Analytical Studies on Basic Creep of Concrete under Multiaxial Stresses

Kwon, Seung-Hee* Kim, Jin-Keun**

ABSTRACT

Creep Poisson's ratio reported by previous experimental studies on multiaxial creep of concrete was controversial. The Poisson's ratio is very sensitive to small experimental error that is inevitably induced, and the sensitivity may cause the controversy. It is difficulty to find out the properties on multiaxial creep of concrete. Therefore, a new approach method to analyze the test results is needed to precisely understand the properties on multiaxial creep of concrete.

In this study, microplane model is used as a new approach method in analyzing the multiaxial creep test data. The six data sets extracted from the literature are fitted from regression analysis. Double-power law as a model representing volumetric and deviatoric creep evolutions on microplane is used, and six parameters in volumetric and deviatoric compliances are determined on the assumption that the volumetric and deviatoric creep strains are linearly proportional to corresponding stresses. The optimum fits give very accurate description of the test data. The Poisson's ratio calculated from the optimum fits varies with time and does not depends on the stress states, namely, uniaxial, biaxial, and triaxial stress states. Regression analysis is also performed on the assumption that the Poisson's ratio remains constant with time. The constant Poisson's ratio can be use in practice without serious error.

1. Introduction

Poisson's ratio due to multiaxial creep reported by some researchers[1-13] was controversial. In Jordaan and Illston's work, creep Poisson's ratio was reported to be similar to elastic Poisson's ratio and to have a different value with stress states, namely uniaxial, biaxial, and triaxial stress states.[4,5] In the study of Gopalakrishnan et al., effective creep Poisson's ratio for each direction differed according to the relative magnitude of stress applied to each direction in a multiaxial stress state.[2,3] Although creep Poisson's ratio varied with time in another study[13], determining whether it varies with time or remains constant is difficult in some studies[1-8] because when calculated from measured strains it is very sensitive to small experimental error and its value fluctuates with time. This discrepancy may be caused by experimental error or by the method of calculating Poisson's ratio. To precisely understand the properties of multiaxial creep of concrete, a new approach is needed.

In this study, the microplane model[14] is used as a new approach to analyzing six multiaxial creep tests. The six sets of data are extracted from the literature[1-4]. They were taken from the basic creep of six different concrete mixes tested under constantly sustained multiaxial stresses at room temperature. Double-power law[15] is used as a mathematical model to represent volumetric and deviatoric creep evolutions on a microplane.

* Dept. of Civil and Environmental Engineering at KAIST, PhD Candidate

** Dept. of Civil and Environmental Engineering at KAIST, Professor

The volumetric and deviatoric compliance functions on a microplane that optimally fits the test data are determined by regression analysis. The optimum fits give an accurate description of the test data. Some Reliable results on Poisson's ratio due to multiaxial creep of concrete are obtained from the regression analysis.

2. Microplane model analysis

The basic concept of microplane model is that the material properties are characterized by relations between the stress and strain components independently for planes of various orientations within the microstructure of the material. In this study, the basic concept of the model and the numerical algorithm are exploited in analyzing multiaxial creep of concrete.

Microplane is a plane of arbitrary orientation in the material and is characterized by the unit normal $\vec{n}$. The stress vector $\vec{\sigma}_N(t_0)$ on the microplane is the projection of the macroscopic stress tensor $\sigma(t_0)$ at the time of initial loading t_0.[14]

$$\vec{\sigma}_N(t_0) = \sigma(t_0)\,\vec{n} \tag{1}$$

The stress vector $\vec{\sigma}_N(t_0)$ can be decomposed by normal stress and tangential stresses on the microplane. When the macrostrain is increased due to creep, the strains on microplane are also increased with time. The normal strain on microplane at arbitrary time, t can be written as

$$\varepsilon_n(t) = \sigma_V(t_0)\,J_V(t, t_0) + \sigma_{Dn}(t_0)\,J_D(t, t_0) \tag{2}$$

where $J_V(t, t_0)$ is volumetric compliance, and $J_D(t, t_0)$ is deviatoric compliance, $\sigma_V(t_0)$ is volumetric component of stress applied at the time t_0, and $\sigma_{Dn}(t_0)$ is deviatoric component of normal stress. At the time t, the shear strain can be considered as two strains of two arbitrary orientations and the corresponding compliances are the same as deviatoric compliance $J_D(t, t_0)$ because shear strain is deviatoric component.

$$\varepsilon_m(t) = \sigma_{Dm}(t_0) J_D(t, t_0) \tag{3}$$

$$\sigma_l(t_0) = \sigma_{Dl}(t_0) J_D(t, t_0) \tag{4}$$

where, $\sigma_{Dm}(t_0)$ and $\sigma_{Dl}(t_0)$are the shear stresses in the directions, m and l, and both stresses are deviatoric components. At time t, the macrostrain can be obtained from the following integral.[14]

$$\varepsilon_{ij}(t) = \frac{3}{2\pi}\int_{\Omega} e_{ij}(t)\,d\Omega = 6\sum_{\mu=0}^{N_m} \omega_\mu e_{ij}(t)^\mu \tag{5}$$

where, $\varepsilon_{ij}(t)$ is the macrostrain in Cartesian coordinate and $e_{ij}(t)$ is a macrostrain transformed from microstrain on an arbitrary microplane. Ω means unit hemisphere and 2π is surface area of unit hemisphere. To calculate the integral over the unit hemisphere, Gaussian integral can be used. In right terms of Eq. (5), the subscript μ represents a chosen set of integration points and N_μ is the number of the integration points. ω_μ is the integration weight associated with microplanes. Bazant and Oh[16] have investigated the optimal numerical integration and twenty five microplanes on the surface of unit hemisphere are used for the integration in this research work. The compliance function generally consists of nonlinear function of time, and Marquardt-Levenberg method[17] known as best nonlinear regression algorithm is used to find the minimum value. Double-power law[15] which was known as very reasonable basic creep model is used as a model to represent volumetric and deviatoric creep evolutions on a microplane.

$$J_V(t,t_0)=\frac{1}{E_V}+\frac{1}{E_V}\phi_V(t,t_0),\quad \phi_V(t,t_0)=\phi_V(t-t_0)^{\alpha} \tag{6}$$

$$J_D(t,t_0)=\frac{1}{E_D}+\frac{1}{E_D}\phi_V(t,t_0),\quad \phi_D(t,t_0)=\phi_D(t-t_0)^{\beta} \tag{7}$$

where, $\phi_V(t,t_0)$ and $\phi_D(t,t_0)$ are volumetric and deviatoric creep coefficient, respectively, and E_V and E_D are volumetric and deviatoric moduli, respectively. In general, it can be assumed that creep strains are linearly related to applied stress up to about 50% of the concrete strength under uniaxial stress state. Also the linear relationships of volumetric and deviatoric components were observed from the experimental studies[1-4]. In Eqs. (2), (3) and (4), volumetric and deviatoric strains are linearly proportional to corresponding stresses, and this assumption on linearity of volumetric and deviatoric components is pertinent. The scope of this study is to investigate the properties of creep of sealed concrete under constantly sustained multiaxial stresses at room temperature. Multiaxial creep at elevated temperature and nonlinear creep effects such as adaptation, flow, and creep recovery are beyond the scope.

3. Microplane model analysis

3.1 Six parameter analysis

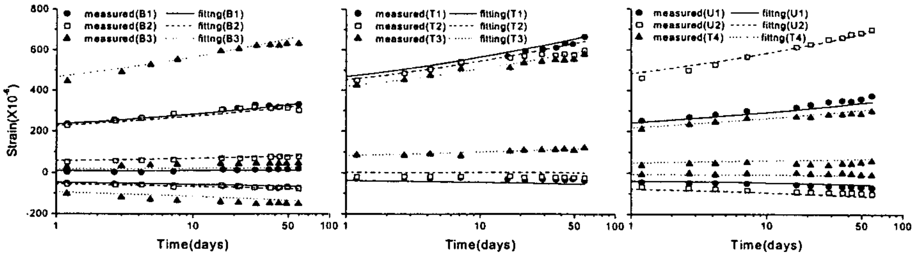
In Eqs. (6) and (7), six parameters, E_V, E_D, ϕ_V, ϕ_D, α, and β, optimally fitting the six data sets are listed in Table 1. Figure1 shows the test data and the optimum fits for the six parameter analysis. In all figures, it is seen that the regression results give very close description of test data. To estimate error between optimum fits and measured strains for six parameter analysis, variance, S^2 were calculated from Eq. (8), and shown in Table 1.

Table 1 Six parameters obtained from regression analysis

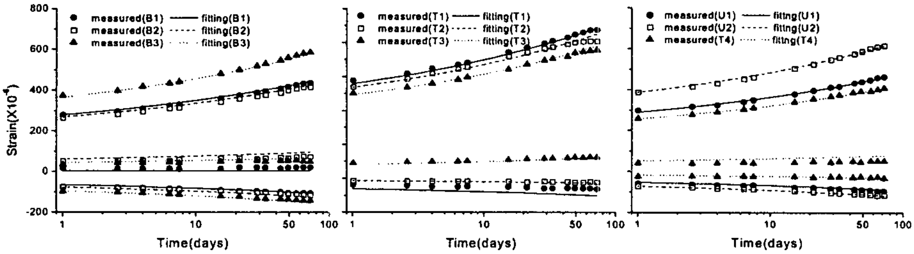
Data	E_V (MPa)	E_D (MPa)	ϕ_V	ϕ_D	α	β	S^2
1. Kim et al.[1] (Concrete CI)	64283.5	34508.9	1.135	0.974	0.141	0.154	140.31
2. Kim et al.[1] (Concrete CII)	70926.9	36072.2	0.745	0.690	0.199	0.210	82.84
3. Kim et al.[1] (Concrete CIII)	79832.2	38374.6	0.666	0.515	0.203	0.227	144.95
4. Gopalakrishnan et al.[2,3]	45740.8	25360.3	0.219	0.202	0.381	0.381	228.62
5. Jordaan and Illston[4] (Series 1)	51822.4	31833.5	0.333	0.313	0.196	0.228	176.82
6. Jordaan and Illston[4] (Series 2)	57828.1	33800.2	0.219	0.205	0.256	0.278	188.74

$$S^2=\frac{1}{n_i-1}\sum_{i=1}^{n}\Delta_i \tag{8}$$

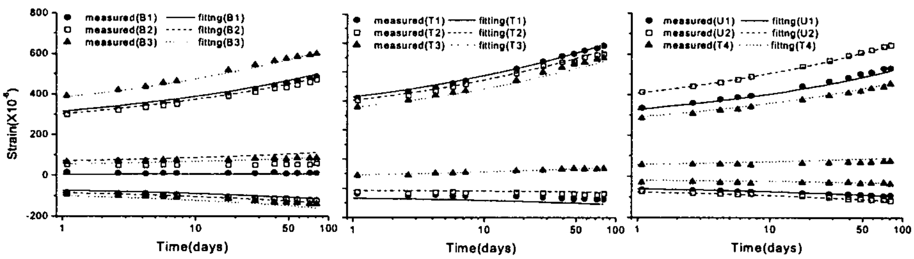
where, Δ_i is the square of the error between optimum fit and measured strain, and n_i is the number of data for i^{th} data set. The standard deviation of square root of variance is ranged from 9.1× 10-6 to 15.1× 10-6, and when inevitable experimental errors such as material variation, gage location, small variation of applied stress, temperature variation are considered, the amount of standard deviation indicates that the regression results very accurately fit the test data.



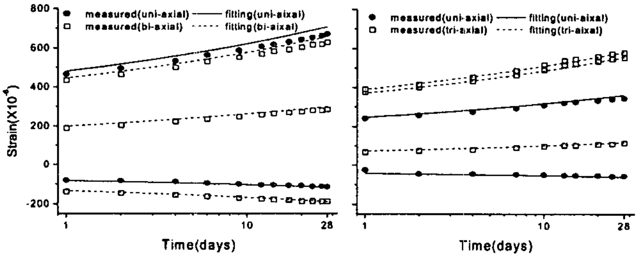
(a) Kim et al.[1](Concrete I)



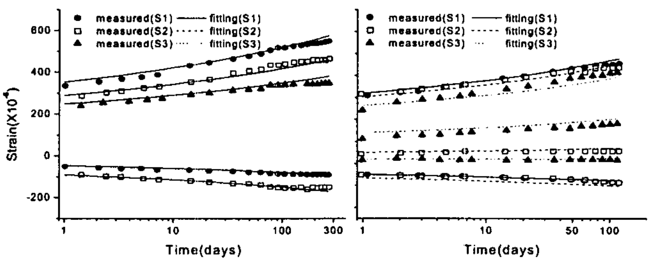
(b) Kim et al.[1](Concrete II)



(c) Kim et al.[1](Concrete III)



(d) Gopalakrishnan et al.[2,3](left : BC-8. right : TC-12)



(e) Jordaan and Illston[4](left : series 1, right : series 2)

Fig. 1 Multiaxial creep tests

3.1.1 Elastic strain and Poisson's ratio at loading

In creep tests, it is very difficult and ambiguous to distinguish between elastic strain and creep strain because significant creep exists even for extremely short load duration. The elastic or instantaneous deformation is generally determined by left horizontal asymptote in creep curves for various ages at loading.[18] A concrete age at loading was unique in the each of six data sets used in this study, and so the instantaneous strain, or the asymptotic strain is very difficult to determine from regression analysis. The time of first data point in time-axis was about 1 hour in the test data sets extracted from Kim et al.. In the case of Gopalakrishnan et al.'s and Jordaan and Illston's works, the strains measured immediately after loading was the first data point in time-axis and was located in the position, $t-t_0=0$. It is not realistic, but there is no difficulty in finding the best possible fits. The elastic modulus can be calculated from simple relationship between bulk and shear modulus in solid mechanics by Eq.(9).

$$E=\frac{3E_V E_d}{2E_V+E_D} \tag{9}$$

where, E is a elastic modulus. The elastic moduli for three concrete mixes of Kim et al. obtained from E_V, E_D listed in Table 1 are about 1.4 to 1.7 times the secant moduli that was experimentally determined at 28 days, and these value is close to generally expected asymptotic modulus, i.e., 1.5 times the secant modulus at 28 days. The elastic moduli calculated for Gopalakrishnan et al., and Jordaan and Illston are close to the secant moduli. The parameters, ϕ_V, and ϕ_D, for Gopalakrishnan et al. and Jordaan and Illston are very smaller that those for Kim et al. and the exponents, α and β, are some larger than those for Kim et al. This tendency is due to overestimation of instantaneous strains for Gopalakrishnan et al. and Jordaan and Illston.

If the value of instantaneous Poisson's ratio, ν_i which means the Poisson's ratio at instantaneous strain or the asymptotic strain is given, one of the two parameters, E_{V0} and E_{D0} can be cancelled out because the volumetric asymptotic modulus, E_{V0} is related to the deviatoric asymptotic modulus, E_{D0} as Eq. (10).

$$\frac{E_{Do}}{E_{V0}}=\frac{1-2\nu_i}{1+\nu_i} \tag{10}$$

However, the difficulty arises in finding accurate instantaneous Poisson's ratio as the difficulty of distinguishing between creep strain and elastic strain, and its value is not needed for static structural analysis. For this purpose, a Poisson's ratio calculated from the conventional elastic strain which corresponds to the secant modulus is more useful in practice, and this Poisson's ratio is defined as static Poisson's ratio in this study. The static Poisson's ratio at loading can be calculated in the location of the first data point in time-axis for each of six data sets using Eq. (11). The strains at the first data point are measured within two hours of load duration and the conventional elastic strains, or secant modulus corresponds to approximately the load duration.

$$\frac{\sigma_D\varepsilon_V}{\sigma_V\varepsilon_D}=\frac{1-2\nu_s}{1+\nu_s} \tag{11}$$

where, σ_V and ε_V are volumetric stress and strain, respectively, and σ_D and ε_D are deviatoric and deviatoric stress and strain, respectively. ν_s is the static Poisson's ratio calculated at the first data point. The static Poisson's ratios for the six data sets are obtained from regression results and measured strains, and are shown in Table 2. The static Poisson's ratios calculated from the optimum fits are very close to those from measured strains at loading. In general, it is obvious that the static Poisson's ratio calculated from optimum fit is more reasonable than that directly obtained from measured strain at one point in time-axis because its value calculated at one data point is more influenced by measurement error or scatter of data than the case of the optimum fit obtained from all the data in time-axis.

Table 2 Static Poisson's ratio

Data	Optimum fits (6 parameters)	Measured
1. Kim et al.[1] (Concrete CI)	0.163	0.168
2. Kim et al.[1] (Concrete CII)	0.188	0.188
3. Kim et al.[1] (Concrete CIII)	0.190	0.190
4. Gopalakrishnan et al.[2,3]	0.174	0.174
5. Jordaan and Illston[4] (Series 1)	0.147	0.147
6. Jordaan and Illston[4] (Series 2)	0.161	0.158

3.1.2 Poisson's ratio with time

The Poisson's ratio with time, ν_t, can be calculated from the optimum fits using Eq. (12).

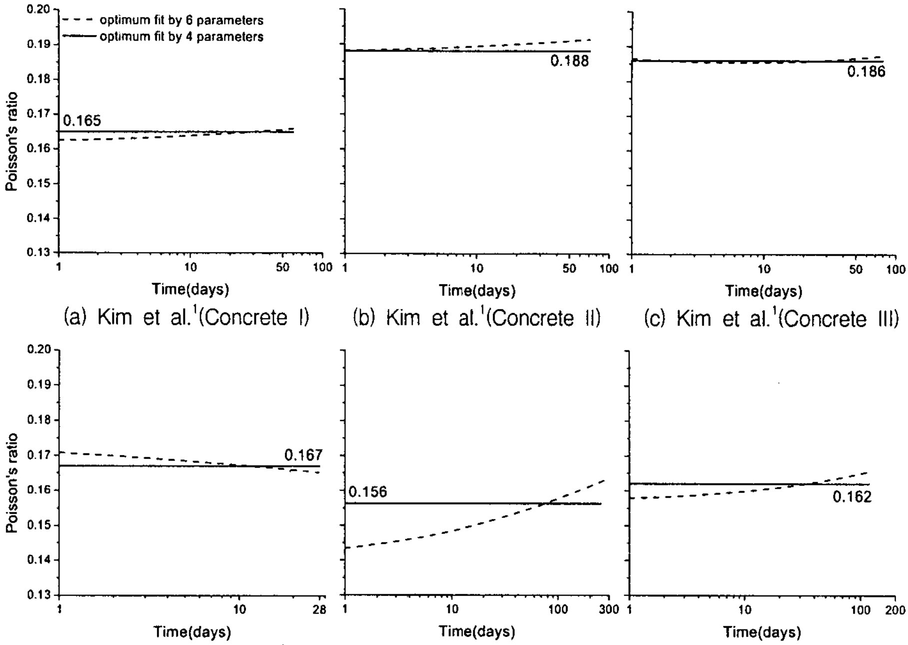
$$\nu_t = \frac{J_D(t, t_0) - J_V(t, t_o)}{J_V(t, t_0) + 2J_D(t, t_0)} \tag{12}$$

The Poisson's ratios are plotted in Figure 2, and are varying with time because the volumetric and deviatoric compliance functions are different to each other as shown in Table 1. As mentioned previously, there was the discrepancy on the tendency of Poisson's ratio with time or with stress states among the previous experimental works on multiaxial creep of concrete. In this study, it is assumed from experimental observation that volumetric and deviatoric compliances are linearly proportional to corresponding stresses. This assumption means that the Poisson's ratio does not depend on stress states, i.e. uniaxial, biaxial and triaxial stress states. The optimum fits using the assumption very accurately simulate the test data as shown in Figure 1. From the above results, it can be seen that Poisson's ratio has no specific relation with stress states and is varying with time.

However, when the Poisson's ratio varying with time is applied in typical structural analysis using conventional and macroscopic constitutive law, much difficulty arises, though there is no difficulty in structural analysis using microplane model as a constitutive law. Therefore, regression analysis is needed for the assumption that multiaxial creep of concrete is isotropic, that is, Poisson's ratio remain constant with time.

3.2 Four parameter analysis

The isotropic assumption The isotropic assumption means that the parameter, ϕ_V, equals to ϕ_D, and the



(a) Kim et al.[1](Concrete I) (b) Kim et al.[1](Concrete II) (c) Kim et al.[1](Concrete III)

(d) Gopalakrishnan et al.[2,3] (e) Jordaan and Illston[4](Series 1) (f) Jordaan and Illston[4](Series 2)

Fig. 2 Poisson's ratio calculated from regression results

parameter, α equals to β, that is, the volumetric and deviatoric creep coefficients must be identical. Regression analysis is carried out again for four parameters, E_V, E_D, $\phi_V = \phi_D$ and $\alpha = \beta$, and the results are listed in Table 3. The Poisson's ratios are calculated from optimum fits for four parameter analysis using Eq. (11), and are shown in Figure 2. In comparison with Table 1, the difference between both variances for six and four parameter analyses is so small. It can be seen that the constant Poisson's ratio over time from the optimum fit for four parameter analysis can be applied in practice without serious error.

Table 3 Four parameters obtained from regression analysis

Data	E_V (MPa)	E_D (MPa)	$\phi_V = \phi_D$	$\alpha = \beta$	S^2
1. Kim et al.[1] (Concrete CI)	60208.0	34643.4	0.985	0.152	140.42
2. Kim et al.[1] (Concrete CII)	70160.4	36809.0	0.727	0.203	82.93
3. Kim et al.[1] (Concrete CIII)	73339.0	38825.5	0.533	0.223	144.97
4. Gopalakrishnan et al.[2,3]	45046.8	25684.8	0.211	0.379	230.06
5. Jordaan and Illston[4] (Series 1)	52935.1	31469.2	0.331	0.209	189.58
6. Jordaan and Illston[4] (Series 2)	58272.3	33883.7	0.214	0.268	192.06

If the static Poisson's ratio is applied in structural analysis for long-period loading considering the multiaxial creep, then the amount of error may become larger because of the difference between the static Poisson's ratio listed in Table 3 and the constant Poisson's ratio calculated from the optimum fits for four parameter analysis. Consequently, the use of the constant Poisson's ratio obtained from the microplane model analysis is more desirable than the use of the static Poisson's ratio.

4. Conclusions

From the statistical regression analysis on multiaxial creep data, the following conclusions can be drawn.

(1) The Poisson's ratio calculated from best possible fits for the multiaxial creep test data is varying with time and does not depend on stress states.
(2) The static Poisson's ratio obtained from microplane model analysis is more reliable than that directly calculated from measured strains.
(3) The constant Poisson's ratio obtained from optimum fits for the isotropic assumption can be used in practice without serious error.

Acknowledgements

This research was supported by the Korea Science and Engineering Foundation, and the National Research Laboratory program from the Ministry of Science and Technology of Korea for the development of cracking control technique of concrete structure. These supports are deeply appreciated.

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