Hausdorffness on Generalized Intuitionistic Fuzzy Filters

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ABSTRACT

The notion of generalized intuitionistic fuzzy sets (GIF sets) was introduced by Mondal and Samanta [J. Fuzzy Math. 10(4) (2002) 839-861]. By this notion, a notion of GIF filters is introduced and studied. Also a new notion of of Hausdorffness is defined on GIF filters and their properties are studied to some extent.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [15]. Atanassov [1] generalized this idea to intuitionistic fuzzy sets, and later there has been much progress in the study of intuitionistic fuzzy sets by many authors [1-7,11,13].
On the other hand, Lowen [11] introduced the concept of fuzzy filter and defined convergence in a fuzzy topological space which enables us to characterize fuzzy compactness. Many results on fuzzy filter are obtained by De Prada and Saralegui [8,9] and Ramakrishnan and Nayagam [13]. More recently, Mondal and Samanta [10] introduced definitions of GIF sets, generalized intuitionistic fuzzy relations and generalized intuitionistic fuzzy topology and studied some of their properties.

In this paper, by using GIF sets, we introduce and study the notion of GIF filters and define the notion of Hausdorffness on GIF filters, and study their properties to some extent.

2. Preliminaries

First we shall present the fundamental definitions given by Atanassov [1]:

Definition 2.1. Let X be a non-empty fixed set. An intuitionistic fuzzy set (shortly, IFS), A is an object having the form

$$A=\{\langle x, \mu_A(x), \gamma_A(x)\rangle : x\in X\}$$

where the functions $\mu_A: X\to[0,1]$ and $\gamma_A: X\to[0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each $x\in X$ to the set A, respectively, and $0\le\mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$. Obviously, every fuzzy set $\{\langle \mu_A(x), x\rangle : x\in X\}$ on X is an IFS of the form

$$A=\{\langle x,\mu_A(x), 1-\mu_A(x)\rangle : x\in X\}.$$

For an IFS $A=\{\langle x, \mu_A(x), \gamma_A(x)\rangle : x\in X\}$, it is observed that $\mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$ and hence $\mu_A(x)\wedge\gamma_A(x)\not> \frac{1}{2}$ for each $x\in X$.

Having motivated from the observation, Monal and Samanta [10] defined a generalized intuitionistic fuzzy sets as following:

Definition 2.2.[10] Let X be a nonempty fixed set. A generalized intuitionistic fuzzy set (GIF set for short) A in X is an object having the form

$$A=\{\langle x, \mu_A(x), \gamma_A(x)\rangle : x\in X\}$$

where the functions $\mu_A: X\to[0,1]$ and $\gamma_A: X\to[0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each $x\in X$ to the set A, respectively, and $\mu_A(x)+\gamma_A(x)\le\frac{1}{2}$ for each $x\in X$.

Definition 2.3.[10] Let A and B be GIF sets in X in the form $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ and $B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$. Then

(a) $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x)$ and $\gamma_A(x)\ge\gamma_B(x)$ for all $x\in X$;
(b) $A=B$ iff $A\subseteq B$ and $B\subseteq A$;
(c) $A^c=\{\langle x, \gamma_A(x), \mu_A(x)\rangle : x\in X\}$;
(d) $A\cap B=\{\langle x, \mu_A(x)\wedge\mu_B(x), \gamma_A(x)\vee\gamma_B(x)\rangle : x\in X\}$;
(e) $A\cup B=\{\langle x, \mu_A(x)\vee\mu_B(x), \gamma_A(x)\wedge\gamma_B(x)\rangle : x\in X\}$;
(f) If $\{A_i : i\in J\}$ is an arbitrary family of GIF sets in X, then

$$\bigcap A_i=\{\langle x, \wedge\mu_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x\in X\},$$
$$\bigcup A_i=\{\langle x, \vee\mu_{A_i}(x), \wedge\gamma_{A_i}(x)\rangle : x\in X\}.$$

(g) $0_\sim=\{\langle x,0,1\rangle : x\in X\}$ and $1_\sim=\{\langle x,1,0\rangle : x\in X\}$.

Definition 2.4.[10] Let X and Y be two non-empty sets and $f: X\to Y$ be a function. Let

$A=\{\langle x, \mu_A(x), \gamma_A(x)\rangle : x\in X\}$ be a GIF set in X and $B=\{\langle y, \mu_B(y), \gamma_B(y)\rangle : y\in Y\}$ be a GIF set in Y.

(a) The inverse image $f^{-1}(B)$ of B under f is the GIF set in X defined by

$$f^{-1}(B)=\{\langle x, \mu_{f^{-1}(B)}(x), \gamma_{f^{-1}(B)}(x)\rangle : x\in X\}.$$

where $\mu_{f^{-1}(B)}(x)=\mu_B(f(x))$ and $\gamma_{f^{-1}(B)}(x)=\gamma_B(f(x))$ for each $x\in X$.

(b) The image $f(A)$ of A under f is the GIF set in Y defined by

$$f(A)=\{\langle y, \mu_{f(A)}(y), \gamma_{f(A)}(y)\rangle : y\in Y\}$$

where

$$\mu_{f(A)}(y)=\begin{cases}\bigvee_{x\in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y)\neq\phi\\ 0, & \text{otherwise},\end{cases}$$

$$\gamma_{f(A)}(y)=\begin{cases}\bigwedge_{x\in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y)\neq\phi\\ 1, & \text{otherwise}.\end{cases}$$

Now we list the properties of images and pre-images, some of which we shall frequently use in Sections 3 and 4.

Theorem 2.5.[10] Let A and A_i $(i\in J)$ be GIF sets in X and B and B_i $(i\in J)$ be GIF in Y and $f: X\to Y$ be a function. Then:

(a) If $A_1\subseteq A_2$, then $f(A_1)\subseteq f(A_2)$.

(b) If $B_1\subseteq B_2$, then $f^{-1}(B_1)\subseteq f^{-1}(B_2)$.

(c) $A\subseteq f^{-1}(f(A))$ (If f is injective, then $A=f^{-1}(f(A))$).

(d) $f(f^{-1}(B))\subseteq B$ (If f is surjective, then $f(f^{-1}(B))=B$).

(e) $f^{-1}(\bigcup B_i)=\bigcup f^{-1}(B_i), f^{-1}(\bigcap B_i)=\bigcap f^{-1}(B_i)$

(f) $f(\bigcup A_i)=\bigcup f(A_i), f(\bigcap A_i)\subseteq\bigcap f(A_i)$ (If f is injective, then $f(\bigcap A_i)=\bigcap f(A_i)$).

(g) $f^{-1}(1_\sim)=1_\sim, f^{-1}(0_\sim)=0_\sim$.

(h) $f(0_\sim)=0_\sim, f(1_\sim)=1_\sim$ if f is surjective.

(i) $f^{-1}(B)^c=f^{-1}(B^c)$, $f(A)^c\subseteq f(A^c)$ if f is surjective.

Definition 2.6.[10] Let $\alpha, \beta\in[0,1]$ and $\alpha\wedge\beta\leq\frac{1}{2}$. A generalized intuitionistic fuzzy point (GIF point for short) $x_{(\alpha,\beta)}$ in X is a GIF set in X defined by

$$x_{(\alpha,\beta)}(y)=\begin{cases}(\alpha,\beta), & \text{if } y=x\\ (0,1), & \text{if } y\neq x.\end{cases}$$

In this case, x is called the support of $x_{(\alpha,\beta)}$ and α and β are called the value and the non-value of $x_{(\alpha,\beta)}$, respectively. An GIF point $x_{(\alpha,\beta)}$ is said to belong to a GIF set $A=\langle x,\mu_A,\gamma_A\rangle$ in X, denoted by $x_{(\alpha,\beta)}\in A$, if $\alpha\leq\mu_{A(x)}$ and $\beta\geq\gamma_{A(x)}$.

Theorem 2.7. Let A, B be GIF sets in X and $x_{(\alpha,\beta)}$ be GIF point in X. Then

(a) $A=\bigcup\{x_{(\alpha,\beta)} : x_{(\alpha,\beta)}\in A\}$.

(b) $A\subset B$ if and only if $x_{(\alpha,\beta)}\in A$ implies $x_{(\alpha,\beta)}\in B$.

Defintion 2.8 Let $A=\{\langle x, \mu_A(x), \gamma_A(x)\rangle : x\in X\}$ and $B=\{\langle y, \mu_B(y), \gamma_B(y)\rangle : y\in Y\}$ be GIF sets in X and Y, respectively. We define the cartesian product of A and B as the GIF set in $X\times Y$ defined as follows:

$$A\times B=\{\langle (x,y), \mu_{A(x)}\wedge\mu_{B(y)}, \gamma_{A(x)}\vee\gamma_{B(y)}\rangle : x\in X, y\in Y\}.$$

Theorem 2.9 Let A and B be GIF sets in X and C and D be GIF sets in Y. Then we have

(a) If $A\subset B$, then $A\times C\subset B\times C$ and $A\times C\subset A\times D$.

(b) $(A\cap B)\times(C\cap D)=(A\times C)\cap(B\times D)$.

(c) $(A\cap B)\times C=(A\times C)\cap(B\times C)$ and $(A\cup B)\times C=(A\times C)\cup(B\times C)$.

3. IF filters

Definition 3.1. A nonvoid family $\boldsymbol{F}$ of GIF sets is called a *fuzzy filter of GIF* or a GIF filter if

(a) $0_\sim\notin\boldsymbol{F}$.

(b) If A, $B\in\boldsymbol{F}$, then $A\cap B\in\boldsymbol{F}$.

(c) If $A\in\boldsymbol{F}$ and $A\subseteq B$, then $B\in\boldsymbol{F}$.

Definition 3.2. (a) A nonvoid family $\boldsymbol{B}$ of GIF set is called a *GIF filter base* if provided $\boldsymbol{B}$ does not contain $0_\sim$ and provided the intersection of any two element of $\boldsymbol{B}$ contains an element of $\boldsymbol{B}$.
(b) A family $\boldsymbol{S}$ is called a *subbase* of GIF filter if it is nonvoid and the intersection of any finite number of elements of $\boldsymbol{S}$ is not $0_\sim$.

Remark 3.3. If $\boldsymbol{S}$ is a subbase of a GIF filter, then the family $\boldsymbol{B}(\boldsymbol{S})$ consisting of all finite intersections of elements of $\boldsymbol{S}$ is a GIF filter base. If $\boldsymbol{B}$ is a GIF filter base, then the family $\boldsymbol{F}(\boldsymbol{B})$, consisting of all GIF sets A such that $A\supseteq B$ for some $B\in\boldsymbol{B}$, is a GIF filter. Furthermore, $\boldsymbol{B}(\boldsymbol{S})$ and $\boldsymbol{F}(\boldsymbol{B})$ are uniquely determined by $\boldsymbol{S}$ and $\boldsymbol{B}$, respectively.

Definition 3.4. The GIF filter $\boldsymbol{F}(\boldsymbol{B})$ and $\boldsymbol{F}(\boldsymbol{B}(\boldsymbol{S}))$ (or, $\boldsymbol{F}(\boldsymbol{S})$) are called, respectively, the GIF filter generated by $\boldsymbol{B}$ and the GIF filter generated by $\boldsymbol{S}$. A family $\boldsymbol{B}$ is called a base of the GIF filter $\boldsymbol{F}$ if $\boldsymbol{B}$ is a GIF filter base and $\boldsymbol{F}=\boldsymbol{F}(\boldsymbol{B})$. Similarly, $\boldsymbol{S}$ is called a subbase of the GIF filter $\boldsymbol{F}$ if S is a subbase and $\boldsymbol{F}=\boldsymbol{F}(S)$.

Remark 3.5. (a) Let Φ be any indexed family of GIF filters on X. Then

(i) $\bigcap_{F\in\Phi} \boldsymbol{F}$ is also GIF filter.

(ii) $\bigcap_{F\in\Phi} \boldsymbol{F}$ is also GIF filter if Φ is directed family of GIF filters under inclusion $\subset$.

(b) Let $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ be two GIF filter bases. Then $\boldsymbol{F}(\boldsymbol{B}_1) \subseteq \boldsymbol{F}(\boldsymbol{B}_2)$ if and only if for any $B \in \boldsymbol{B}_1$ there exists $A \in \boldsymbol{B}_2$ such that $A \subseteq B$.

Theorem 3.6. Let $\boldsymbol{F}$ be a GIF filter on X and $Y \subseteq X$. Then $\boldsymbol{F}|Y$ is a GIF filter on Y if $A|Y \neq 0_{\sim}$ for any $A \in \boldsymbol{F}$.

Theorem 3.7. Let $f: X \to Y$ be a function and $\boldsymbol{F}$ be a GIF filter on X. Then $f(\boldsymbol{F})=\{f(F): F \in \boldsymbol{F}\}$ is a GIF filter base on Y.

Theorem 3.8. Let $f: X \to Y$ be a surjection and $\boldsymbol{G}$ be a GIF filter on Y. Then $f^{-1}(\boldsymbol{G})=\{f^{-1}(G) : G \in \boldsymbol{G}\}$ is a GIF filter base on X.

Definition 3.9. Let $(X_i, \boldsymbol{F}_i)$ be GIF filters (i=1,2). A function $f: (X, \boldsymbol{F}_1) \to (Y, \boldsymbol{F}_2)$ is called GIF *filter continuous* if for every $F \in \boldsymbol{F}_2$, $f^{-1}(F) \in \boldsymbol{F}_1$.

Example 3.10. Let $X=\{a,b,c\}$ and $Y=\{d,e,f\}$. Let A and B be GIF sets in X and Y respectively defined as follows:

$$A=\left\langle x, \left(\frac{a}{\alpha_1}, \frac{b}{\alpha_1}, \frac{c}{\beta_1}\right), \left(\frac{a}{\alpha_2}, \frac{b}{\alpha_2}, \frac{c}{\beta_2}\right)\right\rangle \text{ and}$$

$$B=\left\langle x, \left(\frac{d}{\alpha_1}, \frac{e}{\beta_1}, \frac{f}{\delta_1}\right), \left(\frac{d}{\alpha_2}, \frac{e}{\beta_2}, \frac{f}{\delta_2}\right)\right\rangle,$$

where $\alpha_1 \wedge \alpha_2 \leq \frac{1}{2}$, $\beta_1+\beta_2 \leq \frac{1}{2}$ and $\delta_1 \wedge \delta_2 \leq 1$.

Then clearly $\boldsymbol{B}_1=\{A\}$ and $\boldsymbol{B}_2=\{B\}$ are GIF filter bases on X and Y respectively. Let $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ be the GIF filter generated by $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ respectively. Let $f: X \to Y$ be a function defined by $f(a)=f(b)=d$ and $f(c)=e$. Then by definition $f^{-1}(\boldsymbol{B}_2)=\boldsymbol{B}_1$ and hence f is GIF filter continuous.

However, constant function $f: (X, \boldsymbol{F}_1) \to (Y, \boldsymbol{F}_2)$ need not be GIF filter continuous as shown by the follo-wing example.

Example 3.11. Let $X=\{a,b,c\}$ and $Y=\{d,e,f\}$. Let A and B be GIF sets in X and Y respectively defined as follows:

$$A=\left\langle x, \left(\frac{a}{\alpha_1}, \frac{b}{\alpha_1}, \frac{c}{\alpha_1}\right), \left(\frac{a}{\alpha_2}, \frac{b}{\alpha_2}, \frac{c}{\alpha_2}\right)\right\rangle \text{ and}$$

$$B=\left\langle x, \left(\frac{d}{\alpha_1}, \frac{e}{\beta_1}, \frac{f}{\delta_1}\right), \left(\frac{d}{\alpha_2}, \frac{e}{\beta_2}, \frac{f}{\delta_2}\right)\right\rangle.$$

where $\alpha_1+\alpha_2 \leq 1$, $\beta_1 \wedge \beta_2 \leq 1/2$, $\delta_1 \wedge \delta_2 \leq 1/2$, $\beta_1 < \alpha_1$ and $\beta_2 > \alpha_2$. Clearly, $\boldsymbol{B}_1=\{A\}$ and $\boldsymbol{B}_2=\{B\}$ are GIF filter bases on X and Y respectively. Let $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ be the GIF filter generated by $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ respectively.

Let $f: X \to Y$ be constant function defined by $f(x)=e$ for all $x \in X$.

Choose σ_i $(i=1,2)$ such that $\beta_1 < \sigma_1 < \alpha_1$ and $\beta_2 > \sigma_2 > \alpha_2$. Then clearly,

$$C=\left\langle x, \left(\frac{d}{\alpha_1}, \frac{e}{\sigma_1}, \frac{f}{\delta_1}\right), \left(\frac{d}{\alpha_2}, \frac{e}{\sigma_2}, \frac{f}{\delta_2}\right)\right\rangle \in \boldsymbol{F}_2 \text{ . But}$$

$$f^{-1}(C)=\left\langle x, \left(\frac{a}{\sigma_1}, \frac{b}{\sigma_1}, \frac{c}{\sigma_1}\right), \left(\frac{a}{\sigma_2}, \frac{b}{\sigma_2}, \frac{c}{\sigma_2}\right)\right\rangle \notin \boldsymbol{F}_1.$$

Hence f is not GIF filter continuous.

The following statement is an immediate consequence of definitions.

Remark 3.12. (i) If $f: (X, \boldsymbol{F}) \to (Y, \boldsymbol{G})$ and $g: (Y, \boldsymbol{G}) \to (Z, \mathrm{H})$ are GIF filter continuous, then the composition $f \circ g : (X, \boldsymbol{F}) \to (Z, \mathrm{H})$ is also GIF filter continuous.

(ii) If $f: (X, \boldsymbol{F}) \to (X, \boldsymbol{F})$ is identity function, then f is GIF filter continuous.

(iii) Let $f: (X, \boldsymbol{F}) \to (Y, \boldsymbol{G})$ be a GIF filter continuous function. If $Z \subseteq X$ such that $F|Z \neq 0_{\sim}$ for any $F \in \boldsymbol{F}$, then the restriction $f|Z : (Z, \boldsymbol{F}|Z) \to (Y, \boldsymbol{G})$ is also GIF filter continuous.

Theorem 3.13. A function $f:(X, \boldsymbol{F}) \to (Y, \boldsymbol{G})$ is GIF filter continuous if and only if for every GIF point $x_{(\alpha,\beta)}$ in X and every $G \in \boldsymbol{G}$ such that $f(x_{(\alpha,\beta)}) \in G$, there exists a $F \in \boldsymbol{F}$ such that $x_{(\alpha,\beta)} \in F$ and $f(F) \subseteq G$.

4. Hausdorff GIF filters

In crisp theory, it is well-known that $A \cap B=\phi \Leftrightarrow A \subset B^c$ for any sets A and B. But it is no longer valid in fuzzy setting. So Ramakrishnan and Nayagam [13] chose the notion of fuzzy disjointness that agrees with ordinary set theoretic disjointness in crisp case as follows: Two fuzzy set A and B in X are said to intersect if $\mu_A(x)+\mu_B(x) > 1$ for some $x \in X$, and A and B are said to be disjoint if these sets do not intersect. Now we extend these notions to GIF sets as follows:

Definition 4.1. Two GIF sets A and B in X are said to *intersect at* $x \in X$ if

$$\mu_A(x)+(1-\gamma_B(x)) > 1 \text{ or } \mu_B(x)+(1-\gamma_A(x)) > 1.$$

Otherwise A and B do not intersect at x. A and B are said to be *disjoint* if these sets do not intersect anywhere.

Definition 4.2. A GIF filter $(X, \boldsymbol{F})$ is called *Hausdorff* if for any $x, y \in X$ with $x \neq y$, there exist $F_1, F_2 \in \boldsymbol{F}$ such that $\gamma_{F_1}(x) < \frac{1}{2}$,

$\gamma_{F_2}(y) < \frac{1}{2}$ and $\mu_{F_1}(z) + (1 - \gamma_{F_2}(z)) \le 1$ and $\mu_{F_2}(z) + (1 - \gamma_{F_1}(z)) \le 1$ for any $z \in X$.

Example 4.3. Let $X=\{a,b,c\}$ and B_i $(i=1,2,3,4)$ be GIF sets in X defined as follows:

$$B_1 = \left\langle x, \left(\frac{a}{\alpha}, \frac{b}{\beta}, \frac{c}{\delta}\right), \left(\frac{a}{1-\alpha}, \frac{b}{1/4}, \frac{c}{1-\delta}\right)\right\rangle,$$
$$B_2 = \left\langle x, \left(\frac{a}{\alpha}, \frac{b}{\beta}, \frac{c}{\delta}\right), \left(\frac{a}{1-\alpha}, \frac{b}{1-\beta}, \frac{c}{1-\delta}\right)\right\rangle,$$
$$B_3 = \left\langle x, \left(\frac{a}{\alpha}, \frac{b}{\beta}, \frac{c}{\delta}\right), \left(\frac{a}{1/4}, \frac{b}{1-\beta}, \frac{c}{1-\delta}\right)\right\rangle,$$
$$B_4 = \left\langle x, \left(\frac{a}{\alpha}, \frac{b}{\beta}, \frac{c}{\delta}\right), \left(\frac{a}{1-\alpha}, \frac{b}{1-\beta}, \frac{c}{1/4}\right)\right\rangle,$$

where $\alpha, \beta, \delta \in (0,1/4)$. Let $\boldsymbol{F}$ be an GIF filter generated by $\boldsymbol{B}=\{B_1, B_2, B_3, B_4\}$. Then $(X, \boldsymbol{F})$ is a Hausdorff GIF filter.

Recall that a sequence $\{x_n\}$ of fuzzy filter $(X, \boldsymbol{F})$ is said to *converge filterly to* $x \in X$ if for every $F \in \boldsymbol{F}$ such that $\mu_F(x) > \frac{1}{2}$, there exists $n_0 \in \mathbf{N}$ such that $\mu_F(x_n) > \frac{1}{2}$ for all $n \ge n_0$, equivalently $1 - \gamma_F(x_n) < \frac{1}{2}$ for all $n \ge n_0$.
Now we extend above definition to GIF filter as follows:

Definition 4.4. Let $(X, \boldsymbol{F})$ be a GIF filter. A sequence $\{x_n\}$ of X is said to *converge* GIF *filterly to* x (denoted by $\{x_n\} \to_{\text{GIF}} x$), and x is called *a* GIF *limit* of $\boldsymbol{F}$, if for every $F \in \boldsymbol{F}$ such that $\gamma_F(x) < \frac{1}{2}$, there exists $n_0 \in \mathbf{N}$ such that $1 - \gamma_F(x_n) < \frac{1}{2}$ for all $n \ge n_0$, equivalently $\mu_F(x_n) > \frac{1}{2}$ for all $n \ge n_0$.

Theorem 4.5. Let $f : (X,\boldsymbol{F}) \to (Y,\boldsymbol{G})$ be a GIF continuous function and $\{x_n\}$be a sequence in X. If $\{x_n\} \to_{\text{GIF}} x$, then $\{f(x_n)\} \to_{\text{GIF}} f(x)$.

Theorem 4.6. In Hausdorff GIF filter $(X,\boldsymbol{F})$, every GIF filterly convergent sequence of points of X has exactly one GIF limit.

Theorem 4.7. Let $(X, \boldsymbol{F})$ be an Hausdorff GIF filter and $Y \subseteq X$. If $A|Y \ne 0_\sim$ for any $A \in \boldsymbol{F}$, then $(Y, \boldsymbol{F}|Y)$ is also Hausdorff GIF filter.

Theorem 4.8. Let $f : (X, \boldsymbol{F}) \to (Y, \boldsymbol{G})$ be a bijective GIF filter open function. If $(X,\boldsymbol{F})$ is Hausdorff GIF filter, then $(Y, \boldsymbol{G})$ is a Hausdorff GIF filter.

Lemma 4.9. Let $f : (X, \boldsymbol{F}) \to Y$ be a surjection. Then $\boldsymbol{G}= \{G \in ([0,1] \times [0,1])^X \mid f^{-1}(G) \in \boldsymbol{F}\}$ is a GIF filter on Y.

Definition 4.10. The GIF filter defined in above lemma is called *Quotient GIF filter determined by the surjective function f.*

Suppose $(X, \boldsymbol{F})$ and $(Y, \boldsymbol{H})$ are GIF filters and $f : (X, \boldsymbol{F}) \to (Y, \boldsymbol{H})$ is surjection. The following theorem gives conditions on f that make $\boldsymbol{H}$ equal to the quotient GIf filter $\boldsymbol{G}$ on Y determined by f.

Theorem 4.11. Let $(X,\boldsymbol{F})$ and $(Y,\boldsymbol{H})$ be GIF filters, $f : (X, \boldsymbol{F}) \to (Y, \boldsymbol{H})$ be a surjective GIF filter continuous function and let $\boldsymbol{G}$ be the quotient GIF filter on Y determined by f. If f is GIF filter open, then $\boldsymbol{G}=\boldsymbol{H}$.

Theorem 4.12. Let $f : (X, \boldsymbol{F}) \to Y$ be bijective function and $\boldsymbol{G}$ be the quotient GIF filter on Y determined by f. If $(X, \boldsymbol{F})$ is a Hausdorff GIF filter, then $(Y, \boldsymbol{G})$ is Hausdorff GIF filter.

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