t \in [kT, kT+T)$.

3. Intelligent Digital Redesign With Dual-Rate Sampling

3.1 Discretization of T-S Fuzzy System

If the state transition matrix $\Phi(t, t_0)$ satisfies $\Phi(t, t_0) = \Phi(t, t_1)\Phi(t_1, t_0)$ with initial condition $\Phi(t_0, t_0) = I$ and $\frac{\partial}{\partial t}\Phi(t, t_0) = \sum_{i=1}^{q}\theta_i(z(t))A_i\Phi(t, t_0)$, the general solution of (6) excited by the initial state $x_d(t_0)$ and the input $u(t)$ is given by

$$x_d(t) = \Phi(t, t_0)x_d(t_0) + \int_{t_0}^{t}\Phi(t, \tau)\left(\sum_{i=1}^{q}\theta_i(z(\tau))B_i\right)u_d(\tau)d\tau \quad (7)$$

If an input $u_d(t)$ is generated by a digital computer followed by a digital-to-analog converter, then $u_d(t)$ will be piecewise constant, *i.e.*, $u_d(t) = u_d(kT)$ for $t \in [kT, kT+T)$. Besides, assume that the firing strength $\theta_i(z(t))$ for $t \in [kT, kT+T)$ is $\theta_i(z(kT))$. For this input and $t \in [kT, kT+T)$, (7) is exactly evaluated by

$$\begin{aligned}
x_d(kT+T) &= \Phi(kT+T, kT)x_d(kT) + \int_{kT}^{kT+T}\Phi(kT+T, \tau) \\
&\quad \times \left(\sum_{i=1}^{q}\theta_i(z(\tau))B_i\right)u_d(\tau)d\tau \\
&= \exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right)x_d(kT) \\
&\quad + \int_{kT}^{kT+T}\exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i(kT+T-\tau)\right) \\
&\quad \times \left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right)u_d(kT)d\tau \\
&= \exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right)x_d(kT) \\
&\quad + \left(\exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right) - I\right) \\
&\quad \times \left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\right)^{-1}\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right) \\
&\quad \times u_d(kT) \\
&\triangleq \mathcal{A}(kT)x_d(kT) + \mathcal{B}(kT)u_d(kT)
\end{aligned} \quad (8)$$

Note that there is no approximation error involved in this derivation and yields the exact solution of (6) at $t = kT$. However, it is impossible to utilize (8) for design of the digital controller in terms of LMIs since it is not represented in the polytopic structure unlike the general T-S fuzzy system.

Our key idea is to utilize the dual-rate sampling schemes. The overall dynamics of the T-S fuzzy system is discretized for the slow-rate sampling period T, and then the approximation for the polytopic structure is performed in the fast-rate sampling period $\frac{T}{n}$, where $n > 1$ and n is constant.

Theorem 1 *The digital TS fuzzy system* (6) *can be converted to the following well-approximated pointwise dynamical behavior:*

$$\begin{aligned}
x_d(kT+T) &\approx \underbrace{\sum_{i_1=1}^{q}\sum_{i_2=1}^{q}\cdots\sum_{i_n=1}^{q}}_{n}\theta_{i_1i_2\cdots i_n}(z(kT))\left(\mathcal{G}_{i_1i_2\cdots i_n}x_d(kT)\right. \\
&\quad \left.+\mathcal{H}_{i_1i_2\cdots i_n}u_d(kT)\right)
\end{aligned} \quad (9)$$

where $G_i^{\frac{1}{n}} = \exp(A_i\frac{T}{n})$, $H_i^{(\frac{1}{n})} = \left(G_i^{\frac{1}{n}} - I\right)A_i^{-1}B_i$, $\mathcal{G}_{i_1i_2\cdots i_n} = \prod_{l=1}^{n}G_{i_l}^{\frac{1}{n}}$, $\mathcal{H}_{i_1i_2\cdots i_n} = H_{i_1}^{(\frac{1}{n})} + \sum_{l=2}^{n}\left(\prod_{m=1}^{l-1}G_{i_m}^{\frac{1}{n}}\right)H_{i_l}^{(\frac{1}{n})}$, $\theta_{i_1i_2\cdots i_n} = \prod_{l=1}^{n}\theta_{i_l}(z(kT))$, *and* $(i_1, i_2, \cdots, i_n) \in \underbrace{\mathcal{I}_q \times \cdots \times \mathcal{I}_q}_{n}$.

Proof: Start from question, namely approximation of (8). $\mathcal{A}(kT)$ and $\mathcal{B}(kT)$ of (8) are finely approximated as follows:

$$\begin{aligned}
\mathcal{A}(kT) &= \exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right) \\
&= \exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\frac{T}{n}\right)^n \\
&\approx \left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}\right)^n \\
&= \underbrace{\sum_{i_1=1}^{q}\sum_{i_2=1}^{q}\cdots\sum_{i_n=1}^{q}}_{n}\theta_{i_1i_2\cdots i_n}\mathcal{G}_{i_1i_2\cdots i_n}
\end{aligned} \quad (10)$$

and

$$\begin{aligned}
&\mathcal{B}(kT) = \\
&\left(\exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right) - I\right) \\
&\times \left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\right)^{-1}\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right) \\
&\approx \left(\left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}\right)^n - I\right)\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\right)^{-1}
\end{aligned}$$

$$\times\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right)$$
$$=\left(\left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}\right)^n-I\right)$$
$$\times\left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}-I\right)^{-1}\left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}-I\right)$$
$$\times\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\right)^{-1}\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right)$$
$$\approx\left(\left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}\right)^n-I\right)\left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}-I\right)^{-1}$$
$$\times\sum_{i=1}^{q}\theta_i(z(kT))\left(G_i^{\frac{1}{n}}-I\right)A_i^{-1}B_i$$
$$=\left(I+\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}+\cdots+\left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}\right)^{n-1}\right)$$
$$\times\sum_{i=1}^{q}\theta_i(z(kT))H_i^{(\frac{1}{n})}$$
$$=\underbrace{\sum_{i_1=1}^{q}\sum_{i_2=1}^{q}\cdots\sum_{i_n=1}^{q}}_{n}\theta_{i_1i_2\cdots i_n}(z(kT))\mathcal{H}_{i_1i_2\cdots i_n} \tag{11}$$

■

Corollary 1 *The continuous-data closed-loop T-S fuzzy system* (5) *can also be the following well-approximated pointwise dynamical behavior:*

$$x_c(kT+T)\approx\underbrace{\sum_{i_1=1}^{q}\sum_{i_2=1}^{q}\cdots\sum_{i_n=1}^{q}}_{n}\underbrace{\sum_{j_1=1}^{q}\sum_{j_2=1}^{q}\cdots\sum_{j_n=1}^{q}}_{n}\theta_{i_1i_2\cdots i_n}$$
$$\times\,\theta_{j_1j_2\cdots j_n}\mathcal{M}_{i_1i_2\cdots i_nj_1j_2\cdots j_n}x_c(kT) \tag{12}$$

where $\mathcal{M}_{i_1i_2\cdots i_nj_1j_2\cdots j_n}=\prod_{l=1}^{n}\exp\left((A_{i_l}+B_{i_l}K_{j_l})\frac{T}{n}\right)$ *and* $(i_1,i_2,\cdots,i_n,j_1,j_2,\cdots,j_n)\in\underbrace{\mathcal{I}_q\times\cdots\times\mathcal{I}_q}_{2n}$.

Proof: It can be straightforwardly proved by Theorem 1.

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3.2 Design of Digital controller Using State Matching

Let the digital control law for (6) take the following form:

$$u_d(kT)=\underbrace{\sum_{i_1=1}^{q}\sum_{i_2=1}^{q}\cdots\sum_{i_n=1}^{q}}_{n}\theta_{i_1i_2\cdots i_n}(z(kT))K_d^{i_1i_2\cdots i_n}x_d(kT) \tag{13}$$

for $t\in[kT,kT+T)$, where $K_d^{i_1i_2\cdots i_n}$ is the digital control gain matrix to be redesigned. Substituting (13) into the discretized version (9) of (6) yields the state equations of the closed-loop system:

$$x_d(kT+T)\approx\underbrace{\sum_{i_1=1}^{q}\sum_{i_2=1}^{q}\cdots\sum_{i_n=1}^{q}}_{n}\underbrace{\sum_{j_1=1}^{q}\sum_{j_2=1}^{q}\cdots\sum_{j_n=1}^{q}}_{n}\theta_{i_1i_2\cdots i_n}(z(k$$
$$\times\,\theta_{j_1j_2\cdots j_n}(z(kT))\left(\mathcal{G}_{i_1i_2\cdots i_n}+\mathcal{H}_{i_1i_2\cdots i_n}K_d^{j_1j_2\cdot}\right.$$
$$\times\,x_d(kT) \tag{14}$$

Therefore, we can attempt to match the state $x(kT+T)$ of (14) and (12) under the assumption that $x_c(kT)=x_d(kT)$. This problem can become a convex optimization problem such as

$$\left\|\mathcal{M}_{i_1i_2\cdots i_nj_1j_2\cdots j_n}-\left(\mathcal{G}_{i_1i_2\cdots i_n}+\mathcal{H}_{i_1i_2\cdots i_n}K^d_{j_1j_2\cdots j_n}\right)\right\|<\gamma I \tag{15}$$

hence can be numerically solved in terms of LMIs.

Theorem 2 *If there exist constant matrixes* $K^d_{i_1i_2\cdots i_n}$ *and a possibly small positive scalars* γ *such the following generalized eigenvalue problem (GEVP) has solutions*

$$\underset{K^d_{i_1i_2\cdots i_n}}{\text{Minimize}}\quad\gamma\quad\text{subject to}$$
$$\begin{bmatrix}-\gamma I & *\\ \mathcal{M}_{i_1i_2\cdots i_nj_1j_2\cdots j_n}-\mathcal{G}_{i_1i_2\cdots i_n}-\mathcal{H}_{i_1i_2\cdots i_n}K^d_{j_1j_2\cdots j_n} & -\gamma I\end{bmatrix}$$
$$<0,\quad i_1,i_2\cdots,i_n,j_1,j_2\cdots,j_n\in\mathcal{I}_q \tag{16}$$

then $x_d(kT+T)$ *of* (14) *closely matches the state* $x_c(kT+T)$ *of* (12), *where '$*$' denotes the transposed element in symmetric positions.*

Proof: Consider the convex optimization problem (15) From the definition of the induced-2 norm, the following inequalities hold

$$\left(\mathcal{M}_{i_1i_2\cdots i_nj_1j_2\cdots j_n}-\mathcal{G}_{i_1i_2\cdots i_n}-\mathcal{H}_{i_1i_2\cdots i_n}K^d_{j_1j_2\cdots j_n}\right)^T$$
$$\times\left(\mathcal{M}_{i_1i_2\cdots i_nj_1j_2\cdots j_n}-\mathcal{G}_{i_1i_2\cdots i_n}-\mathcal{H}_{i_1i_2\cdots i_n}K^d_{j_1j_2\cdots j_n}\right)<\gamma \tag{17}$$

Using Schur complement, can be represented by

$$\begin{bmatrix}-\gamma I & *\\ \mathcal{M}_{i_1i_2\cdots i_nj_1j_2\cdots j_n}-\mathcal{G}_{i_1i_2\cdots i_n}-\mathcal{H}_{i_1i_2\cdots i_n}K^d_{j_1j_2\cdots j_n} & -\gamma I\end{bmatrix}$$
$$<0 \tag{18}$$

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4. The Chen's Chaotic Attractor

In this section, we use the results in Section 3. to the digital control problem of the Chen's chaotic attractor. The dynamics of the Chen's chaotic attractor are as follows:

$$\frac{d}{dt}\begin{bmatrix}x_1(t)\\x_2(t)\\x_3(t)\end{bmatrix}=\begin{bmatrix}a(x_2(t)-x_1(t))\\(c-a)x_1(t)-x_1(t)x_3(t)+cx_2(t)\\x_1(t)x_2(t)-bx_3(t)\end{bmatrix} \tag{19}$$

where $a = 35$, $b = 3$, and $c = 28$. The corresponding T-S fuzzy model of the system in (19) is expressed as follows:

R^1 :IF $x_1(t)$ is about Γ_1^1,

$$\text{THEN}\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = A_1 \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + B_1 u_d(t)$$

R^2 :IF $x_1(t)$ is about Γ_1^2,

$$\text{THEN}\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = A_2 \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + B_2 u_d(t)$$

where

$$A_1 = \begin{bmatrix} -a & a & 0 \\ c-a & c & -x_{1min} \\ 0 & x_{1min} & -b \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -a & a & 0 \\ c-a & c & -x_{1max} \\ 0 & x_{1max} & -b \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

and the membership functions are $\Gamma_1^1(x_1(t)) = \frac{-x_1(t)+x_{1max}}{x_{1max}-x_{1min}}$, and $\Gamma_1^2(x_2(t)) = \frac{x_1(t)-x_{1min}}{x_{1max}-x_{1min}}$, where Γ_j^i are positive semi-definite for all $x \in [x_{1min}, x_{1max}](= [-30, 30])$. The gain matrices for the analog fuzzy model-based controller is obtained as follows:

$$K_c^1 = \begin{bmatrix} -5.1763 & 204.8118 & 131.8175 \end{bmatrix}$$
$$K_c^2 = \begin{bmatrix} -5.1763 & 204.8118 & -131.8175 \end{bmatrix}$$

Applying the digital redesign method, where n is selected as 3, discussed in the preceding section yields the following gain matrices for the sampling period $T = 0.02s$.

$$K_d^{111} = \begin{bmatrix} -14.9745 & 179.1430 & 173.8455 \end{bmatrix}$$
$$K_d^{211} = \begin{bmatrix} -17.7989 & 251.3367 & 63.7664 \end{bmatrix}$$
$$K_d^{121} = \begin{bmatrix} -18.0453 & 251.9124 & 64.9188 \end{bmatrix}$$
$$K_d^{221} = \begin{bmatrix} -17.8002 & 249.9941 & -64.1442 \end{bmatrix}$$
$$K_d^{112} = \begin{bmatrix} -17.6624 & 254.4623 & 64.6591 \end{bmatrix}$$
$$K_d^{212} = \begin{bmatrix} -18.0192 & 250.9625 & -64.2822 \end{bmatrix}$$
$$K_d^{122} = \begin{bmatrix} -17.6873 & 256.7548 & -64.9179 \end{bmatrix}$$
$$K_d^{222} = \begin{bmatrix} -14.9745 & 179.1430 & -173.8455 \end{bmatrix}$$

With $x_1(0) = 1$, $x_2(0) = 1$, and $x_3(0) = -1$, the digital control system with the obtained gain matrices is simulated on the digital computer for $T = 0.02$ sec.. Other available intelligent digital redesign method [2] is also simulated for the comparison. Figure 1 shows the comparisons for $T = 0.02$s.

5. Conclusions

In this paper, a new intelligent digital redesign method has been proposed for the digital control of the continuous-time fuzzy system. Unlike other methods, the proposed method utilizes the discrete-time T-S fuzzy model with the dual-rate sampling. The dual-rate scheme is major factor that improves the accuracy of the state matching as well as the discretization.

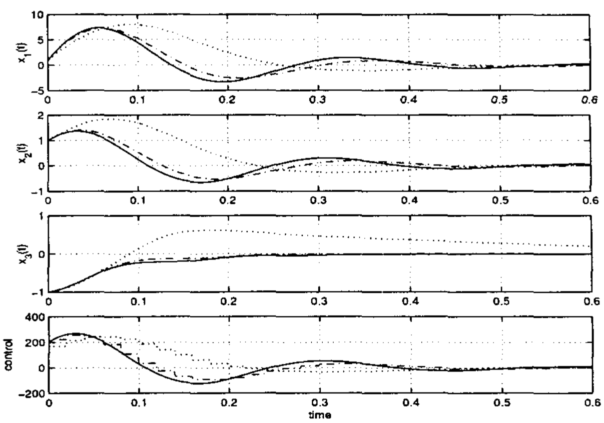

Figure 1: The controlled Chen's chaotic attractor (Solid line: continuous-time control, dotted line: digital control by [2], and dash-dotted line: digital control by the proposed method.)

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