직관적 H-퍼지 관계

Intuitionistic H-Fuzzy Relations

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Abstract

We introduce the category **IRel** (H) consisting of intuitionistic fuzzy relational spaces on sets and we study structures of the category **IRel** (H) in the viewpoint of the topological universe introduced by L.D.Nel. Thus we show that **IRel** (H) satisfies all the conditions of a topological universe over **Set** except the terminal separator property and **IRel** (H) is cartesian closed over **Set**.

Key words and phrases : intuitionistic H-fuzzy relation, (co)topological category, cartesian closed category, topological universe over **Set**.

0. Introduction

L.A.Zadeh[17] introduced a concept of a fuzzy relation naturally, as a generalization of crisp relation in fuzzy set theory. L.D.Nel[14] introduced the notion of a topological universe which implies concrete quasitopos[1]. Every topological universe satisfies all the properties of a topos except one condition of the sub-object classifier. The notion of a topological universe has already been put to effective use in several areas of mathematics[12,13,15]. U.Cerruti[5] and K.Hur[9] investigated the category **Rel** (H) of the fuzzy relational spaces. In particular, K. Hur studied the category **Rel** (H)in the sense of a topological universe.

In 1986, K.T. Atanassov introduced the concept of an intuitionistic fuzzy set. In this paper, we introduce the category **IRel** (H) consisting of intuitionistic H-fuzzy relational spaces and study the category **IRel** (H) in a topological universe viewpoint. In particular, we show that **IRel** (H) satisfies all the conditions of a topological universe over **Set** except the terminal separator property. And **IRel** (H) is shown to be cartesian closed over **Set**. For general categorical background we refer to H.Herrlich and G.E.Strecker[8,11].

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1. Preliminaries

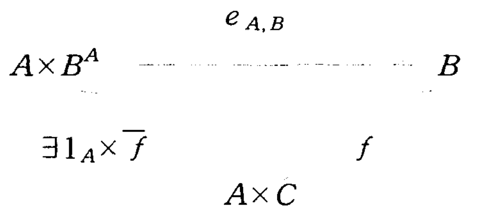
In this section, we will introduce some basic definitions and well-known results from

[7,11,14,16] which are needed in the next section.

Definition 1.1[7]. A category {\bf A} is called *cartesian closed* provided that the following conditions hold :

(1) For any **A**-objects A and B, there exists a product $A \times B$ in **A**.

(2) Exponential exist in **A**, i.e., for any **A**-object A, the functor $A \times - : \mathbf{A} \to \mathbf{A}$ has a right adjoint, i.e., for any **A**-object B, there exists an **A**-object B^A and a **A**-morphism $e_{A,B}: A \times B^A \to B$(called the *evaluation*) such that for any **A**-object C and any **A**-morphism $f: A \times C \to B$, there exists a unique **A**-morphism $\overline{f}: C \to B^A$ such that the diagram



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Definition 1.2[14]. A category **A** is called a *topological universe over* **Set** provided that the following conditions hold :

(1) **A** is well-structured over **Set**, i.e.,

(i) **A** is a concrete category ;

(ii) **A** has the fibre-smallness condition;

(iii) **A** has the terminal separator property.

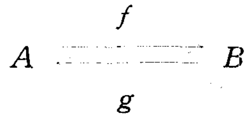
(2) **A** is cotopological over **Set**.

(3) Final epi-sinks in **A** are preserved by pullbacks, i.e.,for any final episink $(g_\lambda : X \to Y)_\Lambda$ and any **A**-morphism $f: W \to Y$, the family $(e_\lambda: U_\lambda \to W)_\Lambda$, obtained by taking the pullback of f and g_λ for each λ, is again a final episink.

Definition 1.3[16]. A category **A** is called a *topos* provided that the following conditions hold :

(1) There is a terminal object U in **A**, i.e., for each **A**-object A, there exists one and only one **A**-morphism from A to U.

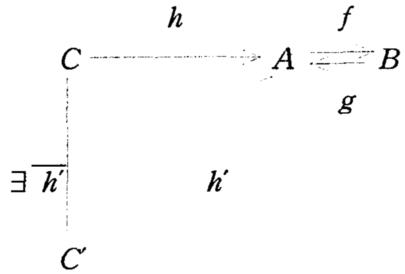
(2) **A** has equalizers i.e., for any **A**-objects A and B and **A**-morphisms



there exist an **A**-object C and an **A**-morphism $h: C \to A$ such that :

(a) $f \circ h = g \circ h$,

(b) for each **A**-object C' and **A**-morphism $h': C' \to A$ with $f \circ h' = g \circ h'$, there exists a unique **A**-morphism $\overline{h'}: C' \to C$ such that $h' = h \circ \overline{h'}$, i.e., the diagram



commutes.

(3) **A** is cartesian closed.

(4) There is a subobject classifier in **A**, i.e., there is an **A**-object Ω and **A**-morphism $v: U \to \Omega$ such that for each **A**-monomorphism $m: A' \to A$, there exists a unique **A**-morphism $\phi_m: A \to \Omega$ such that the following diagram is a pullback :

Remark. Let **A** be any category with a subobject classifier. If f is any biomorphism in **A**, then f is an isomorphism in **A**(cf. [4]).

Definition 1.4[3,10]. A lattice H is called a *complete Heyting algebra*, if H satisfies the following conditions hold :

(1) H is a complete lattice.

(2) For any $a, b \in H$, the set $\{x \in H: x \wedge a \leq b\}$ has a greatest element denoted by $a \to b$(called *pseudo-complement of a and b*), i.e., $x \wedge a \leq b$ if and only if $x \leq (a \to b)$.

In particular, for each $a \in H$, $N(a) = a \to o$ is called the *negation* or the *pseudocomplement* of a.

Result 1.C[3]. Let H be a complete Heyting algebra and let $a, b \in H$. Then :

(1) If $a \le b$, then $N(b) \le N(a)$, i.e., $N: H \to H$ is an involutive order reversing operation in $(H, \le)$.

(2) $a \le NN(a)$.

(3) $N(a) = NNN(a)$.

(4) $N(a \vee b) = N(a) \wedge N(b)$ and
$N(a \wedge b) = N(a) \wedge N(b)$.

Throughout this paper, we use H as a complete Heyting algebra.

Definition 1.5[9]. (1) Let X be a set. R is called an *H-fuzzy relation*}(or simply, a *fuzzy relation*) on X if $\mu_R: X \times X \to H$ is a map.

In this case, (X, R) is called an *H-fuzzy relational space*(or simply, a *fuzzy relational space*).

(2) Let (X, R_X) and (Y, R_Y) be any fuzzy relational spaces. A map $f: X \to Y$ is called a *relation preserving map* provided that $\mu_R \le \mu_R \circ f^2$, where $f^2 = f \times f$.

From the above definitions, we can form a concrete category **Rel** (H) consisting of all relational spaces and relation preserving maps between them. Every **Rel** (H)-morphism will be called a **Rel** (H)-*map*.

2. The Category IRel (H)

In this section, we introduce the category **IRel** (H) of intuitionistic H-fuzzy relational spaces and show that it has similar structures as those of **ISet** (H).

Definition 2.1. Let X be a set. A pair $R = (\mu_{R,} \nu_R)$ is called an *intuitionistic H-fuzzy relation*(in shot, *IHFR*) on X if it satisfies the following conditions :

(i) $\mu_R: X \times X \to H$ and $\nu_R: X \times X \to H$ are mappings, where μ_R and ν_R denote the degree of membership(namely $\mu_R(x, y)$) and the degree of nonmembership (namely $\nu_R(x, y)$) of each $(x, y) \in X \times X$ to R.

(ii) $\mu_R \le N(\nu_R)$, i.e., $\mu_R(x, y) \le N(\nu_R(x, y))$ for each $(x, y) \in X \times X$.

In this case, (X, R) or (X, μ_R, ν_R) is called an *intuitionistic H-fuzzy relatinal space* (in short, *IHFRS*).

Definition 2.2. Let (X, R_X) and (Y, R_Y) be an IHFRSs. A mapping $f: X \to Y$ is called a *relation preserving mapping* if $\mu_{R_X} \le \mu_{R_Y} \circ f^2$ and $\nu_{R_X} \ge \nu_{R_Y} \circ f^2$, where $f^2 = f \times f$.

The following are the immediate results of Definition 2.2 :

Proposition 2.3. Let (X, R_X), (Y, R_Y) and (Z, R_Z) be IHFRSs .

(1) $1_X: (X, R_X) \to (X, R_X)$ is a relation preserving mapping.

(2) If $f: (X, R_X) \to (Y, R_Y)$ and $g: (Y, R_Y) \to (Z, R_Z)$ are relation preserving mappings, then $g \circ f: (X, R_X) \to (Z, R_Z)$ is a relation preserving mapping.

From the above definitions and Proposition2.3, we can form a concrete category **IRel** (H) consisting of all IHFRSs and relation preserving mappings between them. Every **IRel** (H)-morphism will be called an ***IRel*** (H)-*mapping*. Moreover, it is clear that if $f: (X, R_X) \to (Y, R_Y)$ is an **IRel** (H)-mapping, then $f: (X, \mu_{R_X}) \to (Y, \mu_{R_Y})$ is a **Rel** (H)-mapping.

Theorem 2.4. **IRel** (H) is topologi- cal over **Set**.

Proposition 2.5. **IRel** (H) is well-powered and co-well-powered.

Theorem 2.6. **IRel** (H) is cotopological over **Set**.

Theorem 2.7. Final episinks in **IRel** (H) are preserved by pullbacks.

For any singleton set $\{a\}$, since the IHFR R on $\{a\}$ is not unique, the category **IRel** (H) is not properly fibred over **Set**. Hence, by Theorem 2.6 and Theorem 2.7, we obtain the following result.

Theorem 2.8. **IRel** (H) satisfies all the conditions of a topological universe over **Set** except the terminal separator property.

Theorem 2.9. **IRel** (H) is cartesian closed over **Set**.

Remark 2.10. **IRel** (H) has no subobject classifier. Hence **IRel** (H) is not topos.

Example 2.11. Let $H = \{0, 1\}$ be the two points chain and let $X = \{a\}$. Let R_1 and R_2 be the IHFRs on X given by $\mu_{R_1}(a, a) = 0$, $\nu_{R_1}(a, a) = 1$ and $\mu_{R_2}(a, a) = 1$, $\nu_{R_2}(a, a) = 0$. Let $1_X: (X, R_1) \to (X, R_2)$ be the identity mapping. Then clearly, 1_X is both a monomorphism and an epimorphism in **IRel** (H). But, 1_X is not an isomorphism in **IRel** (H).

Hence **IRel** (H) has no subobject classifier (See [4]).

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