H_2, H_∞, and mixed H_2/H_∞ FIR Filters for Discrete-time State Space Models

*Young Sam Lee, *Soo Yul Jung, *Joong Eon Seo,
°Soo Hee Han, and °Wook Hyun Kwon,

* Digital Media R&D Center, Samsung Electronics Co., Ltd.
° School of Electrical Eng. and Computer Science, Seoul National Univ.

Abstract:

In this paper, H_2, H_∞, and mixed H_2/H_∞ FIR filters are newly proposed for discrete-time state space signal models. The proposed filters require linearity, unbiased property, FIR structure, and independence of the initial state information in addition to the performance criteria in both H_2 and H_∞ sense. It is shown that H_2, H_∞, and mixed H_2/H_∞ FIR filter design problems can be converted into convex programming problems via linear matrix inequalities (LMIs) with a linear equality constraint. Simulation studies illustrat that the proposed FIR filter is more robust against uncertainties and has faster convergence than the conventional IIR filters.

초록

이 논문에서는 이산형 상태공간 모델에 대한 H_2, H_∞, 및 혼합 H_∞ FIR 필터를 선형행렬부등식(LMI)를 이용하여 제안한다. 제안되는 필터는 FIR 구조로서 H_2 및 H_∞ 관점에서의 성능기준을 만족함과 더불어 선형성 및 불편향성의 특성을 지니고, 초기 상태에 관한 정보를 필요로 하지 않는다. 그리고 FIR 구조로 인해 기존의 IIR 형태의 필터에 비해 불확실성에 대해 보다 견실하며 빠른 수렴성을 갖는다. 모의 실험을 통해 이러한 장점을 예시한다.

I. Introduction

The estimation problem deals with recovering some unknown parameters or variables from measured information in physical or mathematical models. Among estimation problems, the state estimator, called the filter, has been widely investigated for wide applications. The performance of the filter is measured by stability, small error, and insensitivity or robustness to signal model uncertainties and disturbances.

For a small error, it is usual to require the filter to be unbiased. For stochastic systems, an unbiased filter means that no matter what the real state is, the filter will follow it on the average. This also means that if there is no noise in the systems the filter will follow the real state exactly. In a similar way to the stochastic case, filters for deterministic systems can adopt the unbiased property in a deterministic sense. The unbiasedness for deterministic systems requires the filters to match exactly the real states of systems with zero disturbances. In short, "the unbiased property" will be used even for deterministic systems throughout this paper. The terminology "deadbeat" has also been used in other studies instead of "unbiased".

Some prefer finite impulse response (FIR) filters to infinite impulse response (IIR) filters for robustness and stability. FIR filters make use of a finite number of measurements and inputs on the most recent time interval $[k-N, k-1]$, called the receding horizon, or the moving window. It has been generally accepted that the FIR structure is more robust to temporary modeling uncertain parameters and numerical errors than the IIR structure. Additionally, bounded input bounded output (BIBO) stability is always guaranteed for FIR filters.

In conventional filters that estimate states, the initial state information is often assumed to be known even if the initial state is also a state to be estimated. This is not reasonable. Therefore, in this paper the initial state information is assumed to be completely unknown. That is, the suggested filters will be obtained independently of the initial state information.

Filter properties depend heavily on the performance criterion. In this paper, to obtain the optimal filter for state space models, two types of performance criterion are considered. In the H_2 performance criterion, the H_2 norm of the transfer function from the disturbance to the estimation error is minimized [1, 2, 3]. This approach has been widely used and researched because it is tractable mathematically. In the H_∞ performance criterion, the worst case gain between disturbance and estimation error is minimized [4, 5, 6, 7]. More recently, there have been approaches that consider both the performance criteria simultaneously [8].

Existing FIR filters are mainly focused on the minimum variance criterion that is a special case of the H_2 performance criterion [9, 10]. The H_∞ FIR filtering problem was first considered in [11]. The H_∞ FIR filter presented in [11] is obtained by repeatedly solving a finite horizon H_∞ filtering problem. However, in practice it neither guarantees the H_∞ norm bound nor has state independence. In this paper, among linear FIR filters with the unbiased property that is also independent of the initial state, an optimal filter will be chosen according to the H_2, H_∞ and H_2/H_∞ performance criteria.

The proposed H_2/H_∞ FIR filter is both unbiased and optimal *by design* for the given performance criterion. The '*by design*' means that the unbiased property and optimality are simultaneously built into the proposed FIR filter during its design. Actually, the unbiased property of the proposed FIR filter avoids the unnecessary large estimation error. The proposed FIR filter is represented as both a standard batch form and an iterative form.

II. Problem Statement

Consider the following linear discrete-time state space signal model

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Gw_k, \\ y_k &= Cx_k + Dw_k \end{aligned} \tag{1}$$

where $x_k \in \mathbf{R}^n$ is the state, $u_k \in \mathbf{R}^l$ is the input, $y_k \in \mathbf{R}^q$ is the measured output, and $w_k \in \mathbf{R}^p$ is the disturbance input (such as a disturbance, process/sensor noise). In the case of no disturbance input, the system (1) becomes

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k \end{aligned} . \tag{2}$$

The system in (2) will be called the nominal system.

Conventional filters of IIR structure are of the following form:

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + K(y_k - C\hat{x}_k), \tag{3}$$

where K is the filter gain matrix. Define $T_K(z)$, the transfer function from the disturbance input w to the estimation error e. Then, depending on estimation performance criterion, three filtering problems of the IIR type are formulated as follows:

- H_2 filtering problem : Find the filter (3) that minimizes $\|T_K(z)\|_2$.
- H_∞ filtering problem : Find the filter (3) that minimizes $\|T_K(z)\|_\infty$.
- Mixed H_2/H_∞ filtering problem : Find the filter (3) that minimizes $\|T_K(z)\|_\infty$ subject to $\|T_K(z)\|_2 < \beta$ (or minimizes $\|T_K(z)\|_2$ subject to $\|T_K(z)\|_\infty < \beta$).

In a similar fashion to the IIR case we can formulate three different FIR filtering problems depending on the performance criterion. The aim of this paper is to develop design methods for FIR filters with a batch form

$$\hat{x}_k = HY_{k-1} + LU_{k-1} \tag{4}$$

as solutions to those three FIR filtering problems.
H and L in (4) are the gain matrices of a linear filter represented by

$$\begin{aligned} H &\triangleq [H_N \ H_{N-1} \ \cdots \ H_1], \\ L &\triangleq [L_N \ L_{N-1} \ \cdots \ L_1]. \end{aligned}$$

U_{k-1} and Y_{k-1} are defined as

$$U_{k-1} \triangleq [u_{k-N}^T \ u_{k-N+1}^T \ \cdots \ u_{k-1}^T]^T, \tag{5}$$

$$Y_{k-1} \triangleq [y_{k-N}^T \ y_{k-N+1}^T \ \cdots \ y_{k-1}^T]^T. \tag{6}$$

u_{k-i} and y_{k-i}, where $i = 1, \cdots, N$, are the inputs and outputs, respectively, at time $k-i$. It is noted that the estimate $\hat{x}_k$ in (4) is a linear function of the finite number of inputs and measurements on the most recent time interval $[k-N, k-1]$, called the horizon. N, which is a positive integer, is a horizon length.
We require that the filter in (4) be independent of any *a priori* information about the horizon initial state, x_{k-N}, by making a filter of FIR structure. Furthermore, we require an unbiased property that the FIR filter in (4) satisfies the following relation for the nominal system (2):

$$\hat{x}_k = x_k \text{ for any } x_{k-N}. \tag{7}$$

To determine the constraint required for (7) to be satisfied, denote the measurements on the most recent time interval $[k-N, k-1]$ in terms of the state x_k at the current time k as

$$Y_{k-1} = \bar{C}_N x_k + \bar{B}_N U_{k-1} + (\bar{G}_N + \bar{D}_N) W_{k-1} \tag{8}$$

where

$$W_{k-1} \triangleq [w_{k-N}^T \ w_{k-N+1}^T \ \cdots \ w_{k-1}^T]^T. \tag{9}$$

$\bar{C}_N$, $\bar{B}_N$, $\bar{G}_N$, and $\bar{D}_N$ are constant matrices obtained either in a batch form or in a recursive form as

$$\bar{C}_i \triangleq \begin{bmatrix} CA^{-i} \\ CA^{-i+1} \\ CA^{-i+2} \\ \vdots \\ CA^{-1} \end{bmatrix},$$

$$\bar{B}_i \triangleq - \begin{bmatrix} CA^{-1}B & CA^{-2}B & \cdots & CA^{-i}B \\ 0 & CA^{-1}B & \cdots & CA^{-i+1}B \\ 0 & 0 & \cdots & CA^{-i+2}B \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & CA^{-1}B \end{bmatrix},$$

$$\bar{G}_i \triangleq - \begin{bmatrix} CA^{-1}G & CA^{-2}G & \cdots & CA^{-i}G \\ 0 & CA^{-1}G & \cdots & CA^{-i+1}G \\ 0 & 0 & \cdots & CA^{-i+2}G \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & CA^{-1}G \end{bmatrix},$$

$$\bar{D}_i \triangleq [\mathrm{diag}(\overbrace{D \ D \ \cdots \ D}^{i})],$$

where $1 \le i \le N$.
For a nominal system (2) we obtain, from (8),

$$\hat{x}_k = HY_{k-1} + LU_{k-1} = H\bar{C}_N x_k + H\bar{B}_N U_{k-1} + LU_{k-1}.$$

Therefore, the constraints on H and L required to satisfy (7) are given by

$$H\bar{C}_N = I, \quad H\bar{B}_N = -L. \tag{10}$$

From (10), we rewrite the FIR filter in (4) as

$$\hat{x}_k = H(Y_{k-1} - \bar{B}_N U_{k-1}), \quad H\bar{C}_N = I. \tag{11}$$

The constraint $H\bar{C}_N = I$ will be called the *unbiased constraint* in the sense that it is an unbiased constraint for the nominal system (2) with zero disturbance, but may not be an unbiased constraint for the system (1) with nonzero disturbance input.
Define $T_H(z)$ as the transfer function from the disturbance input w to the estimation error e of an FIR filter (11). Then we can formulate three FIR filtering problems as follows:

- H_2 FIR filtering problem : Find the filter (11) that minimizes $\|T_H(z)\|_2$.
- H_∞ FIR filtering problem : Find the filter (11) that minimizes $\|T_H(z)\|_\infty$.
- H_2/H_∞ FIR filtering problem : Find the filter (11) that minimizes $\|T_H(z)\|_\infty$ subject to $\|T_H(z)\|_2 < \beta$ (or minimizes $\|T_H(z)\|_2$ subject to $\|T_H(z)\|_\infty < \beta$).

In the next section, we present the formulation of the above FIR filtering problems in terms of LMIs.

III. H_2/H_∞ FIR Filtering via LMIs

3.1 Error Dynamics of FIR Filters

As a starting point we derive the transfer function $T_H(z)$. The disturbance input w_k satisfies the following state model on W_{k-1}

$$W_k = A_u W_{k-1} + B_u w_k, \tag{12}$$

where

$$A_u = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \quad B_u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I \end{bmatrix}.$$

It is noted that $A_u \in \mathbf{R}^{pN \times pN}$ and $B_u \in \mathbf{R}^{pN \times p}$. It follows from (8) that

$$Y_{k-1} - \bar{B}_N U_{k-1} - \bar{C}_N x_k = (\bar{G}_N + \bar{D}_N) W_{k-1}. \tag{13}$$

Pre-multiply (13) by H. From (11), we obtain

$$e_k = \hat{x}_k - x_k = H(\bar{G}_N + \bar{D}_N) W_{k-1}. \tag{14}$$

From (12) and (14), $T_H(z)$ is given by

$$T_H(z) = H(\bar{G}_N + \bar{D}_N)(zI - A_u)^{-1} B_u. \tag{15}$$

3.2 H_2 FIR Filtering

Given a system transfer function

$$G(z) \triangleq \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] = C(zI - A)^{-1}B,$$

it is well-known that $\|G(z)\|_2$ is given by

$$\|G(z)\|_2 = \sqrt{\mathrm{tr}(CPC^T)}, \tag{16}$$

where P is the controllability Grammian given by

$$P = \sum_{i=0}^{\infty} A^i BB^T (A^T)^i,$$

and obtained as the solution to the following Lyapunov equation

$$APA^T - P + BB^T = 0.$$

Therefore, we have the following theorem for the H_2 FIR filter:

Theorem 1. *Assume that the following LMI problem is feasible:*

$$\min_{F,W} \ \mathrm{tr}(W) \text{ subject to}$$

$$\begin{bmatrix} W & (FM + H_0)(\bar{G}_N + \bar{D}_N) \\ \star & I \end{bmatrix} > 0 \tag{17}$$

where $H_0 = (\bar{C}_N^T \bar{C}_N)^{-1} \bar{C}_N^T$ and M^T is the base of the null space of $\bar{C}_N^T$. Then the optimal gain matrix of the H_2 FIR filter of the form (11) is given by

$$H = FM + H_0.$$

Proof. The constraint $H\bar{C}_N = I$ is required for the FIR filter to be of the form (11). H_2 norm of the transfer function $T_H(z)$ in (15) is obtained by

$$\|T_H(z)\|_2^2 = \mathrm{tr}\big(H(\bar{G}_N + \bar{D}_N)P(\bar{G}_N + \bar{D}_N)^T H^T\big),$$

where

$$P = \sum_{i=0}^{\infty} A_u^i B_u B_u^T (A_u^T)^i.$$

Because $A_u^i = 0$ for $i \geq N$, we obtain

$$P = \sum_{i=0}^{\infty} A_u^i B_u B_u^T (A_u^T)^i = \sum_{i=0}^{N-1} A_u^i B_u B_u^T (A_u^T)^i = I.$$

Therefore

$$\|T_H(z)\|_2^2 = \mathrm{tr}\big(H(\bar{G}_N + \bar{D}_N)(\bar{G}_N + \bar{D}_N)^T H^T\big). \tag{18}$$

Introduce a matrix variable W such that

$$W > H(\bar{G}_N + \bar{D}_N)(\bar{G}_N + \bar{D}_N)^T H^T. \tag{19}$$

Then $\mathrm{tr}(W) > \|T_H(z)\|_2^2$. By the Schur complement, (19) is equivalent to

$$\begin{bmatrix} W & H(\bar{G}_N + \bar{D}_N) \\ (\bar{G}_N + \bar{D}_N)^T H^T & I \end{bmatrix} > 0. \tag{20}$$

Therefore, by minimizing $\mathrm{tr}(W)$ subject to the equality constraint $H\bar{C}_N = I$ and the above LMI, we obtain the optimal gain matrix H of the H_2 FIR filter. The equality constraint $H\bar{C}_N = I$ can be eliminated by computing the null space of $\bar{C}_N^T$. All solutions to the equality constraint $H\bar{C}_N = I$ are parameterized by

$$H = FM + H_0, \tag{21}$$

where F is a matrix containing the independent variables. Replacing H by $FM + H_0$, the LMI condition in (20) is changed into (17) This completes the proof. □

3.3 H_∞ FIR Filtering

For the system transfer function

$$G(z) \triangleq \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = C(zI - A)^{-1}A + D,$$

it is well known from the *bounded real lemma* that, given $\gamma > 0$, the following two conditions are equivalent:

(1) $\|G(z)\|_\infty < \gamma$.

(2) There exists an $X > 0$ such that

$$\begin{bmatrix} -X & XA & XB & 0 \\ A^T X & -X & 0 & C^T \\ B^T X & 0 & -\gamma I & D^T \\ 0 & C & D & -\gamma I \end{bmatrix} < 0.$$

From this, we obtain the following theorem for the optimal H_∞ FIR filter.

Theorem 2. *Assume that the following LMI problem is feasible:*

$$\min_{F,X} \ \gamma_\infty \text{ subject to}$$

$$\begin{bmatrix} -X & XA_u & XB_u & 0 \\ \star & -X & 0 & (\bar{G}_N + \bar{D}_N)^T (FM + H_0)^T \\ \star & \star & -\gamma_\infty I & 0 \\ \star & \star & \star & -\gamma_\infty I \end{bmatrix} < 0.$$

where $H_0 = (\bar{C}_N^T \bar{C}_N)^{-1} \bar{C}_N^T$ and M^T is the base of the null space of $\bar{C}_N^T$. Then, the optimal gain matrix of the H_∞ FIR filter of the form (11) is given by

$$H = FM + H_0.$$

Proof. From the bounded real lemma, the condition for $\|T_H(z)\|_\infty < \gamma_\infty$ is equivalent to the condition under which there exists $X > 0$ such that

$$\begin{bmatrix} -X & XA_u & XB_u & 0 \\ A_u{}^T X & -X & 0 & (\bar{G}_N + \bar{D}_N)^T H^T \\ B_u{}^T X & 0 & -\gamma_\infty I & 0 \\ 0 & H(\bar{G}_N + \bar{D}_N) & 0 & -\gamma_\infty I \end{bmatrix} < 0.$$

The equality constraint $H\bar{C}_N = I$ can be eliminated in exactly the same way as in H_2 FIR filter. This completes the proof. □

3.4 Mixed H_2/H_∞ FIR Filtering

From the previous two subsections, the formulation of the H_2/H_∞ FIR filtering problem via LMIs is obvious. Therefore, we obtain the following theorem for the H_2/H_∞ FIR filter:

Theorem 3. *Given $\alpha > 1$, assume that the following LMI problem is feasible:*

$$\min_{W,X,F} \ \gamma_\infty \quad \text{subject to}$$

$$\mathrm{tr}(W) < \alpha \gamma_2^*$$

$$\begin{bmatrix} W & (FM + H_0)(\bar{G}_N + \bar{D}_N) \\ \star & I \end{bmatrix} > 0,$$

$$\begin{bmatrix} -X & XA_u & XB_u & 0 \\ \star & -X & 0 & (\bar{G}_N + \bar{D}_N)^T (FH + H_0)^T \\ \star & \star & -\gamma_\infty I & 0 \\ \star & \star & \star & -\gamma_\infty I \end{bmatrix} < 0,$$

where $H_0 = (\bar{C}_N^T \bar{C}_N)^{-1} \bar{C}_N^T$ and M^T is the base of the null space of $\bar{C}_N^T$. Then, the gain matrix of the H_2/H_∞ FIR filter of the form (11) is given by

$$H = FM + H_0.$$

The above mixed H_2/H_∞ FIR filtering problem allows us to design the optimal FIR filter with respect to the H_∞ norm while assuring a prescribed performance level in the H_2 sense. By adjusting $\alpha > 0$, we can trade off the H_∞ performance against the H_2 performance.

Table 1: H_2 and H_∞ norm for $N = 10$ and $\alpha = 1.3$

	H_∞ norm	H_2 norm
H_∞ IIR filter	2.0009	2.0223
H_2 IIR filter	2.9043	1.8226
H_∞ FIR filter	4.2891	3.7295
mixed H_2 FIR filter	5.4287	2.7624
H_2/H_∞ FIR filter	4.4827	3.1497

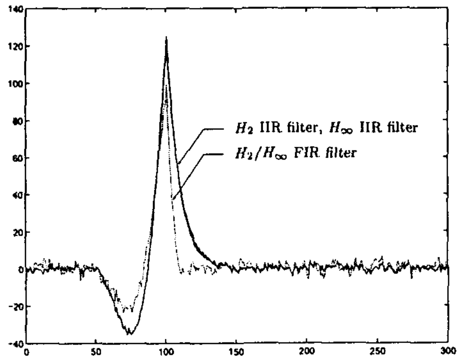

Figure 1: Estimation error in the state x_2

IV. Numerical Example

To illustrate the validity of the proposed FIR filter, numerical examples are given for a linear discrete-time invariant state space model from [9]

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} 0.9950 & 0.0998 \\ -0.0998 & 0.9950+\delta(k) \end{bmatrix} x_k + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} u_k \\ &\quad + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} w_k \\ y_k &= [1 \quad 0]x_k + [0 \quad 1]w_k \end{aligned}$$

where δ_k is an model uncertain parameter.

We have designed a mixed H_2/H_∞ filter with $N = 10$, $\alpha = 1.3$, and $\delta_k = 0$. Table 1 compares the H_2 and H_∞ norms of the conventional IIR filters and the proposed FIR filters. It is shown that the performances of the proposed H_2, H_∞, and mixed H_2/H_∞ FIR filter are worse than those of conventional IIR filters. However, this is not necessarily true in real applications. As mentioned previously, the FIR filters are more robust against temporary modeling uncertainties because they utilize only finite measurements on the most recent horizon. To illustrate this feature and the fast convergence, we applied the mixed H_2/H_∞ FIR filter to a system that has temporary uncertainty. The uncertain parameter δ_k is assumed to be

$$\delta(k) = \begin{cases} 1, & 50 \le k \le 100 \\ 0, & \text{otherwise} \end{cases} .$$

Figure 1 compares the estimation errors where the disturbance input w_k is stochastic noise given by

$$w(k) = \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix}, \text{ where } w_1 \sim (0,1), w_2 \sim (0,1).$$

It is clearly shown that the proposed H_2/H_∞ FIR filter is more robust for the uncertainty and faster in convergence. Therefore, it is expected that the proposed FIR filter can be usefully used in real applications.

V. Conclusions

In this paper, H_2, H_∞, and mixed H_2/H_∞ filters are proposed for discrete-time state space signal models. The filtering problems are formulated in terms of linear matrix inequalities. The proposed filters have many desirable properties, that is, the filters are linear with the most recent finite measurements and inputs, do not require *a priori* information of the horizon initial state, and have the unbiased property for zero disturbance. Furthermore, due to the FIR structure, the proposed FIR filters are believed to be robust against temporary modeling uncertainties or numerical errors, while other IIR filters, such as Kalman filters and H_∞ filters, may show poor robustness in these cases. The proposed FIR filters will be useful for many signal processing problems where signals are represented by state space models.

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