Proceedings of KFIS Fall Conference, 2003. 5. 3

직관적 H-퍼지 반사관계

Intuitionistic H-Fuzzy Reflexive Relations

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Abstract

We introduce the subcategory **IRel**$_R(H)$ of **IRel**(H) consisting of intuitionistic H-fuzzy reflexive relational spaces on sets and we study structures of **IRel**$_R(H)$ in a viewpoint of the topological universe introduce by L.D.Nel. We show that **IRel**$_R(H)$ is a topological universe over **Set**. Moreover, we show that exponential objects in **IRel**$_R(H)$ are quite different from those in **IRel**(H) constructed in [7].

Key words and phrases : intuitionistic H-fuzzy relation, Cartesian closed category topological universe.

0. Introduction

In [7,8], we studied categorical structures of the category **ISet**(H) consisting of intuitioninstic H-fuzzy sets and the category **IRel**(H) consisting of intuitinistic H-fuzzy relational spaces in a viewpoint of topological universe, defined by L.D.Nel(cf. [11]).

In this paper, we study categorical structures

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2000 Mathematics Subject Classification of AMS: 54C05, 54C60

of the subcategory **IRel**$_R$ (H) of **IRel** (H) consisting of intuitionistic H-fuzzy reflexive relational spaces on sets in a viewpoint of a topological universe. In particular, it is very interesting that exponential objects in **IRel**$_R$ (H) are shown to be quite different from those in **IRel** (H)(see [7]).

For general background for lattice theory, we refer to [1,9] and for general categorical background to [4,5,10,11].

1. Preliminaries

We will introduce some well-known results [5,9] which are needed in a later section.

Result 1.A[10, Theorem 2.4 ; 5, Proposition 36.10 and 36.11]. Let **A** be a well-powered and co-(well-powered) topological category and let **B** a subcategory of **A**. Then the following are equivalent :

(1) **B** is epireflective in **A**.

(2) **B** is closed under the formation of initial monosources.

(3) **B** is closed under the formation of products and pullbacks in **A**.

Result 1.B[10, Theorem 2.5]. Let **A** be a well-powered and co-(well-powered) topol- ogical category and let **B** a subcategory of **A**. Then the following are equivalent :

(1) **B** is bireflective in **A**.

(2) **B** is closed under the formation of initial sources.

Result 1.C[10, Theorem 2.6]. If **A** is a (property fibred, resp.) topological category and **B** is a bireflective subcategory of **A**, then **B** is also a (property fibred, resp.) topological category. Moreover, every source in **B** which is initial in **A** is initial in **B**.

Throughout this paper, we use H as a complete Heyting algebra.

2. The Category IRel$_R$ (H)

In this section, we obtain a subcategory **IRel**$_R$ (H) of **IRel** (H) which is a topological universe over **Set**. It is very interesting that final structures and exponential objects in **IRel**$_R$ (H) are shown to be quite different from those in **IRel** (H).

Definition 2.1[4]. An IHFR R on a set X is said to be *reflexive* if $\mu_R(x,x)=1$ and $\nu_R(x,x)=0$ for each $x \in X$.

The class of all intuitionistic H-fuzzy reflexive relational spaces and **IRel** (H)-mappings between them form a subcategory of **IRel** (H) and denoted by **IRel**$_R$ (H).

It is clear that **IRel**$_R$ (H) is a full and isomorphism-closed subcategory of **IRel** (H).

We can easily obtain the following.

Proposition 2.2. **IRel**$_R$ (H) is properly fibred over **Set**.

Lemma 2.3. **IRel**$_R$ (H) is closed under the formation of initial sources in **IRel** (H).

Theorem 2.4. (1) $\mathbf{IRel_R}$ (H) is a bireflexive subcategory of **IRel** (H).

(2) $\mathbf{IRel_R}$ (H) is topological over **Set**.

Theorem 2.5. $\mathbf{IRel_R}$ (H) has final structures over **Set**.

Theorem 2.6. Final episinks in $\mathbf{IRel_R}$ (H) are preserved by pullbacks.

Theorem 2.7. $\mathbf{IRel_R}$ (H) is a topological universe over **Set**. Hence $\mathbf{IRel_R}$ (H) is a concrete quasitopos in the sense of E.J.Dubuc [11].

Theorem 2.8. $\mathbf{IRel_R}$ (H) has exponential objects. Hence $\mathbf{IRel_R}$ (H) is cartesian closed over **Set**.

Remark 2.9. (1) In [12], Y.Noh obtained exponential objects in $\mathbf{Rel_R}$ (I), where $I=[0,1]$. In Theorem 2.8, we showed that the construction of an exponential object in $\mathbf{Rel_R}$ (I) is applicable to the case of $\mathbf{IRel_R}$ (H).

(2) We note that exponential objects in $\mathbf{IRel_R}$ (H) are quite different from those in **IRel** (H) constructed in Theorem 3.9.

(3) $\mathbf{IRel_R}$ (H) has no subobject classifier.

Example 2.10. Let $H=\{0,1\}$ be the two points chain and let $X=\{a,b\}$. Let R_1 and R_2 be the intuitionistic H-fuzzy reflexive relations on X given by :

$$\mu_{R_1}(a,a)=\mu_{R_1}(b,b)=1,$$

$$\mu_{R_1}(a,b)=\mu_{R_1}(b,a)=0;$$

$$\nu_{R_1}(a,a)=\nu_{R_1}(b,b)=0,$$

$$\nu_{R_1}(a,b)=\nu_{R_1}(b,a)=1,$$

$$\mu_{R_2}(a,a)=\mu_{R_2}(b,b)=1,$$

$$\mu_{R_2}(a,b)=\mu_{R_2}(b,a)=0;$$

$$\nu_{R_2}(a,a)=\nu_{R_2}(b,b)=0,$$

$$\nu_{R_2}(a,b)=\nu_{R_2}(b,a)=1.$$

Let $1_X: (X,R_1)\rightarrow(X,R_2)$ be the identity mapping. Then clearly 1_X is both a monomorphism and an epimorphism in $\mathbf{IRel_R}$ (H). But 1_X is not an isomorphism in $\mathbf{IRel_R}$ (H). Hence $\mathbf{IRel_R}$ (H) has no subobject classifier (See [5]).

References

[1] G.Birkhoff, Lattice Theory Space(A.M.S. Colloquium Publication Vol XXV, 1967).

[2] J.C.Carrega, The category **Set**(H) and **Fuz**(H), Fuzzy Sets and Systems 9(1983) 327-332.

[3] E.J.Dubuc, Concrete quasitopoi, Applications of Sheaves. Proc. Dunham 1977, Lect. Notes in Math. 753(1979) 239-254.

[4] H.Herrlich, Cartesian closed topological categories, Math.Coll.Univ.Cape Town 9 (1974) 1-16.

[5] H.Herrlich and G.E.Strecker, Category Theory space(Allyn and Bacon, Newton, Ma, 1973).

[6] K.Hur, H-fuzzy relation (II) : A topological universe viewpoint, Fuzzy Set and Systems 63(1994) 73-79.

[7]______, Intuitionistic H-fuzzy relations, To appear.

[8] ______, Intuitionistic H-fuzzy sets, To

appear.

[9] P.T.Johnstone, Stone spaces, Cambridge University Press(1982).

[10] C.Y. Kim, S.S.Hong, Y.H.Hong and P.H. Park, Algebras in Cartesian closed topological categories, Lecture Note Series 1, 26(1985) 273-309.

[11] L.D.Nel, Topological universes and smooth Gelfand-Naimark duality, mathematical applications of category theory, Proc.A.M.S.Spec. Session Denver, 1983,Contemporary Mathematics 30(1984) 224-276.

[12] Y.Noh, Categorical aspects of fuzzy relations, M.A. Thesis, Yon Sei university

[13.] D.Ponasse, Some remarks on the category **Fuz**(H) of M.Eytan, Fuzzy Sets and Systems 9(1983) 199-204.