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Fractal Scaling of Permeability in Unsaturated Fractured Tuff: Wavelet-Based Approach

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Abstract

Air permeabilities in unsaturated fractured tuff at the Apache Leap Research Site (ALRS) near Superior, Arizona, exhibit a self-affine behavior, thus renders a field random fractal. Based up fractal scaling, the observed scale effect has been interpreted [*Hyun et al.*, 2002]. Recently, *Frantziskonis and Hansen* [2000] presented that fractal scaling can be represented based on wavelets. This study deals with the way of using wavelets for fractal scaling. A numerical study is presented to examine the applicability of wavelet-based approach to determining upscaled air permeability values on various data supports at the site. To characterize the scaling property of self-affine fields generated based upon wavelets, Hurst coefficient, *H*. was inferred by applying the average wavelet coefficient (AWC) method. The result yielded $H = 0.24$, which is very close to the result of geostatistical analysis using a power variogram ($H = 0.22$). The study concludes that wavelet-based scaling is a useful way of determining parameter values on different data supports, which is an essential task for modeling of subsurface flow and mass transport in a numeric grid with different resolutions (grid size).

key word : permeability, wavelet, data support, scaling, fractured

1. Introduction

Numerous single-hole and cross-hole pneumatic injection tests have been conducted in unsaturated fractured tuff at the Apache Leap Research Site (ALRS) near Superior, Arizona. Analyses of these pneumatic test results have revealed a highly pronounced scale effect in permeability (and porosity) at the ALRS. The permeability scale effect has been interpreted on the basis of a recent stochastic scaling theory, which treats the rock as a truncated random fractal [Hyun et al., 2002].

Recently, *Frantziskonis and Hansen* [2000] proposed a new scaling theory based upon wavelets, and demonstrated it complements with scaling based on a truncated random fractal. Though many applications of wavelets are found to be very useful in geophysics and other earth sciences, little have been done in subsurface hydrology. This paper uses wavelets for investigating the scaling property and determining scaled aquifer parameters in a way of demonstrating the applicability of wavelets to subsurface hydrology studies.

2. Theory

The wavelet transform is an integral transformation whose basis functions are known as wavelets. The (continuous) wavelet transform of a function $f(x)$ is defined as

$$W_{a,b}(f(x)) = \int_{-\infty}^{\infty} f(x) \quad \Psi^{*}_{a,b}(x)\,dx \tag{1}$$

where

$$\Psi_{a,b}(x) = \frac{1}{\sqrt{a}}\,\Psi\left(\frac{x-b}{a}\right) \tag{2}$$

represents the two-parameter family of wavelet basis functions called wavelets [Daubechies, 1992]. Here $W_{a,b}(f)$ is a wavelet coefficient, a > 0 is a scale parameter, b $(-\infty < b < \infty)$ is a (space or time) translation parameter, and $\Psi^{*}_{a,b}(x)$ corresponds to the complex conjugate of $\Psi_{a,b}(x)$.

Consider a self-affine function $f(x)$, characterized by a power variogram, filterd by a lower cutoff, l > 0, $f_l(x)$, in one dimension. A lower cutoff corresponds to a support scale (data support). $f_l(x)$ can be obtained according to the inverse wavelet transform as [*Frantziskonis and Hansen*, 2000]

$$f_l(x) \overset{d}{=} \frac{1}{C_{\Psi}} \int_{l}^{\infty} \int_{-\infty}^{\infty} W_{a,b}(f(x)) \times \frac{1}{\sqrt{a}}\,\Psi\left(\frac{x-b}{a}\right) db\,\frac{da}{a^2} \tag{3}$$

Eq.(3) implies that one can estimate $f_l(x)$ for any l given $W_{a,b}(f)$.

3. Data Analysis and Numerical Study

3.1. Geostatistical Analysis

Sample variogram was obtained from 184 $\log_{10} k$ data obtained from single hole tests with 1-m interval. Figure 1 shows that the data are characterized by a power variogram, $\gamma = C_0 s^{2H}$ with C_o =0.27 and H=0.22, which renders an underlying field self-affine.

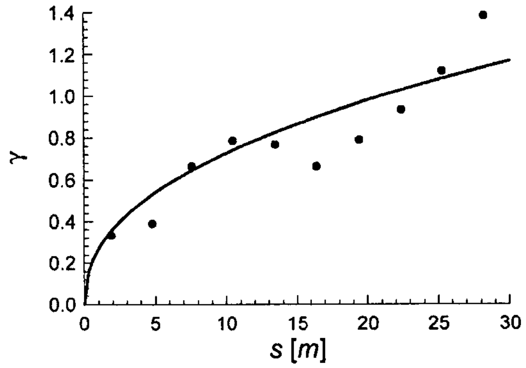

Figure 1. Omni-directional sample variogram and power model of 1-m scale single-hole log10 *k* data.

3.2. Numerical Study: Wavelet-based Scaling Analysis

For purposes of investigating fractal scaling of single-hole test data by using wavelets, two-dimensional $\log_{10}k$ random fields on a (600×600) grid were generated by using a power variogram with estimated parameters from the geostatistical analysis. Figure 2(a) shows an example of generated fields. This field has a resolution of (1×1), which is negligible compared to the domain size. Thus, this field is considered to have a zero data support. For nonzero data supports, I performed wavelet-based scaling upon (3), and obtained upscaled $\log_{10}k$ fields. For wavelet analysis, Daubechies 12 wavelets were used. The upscaled field for (4×4) support scale is presented in Figure 2(b).

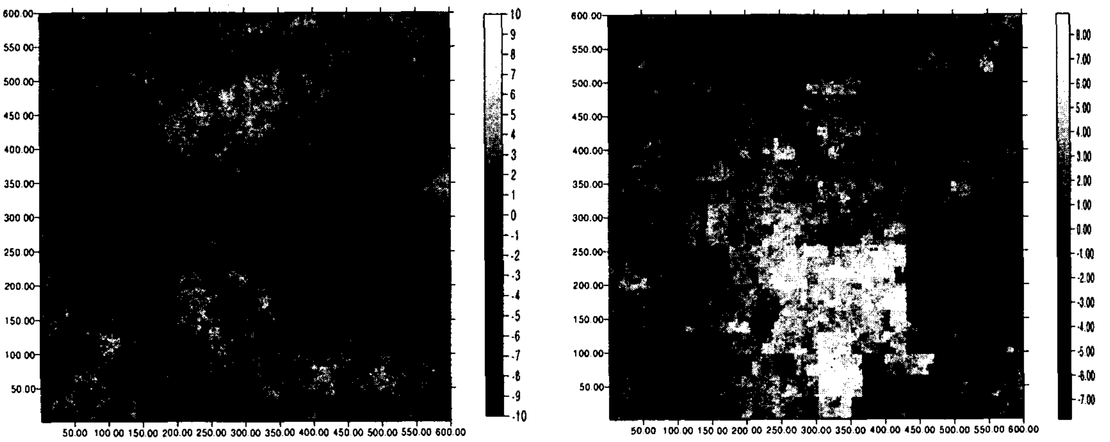

Figure 2. (a) Realization of a random field associated with a power variogram on a (600×600) grid with a resolution of (1×1); (b) Upscaled field defined on data support of size (4×4) by wavelet-based scaling.

To examine the scaling property between the fields (Figures 2(a) and 2(b)) Hurst coefficient was inferred by applying the average wavelet coefficient method, $W_{f_{rl}}(ra, rb) \overset{d}{=} r^{1+H} W_{f_l}(a, b)$ (for any scaling ratio, $r > 1$) for two dimensions. Average wavelet coefficients of Figures 2(a) and 2(b) were calculated and plotted in Figure 3.

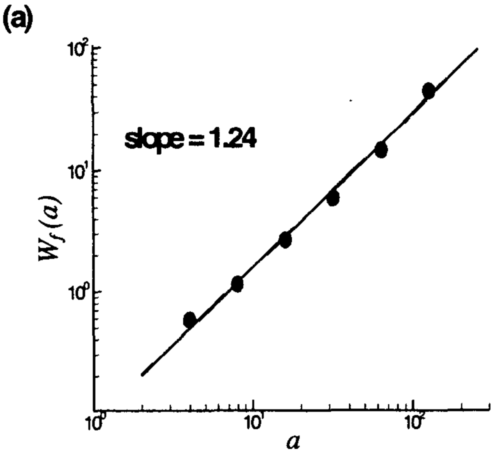

Figure 3. Estimation of Hurst coefficient by means of the averaged wavelet coefficient (AWC) method.

The result yields that H = 0.24, which is very close to H obtained in the geostatistical analysis.

4. Summary and Conclusions

Shown in a numerical study, wavelet-based fractal scaling enable us to determine the air permeability values on various data supports. The Hurst coefficient H obtained based on wavelet models (average wavelet coefficient method) is consistent with that obtained from variogram analysis with a power variogram. Due to *Simonsen and Hansen* [1998], this approach avoids the uncertainty caused by variogram analysis, which requires the significant number of data pairs. Therefore, we may conclude that wavelet-based approach is a promising way of fractal scaling. Although the applications of the wavelet-based model to subsurface physical properties are feasible in principle, they have not been yet performed. Thus, wavelet-based approaches are anticipated to be useful for further scaling studies.

5. References

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