Robust Output-Tracking Control of Uncertain Takagi-Sugeno Fuzzy Systems

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ABSTRACT

A systematic output-tracking control design technique for robust control of Takagi-Sugeno (T-S) fuzzy systems with norm-bounded uncertainties is developed. The uncertain T-S fuzzy system is first represented as a set of uncertain local linear systems. The tracking problem is then converted into the stabilization problem for a set of uncertain local linear systems thereby leading to a more feasible controller design procedure. A sufficient condition for robust asymptotic output tracking is derived in terms of a set of linear matrix inequalities (LMIs). A stability condition on the traversing time-instances is also established. The output tracking control simulation for a flexible-joint robot-arm model is demonstrated, to convincingly show the effectiveness of the proposed system modeling and controller design method.

1. Introduction

Fuzzy control technology has become quite popular today in the control of nonlinear dynamical systems such as robotic systems [1] and even complex chaotic systems [2, 3]. There have been many successful applications of fuzzy control in industry to date. As the first example, Mamdani and his colleagues suggested and applied a fuzzy system method to the control of a steam engine [4], where the applied control law consists of a set of linguistic control rules based on domain experts' knowledge about the physical plant.

For practical applications tracking control is as important as stabilization. Examples include robot tracking control, chemical process control, and aircraft attitude control. In general, it is believed that this kind of tracking control problems are more difficult than the stabilization problems.

Motivated by the above observations, this paper aims at studying the robust output-tracking control problem for a class of continuous-time T-S fuzzy systems in the presence of norm-bounded time-varying uncertainties. The uncertainties may be caused by inaccurate measurements of system parameters, or model variations due to aging of the systems, which degrade the output-tracking control performance and enfeeble the initial system stability. Knowing these, the so-called structured uncertainty issue in tracking control has been discussed in the literature [5]. This issue must also be carefully handled in all the T-S fuzzy-model-based systems for safety and for improved operational performance of the controlled systems.

2. T-S Fuzzy Systems

Consider the following uncertain T-S fuzzy systems

R^i : IF $x_1(t)$ is about Γ_1^i and $\cdots$ and $x_n(t)$ is about Γ_n^i

$$\text{THEN} \begin{cases} \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \\ y(t) = C_i x(t) \end{cases} \tag{1}$$

where ΔA_i and ΔB_i are real matrix functions with appropriate dimensions representing uncertainties. Using the center-average defuzzification, product inference, and singletone fuzzifier, the global dynamics of (1) is given by

$$\dot{x}(t) = \sum_{i=1}^{q} \theta_i(z(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)) \tag{2}$$

$$y(t) = \sum_{i=1}^{q} \theta_i(z(t))C_i x(t) \tag{3}$$

where $\omega_i(x(t)) = \prod_{h=1}^{n} \Gamma_h^i(x_h(t))\theta_i(z(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{q}\omega_i(x(t))}$ and $\Gamma_h^i(x_h(t))$ is the membership value of $x_h(t)$ in Γ_h^i.

Assumption 1 *The employed fuzzy sets are assumed to be consistent, normal with pseudo-trapezoid membership functions. In addition, they are assumed to be transparent [6].*

The global dynamical behavior of (2) is represented as a set of uncertain local linear systems, with the corresponding operating region defined as follows:

$$\Theta_i = \{x(t)|\theta_i(z(t)) \geq \theta_j(z(t))j \in \mathcal{I}_Q - \{i\}\}_{i\in\mathcal{I}_Q} \tag{4}$$

The boundary of Θ_i, denoted by Bdy Θ_i, that occurs the trajectory $x(t)$ passes from Θ_i to $\Theta_j, j \in \mathcal{I}_Q - \{i\}$, is defined by

$$\text{Bdy}\,\Theta_i = \{x(t) | x(t^-) \in \Theta_i, x(t^+) \in \Theta_j, j \in \mathcal{I}_Q - \{i\}\}_{i \in \mathcal{I}_Q} \quad (5)$$

where $(\cdot)(t^-)$ and $(\cdot)(t^+)$ denote $\lim_{\delta \to 0}(\cdot)(t-\delta)$ and $\lim_{\delta \to 0}(\cdot)(t+\delta)$, respectively. This partitioned sets Θ_i satisfies i) $\cup_{i=1}^q \Theta_i \supseteq U_x$; ii) $\Theta_i \cap (\cup_{j=1, j\neq i}^q \Theta_j) = \text{Bdy}\,\Theta_i$; iii) $\text{Int}\,\Theta_i \cap \text{Int}\,\Theta_j = \emptyset$ for any $i \in \mathcal{I}_Q, i \neq j$. Then, in the whole space of interest U_x, (2) can be represented as an uncertain linear system:

$$\dot{x}(t) = \sum_{i=1}^q \lambda_i(x(t))((A_i + \Delta_{ai})x(t) + (B_i + \Delta_{bi})u(t)) \quad (6)$$

where the characteristic function $\lambda_i(x(t))$ that activation of Θ_i is defined by

$$\lambda_i(x(t)) = \begin{cases} 1, & x(t) \in \Theta_i, \\ 0, & x(t^-) \in \text{Bdy}\,\Theta_i, x(t) \notin \Theta_i \end{cases} \quad (7)$$

and $\Delta_{ai} = \sum_{j=1}^q \theta_j(z(t))\Delta A_i + \sum_{j=1}^q \theta_j(z(t))\Delta A_{ij}, \Delta_{bi} = \sum_{j=1}^q \theta_j(z(t))\Delta B_i + \sum_{j=1}^q \theta_j(z(t))\Delta B_{ij}, \Delta A_{ij} = A_j - A_i, \Delta B_{ij} = B_j - B_i$. This representation of the uncertain T-S fuzzy system yields a set of independent uncertain local linear systems, suitable for controller synthesis.

Assumption 2 *The uncertainties considered here are norm-bounded of the form:*

$$\begin{bmatrix} \Delta A_i & \Delta B_i \end{bmatrix} = H_i F_i(t) \begin{bmatrix} E_{ai} & E_{bi} \end{bmatrix}$$

where $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $F_i(t)^T F_i(t) \leq I, \forall t \in [0, T_f)$ in which I is the identity matrix of appropriate dimension, and H_i, E_{ai}, and E_{bi} are known real constant matrices of appropriate dimensions that characterize how the uncertain elements affect the nominal matrices A_i and B_i.

Remark 1 *The matrices $\sum_{i=1}^q \theta_i(z(t))\Delta A_{ij}$ and $\sum_{i=1}^q \theta_i(z(t))\Delta B_{ij}$ in (13) represent the highly complex and nonlinear interactions among the subsystems of the overall T-S fuzzy system through the fuzzy inference rules. It also can be viewed as the model validity error of the governing system (A_i, B_i) on the operating region Θ_i. Since matrices ΔA_{ij} and ΔB_{ij} are known, hence can be decomposed, as is the case of ΔA_i and ΔB_i. Then one can see that these can be lumped with the parametric uncertainty, thus can be decomposed into the specific form $\begin{bmatrix} \Delta_{ai} & \Delta_{bi} \end{bmatrix} = \mathcal{H}_i \mathcal{F}_i(t) \begin{bmatrix} \mathcal{E}_{ai} & \mathcal{E}_{bi} \end{bmatrix}$, for all $t \in [0, \infty)$, where $\mathcal{F}_i(t)$ satisfies $\mathcal{F}_i(t)^T \mathcal{F}_i(t) \leq I$.*

3. Problem Statement

This section formulates the output tracking problem of the continuous-time T-S fuzzy system described previously.

Throughout this paper, the reference signal to be tracked by (3) is assumed to be the output $v(t) \in \mathbb{R}^p$ of the following exogenous signal system:

$$\dot{\zeta}(t) = \Psi \zeta(t) \quad (8)$$

$$v(t) = \Phi \zeta(t) \quad (9)$$

where $\zeta(t) \in \mathbb{R}^k$ is the state of the exogenous signal system.

Assumption 3 *To be practical, it is assumed that $\zeta(t)$ belongs to $\mathcal{L}_\infty[0, T_f)$, i.e., $\sup_{t \in [0, T_f)} \|\zeta(t)\| = \zeta_M$, where T_f is the terminal time of control.*

Definition 1 *Let $e(t)$ be the difference between output of a dynamical system and the reference signal. Given $\rho > 0$, the system of interest is said to be ρ-trackable if there is a control law $u(t)$ implying the existence of positive constants ν for all $\eta \in (0, \nu)$, and $T = T(\eta)$ independent of t_0 such that $\|e(t_0)\| < \eta \Rightarrow \|e(t)\| \leq \rho$ for all $t \geq t_0 + T$.*

Problem 1 (ρ-tracking controller design) *Let the output tracking error be $e(t) = y(t) - v(t)$. The objective is to design a T-S fuzzy-model-based controller such that (3) tracks (9) with the tracking error to be UUB, and the controlled system is robustly stable against the admissible uncertainty.*

In order to construct the error dynamics, a new state is defined as

$$\chi(t) := x(t) - T_i \zeta(t) \quad \text{for } \lambda_i(x(t)) = 1, i \in \mathcal{I}_Q \quad (10)$$

where T_i is a solution to the following matrix equations:

$$\begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix} \begin{bmatrix} T_i \\ L_i \end{bmatrix} = \begin{bmatrix} T_i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \Psi \\ \Phi \end{bmatrix}, i \in \mathcal{I}_Q. \quad (11)$$

For a detail discussion of the solution to (11), readers may refer [5].

Assumption 4 *Assume that*

$$\text{rank} \begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix} = n + l,$$

so that (11) is solvable [5]. It is satisfied if each nominal subsystem in (6) is controllable and the number of outputs is less than or equal to the number of the inputs, i.e., $m \geq p$.

In this study, the following output-tracking controller is employed:

$$u(t) = \sum_{i=1}^q \lambda_i(x(t))(L_i \zeta(t) + v(t)) \quad (12)$$

where $v(t)$ remains to be determined.

Remark 2 *It is noted that usage of* (10) *brings about finite impulses on* $\chi(t)$ *at the instances of beating the closure of the subspaces. Therefore, in the following discussion the dynamical behavior is described by the uncertain linear differential equations with finite jumps.*

After some algebraic manipulations using (11), $\chi(t)$ defined in (10) satisfies

$$\dot{\chi}(t) = \sum_{i=1}^{q} \lambda_i(x(t))((A_i + \Delta_{ai})\chi(t) + (B_i + \Delta_{bi})v(t) + \Delta_{di}\zeta(t)), \quad \text{for } x(t) \in \cup \operatorname{Int} \Theta_i \tag{13}$$

$$\chi(t^+) = I\chi(t) + \sum_{k=1}^{q} \theta_k(x(t))(B_k + \Delta B_k)v(t) + \Delta_{tij}\zeta(t), \quad \text{for } x(t) \in \operatorname{Bdy} \Theta_i, x(t^+) \in \Theta_j \tag{14}$$

$$e(t) = \sum_{i=1}^{q} \theta_i(z(t))C_i\chi(t) \tag{15}$$

where $\Delta_{di} = \Delta_{ai}T_i + \Delta_{bi}L_i$ and $\Delta_{tij} = T_i - T_j$.

If systems (13) and (14) are robustly globally stable, then $e(t)$ is UUB, which means that the γ-output tracking control of (2) is achieved. Therefore, Problem 1 can be reformulated as follows.

Problem 2 *Find a control law* $v(t)$ *for* (13) *such that the resulting closed-loop system is robustly UUB in the presence of the norm-bounded uncertainties and the disturbance which is belong to* $\mathcal{L}_\infty[0, T_f)$*. In this case the system* (13) *is said to be robustly stabilizable in the presence of structured uncertainties. Furthermore,* (2) *is said to be robustly* γ*-trackable.*

Remark 3 *Since* $\Delta_{di} = \Delta_{ai}T_i + \Delta_{bi}L_i$*, this uncertain matrix can also be decomposed as*

$$\Delta_{di} = \mathcal{H}_i\mathcal{F}_i(t)\mathcal{E}_{di}$$

for all $t \in [0, \infty)$*, where* $\mathcal{E}_{di} = \mathcal{E}_{ai}T_i + \mathcal{E}_{bi}L_i$.

4. Robust Output-Tracking Controller Design

As an additional control law, the following is introduced:

$$v(t) = \begin{cases} \sum_{i=1}^{q} \lambda_i(x(t))k_i\chi(t), & \text{for } x(t) \in \operatorname{Int} \Theta_i \\ k_{ij}\chi(t), & \text{for } x(t) \in \operatorname{Bdy} \Theta_i, x(t^+) \in \Theta_j \end{cases} \tag{16}$$

The main result on robust output-tracking, with the guaranteed cost for the continuous-time T-S fuzzy system with norm-bounded uncertainties, is now summarized in the following theorem.

Theorem 1 *If there exist some symmetric and positive definite matrices* P_i*, a symmetric and positive definite matrix* Q*, and constant matrices* k_i*, such that the following LMIs are satisfied, then the T-S fuzzy system* (1) *is globally stable, and the output tracking error* (15) *of the T-S system* (1) *is globally UUB*

$$\begin{bmatrix} \begin{pmatrix} W_iA_i^T + A_iW_i + M_i^TB_i^T \\ +B_iM_i + \gamma_iW_i + \epsilon_i\mathcal{H}_i\mathcal{H}_i^T \end{pmatrix} & (\bullet)^T \\ \mathcal{E}_{ai}W_i + \mathcal{E}_{bi}M_i & -\epsilon_i^{-1}I \end{bmatrix} < 0 \tag{17}$$

$$\begin{bmatrix} -W_j + \epsilon_{ij}H_iH_i^T & (\bullet)^T & (\bullet)^T \\ W_j + \frac{B_ik_{ij}+B_jk_{ij}}{2} & -W_i & (\bullet)^T \\ E_{bi}M_i & 0 & \epsilon_{ij}I \end{bmatrix} \leq 0, \quad i,j \in \mathcal{I}_Q, i \neq j \tag{18}$$

with $W_i = P_i^{-1}$*,* $M_i = k_iP_i^{-1}$*, and* $(\bullet)^T$ *denotes the transposed elements in the symmetric positions.*

Proof: The proof is omitted due to lack of space. ■

5. An Application

This section presents an illustrative example, to show the effectiveness of the proposed tracking controller design technique. More precisely, the output tracking problem for a flexible-joint robot arm model is considered. Now, an T-S fuzzy system illustrated in Fig. 1 can be modelled as follows:

$$R^1: \text{IF } x_1(t) \text{ is about } \Gamma_1^1 \text{ THEN } \begin{cases} \dot{x}(t) = A_1x(t) + B_1u(t) \\ y(t) = C_1x(t) \end{cases}$$

$$R^2: \text{IF } x_1(t) \text{ is about } \Gamma_1^2 \text{ THEN } \begin{cases} \dot{x}(t) = A_2x(t) + B_2u(t) \\ y(t) = C_2x(t) \end{cases}$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ (2,1)_i & 0 & \frac{k}{I} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J} & 0 & -\frac{k}{J} & 0 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J} \end{bmatrix}^T$$

where $(2,1)_1 = -\frac{Mgl+k}{I}$ and $(2,1)_2 = -\frac{\alpha Mgl+k}{I}$. Let the output matrices be $C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$, which satisfy Assumption 4. The membership functions are shown in Fig 2.

The simulation results are shown in Fig. 3: (a) illustrates the controlled output of the T-S fuzzy system (solid line) and the output of the exogenous system (dashed line) (b) depicts the output tracking error. For the purpose of a clearer comparison, the control input is activated at $t = 0.5$ sec. Before the control input is activated, the output of the system, $x_1(t)$ (solid line) does not follow the output of the exogenous signal system (dashed line). After $t = 0.5$ sec., the output of the controlled system is quickly guided to the output of the exogenous system. Indeed, from the the simulation results, one can see that the T-S fuzzy-model-based controller has a good tracking performance as well as a strong robustness against the admissible norm-bounded uncertainties.

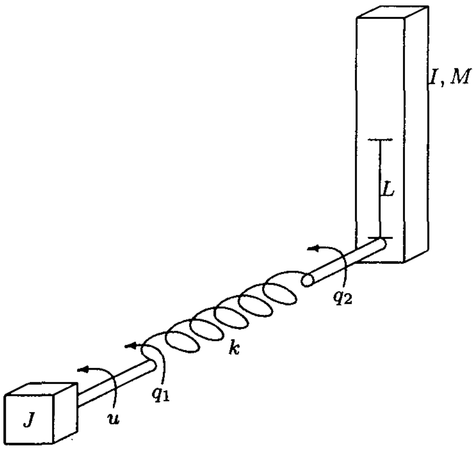

Figure 1: Illustration of the flexible-joint robot arm.

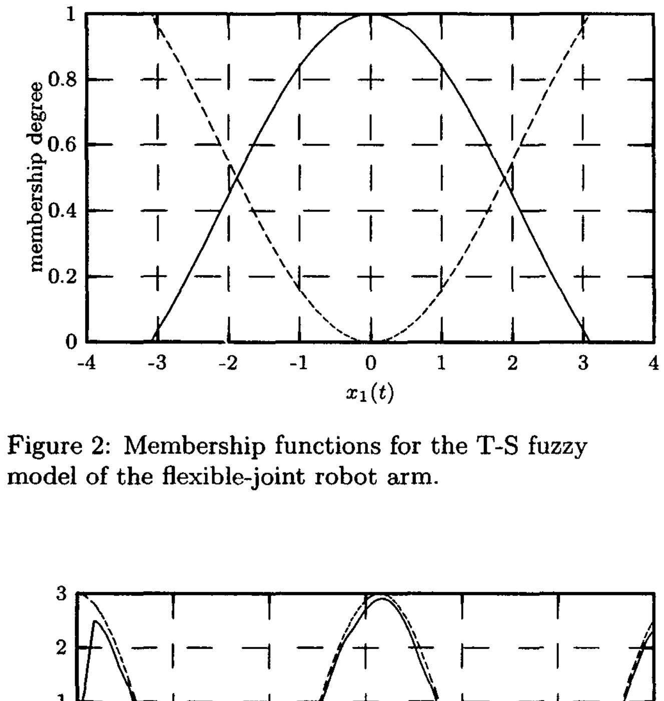

Figure 2: Membership functions for the T-S fuzzy model of the flexible-joint robot arm.

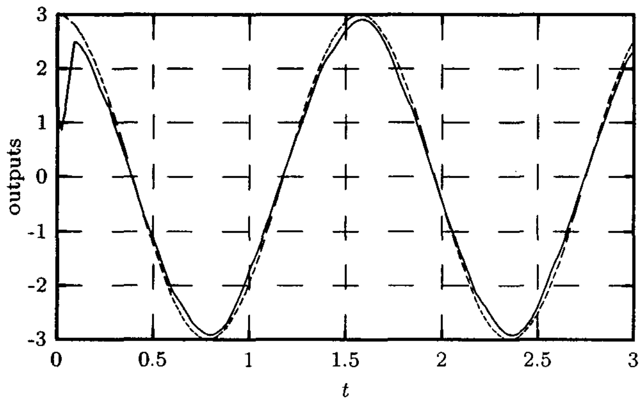

Figure 3: The controlled response of the flexible-joint robot arm in the presence of norm-bounded uncertainties: the output of the controlled system $y(t)$ (solid line) and the output of the exogenous system $v(t)$ (dashed line) in seconds (the total masses including the loads are varied within 200% of their nominal values).

6. Conclusion

In this paper, a new and systematic design procedure has been presented for robust output-tracking control for a continuous-time T-S fuzzy system in the presence of norm-bounded uncertainties. This design procedure provides a tracking performance with global UUB error against the significant but admissible norm-bounded and time-varying uncertainties. The output tracking problem of this set of local linear systems is then converted into the stabilization problem by using a simple affine transformation. Some sufficient condition for robust output tracking has been formulated in terms of LMIs.

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