# 퍼지뉴럴 네트워크를 이용한 불확실한 비선형 시스템의 출력 피드백 강인 적응 제어

# Robust Adaptive Output Feedback Controller Using Fuzzy-Neural Networks for a Class of Uncertain Nonlinear Systems

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**Abstract** : In this paper, we address the robust adaptive backstepping controller using fuzzy neural network (FNN) for a class of uncertain output feedback nonlinear systems with disturbance. A new algorithm is proposed for estimation of unknown bounds and adaptive control of the uncertain nonlinear systems. The state estimation is solved using K-filters. All unknown nonlinear functions are approximated by FNN. The FNN weight adaptation rule is derived from Lyapunov stability analysis and guarantees that the adapted weight error and tracking error are bounded. The compensated controller is designed to compensate the FNN approximation error and external disturbance. Finally, simulation results show that the proposed controller can achieve favorable tracking performance and robustness with regard to unknown function and external disturbance.

**Keywords**: Nonlinear systems, Adaptive backstepping, Fuzzy neural networks

## Ⅰ. Introduction

The backstepping is one of the most important results, which provides a powerful design tool, for nonlinear (and linear) systems in the pure feedback and strict feedback forms[1][2].

Recently, active research has been carried out in fuzzy-neural control. It has been that fuzzy-neural network(FNN)[3] can approximate any nonlinear function to any desired accuracy because of the universal approximation theorem.

This paper deals with robust adaptive control design of nonlinear output feedback systems under bounded disturbance whose bounds are unknown. The compensated controller using FNN is designed to compensate the approximation error and external disturbance. The design procedure follows the standard backstepping with adaptive estimation strategy for the upper bound of disturbances. The new algorithm is a combination of adaptive backstepping and fuzzy-neural network based design techniques.

## 2.Problemstatement

We consider a SISO nonlinear systems which can be transformed into the output feedback form [4].

$$\begin{aligned} \dot{x} &= A_c x + b\sigma(y)u + \phi_0(y) + \sum_{i=1}^{m}\phi_i(y)a_i + \sum_{i=1}^{l}\psi_i(y)w_i \\ &= A_c x + b\sigma(y)u + \phi_0(y) + \Phi(y)a + \Psi(y)W \\ y &= e_1^T x \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R$ is the control, $y \in R$ is the output, $a = [a_1, \cdots\cdots, a_m]^T \in R^m$, $b = [b_1, \cdots\cdots, b_n]^T \in R^n$ are constant uncertain parameter vectors, $W = [w_1, \cdots\cdots, w_l]^T \in R^l$ is a bounded time-varying disturbance vector, $\phi_0$, $\phi_i$, $\psi_i$, $1 \le i \le l$, are smooth vector fields in $R^n$, $\sigma : R \to R$ is a smooth function, $\sigma(y) \ne 0$, $\forall y \in R$, $e_i$ denotes the $i$ column of the identity matrix $I$, and

$$A_c = \begin{bmatrix} 0 & & I_{n-1} \\ \vdots & & \\ 0 & \cdots & 0 \end{bmatrix}, \quad \phi_0 = \begin{bmatrix} \phi_{0,1} \\ \vdots \\ \phi_{0,n} \end{bmatrix}, \quad \Phi(y) = \begin{bmatrix} \phi_{1,1} & \cdots & \phi_{m,1} \\ \vdots & & \vdots \\ \phi_{1,n} & \cdots & \phi_{m,n} \end{bmatrix}.$$

**Assumption 1**: The polynomial $B(s) = \sum_{i=p}^{n} b_i s^{(n-1)}$ is Hurwitz, i.e., the system is of minimum phase.

**Assumption 2**: The sign of $b_p$ is known.

**Assumption 3**: The disturbance $W(t)$ is bounded, $W = W^* + \varepsilon_r$, where $W^*$ is optimal value and $\varepsilon_r$ is reconstruction error.

The T-S FNN[5] system includes a fuzzy rule base, which consists of a collection of fuzzy IF-THEN rule in the following form:

$$Rule\ i: IF\ z\ is\ F_z^i,\ THEN\ \hat{W}\ is\ \hat{V}^i$$

where $F_z^i$ and $\hat{V}^i$ are fuzzy sets and $\hat{W}(t;\hat{V})$ are output. The center of gravity method is used for the defuzzification

$$\hat{W}=\frac{\sum_{i=1}^{m}\hat{V}^i\eta^i(z)}{\sum_{i=1}^{m}\eta^i(z)}=\hat{V}\mathrm{H}(z)$$

where H(z) is fuzzy basis function vector.

## 3. Filters and control design

### 3.1 K-filters for state estimation

Fallowing the filter and observer design in [2], we have

$$\dot{\xi}=A_0\xi+ky+\phi_0(y), \quad (2)$$

$$\dot{\Omega}^T=A_0\Omega^T+F(y,u)^T, \quad (3)$$

where

$$F(y,u)^T=\left[\begin{bmatrix}0_{(\rho-1)\times(n-\rho+1)}\\ I_{n-\rho+1}\end{bmatrix}\sigma(y)u,\Phi(y)\right], \quad (4)$$

$$k=[k_1,\cdots\cdots,k_n]^T,\ A_0=A_c-ke_1^T \quad (5)$$

with $k$ chosen so that $A_0$ is Hurwitz.

We define the state estimate $\hat{x}=\xi+\Omega^T\theta$, $\theta=[b_\rho,\cdots,b_n,a_1,\cdots,a_m]^T\in R^{m+n-\rho+1}$.

To reduce the dynamic order of the $\Omega$-filter (3), $\Omega$ is generated by

$$\Omega^T=[v_\rho,\cdots,v_n,\Xi], \quad (6)$$

$$\dot{\Xi}=A_0\,\Xi+\Phi(y) \quad (7)$$

$$\dot{\lambda}=A_0\lambda+e_n\sigma(y)u, \quad (8)$$

$$v_j=A_i^{n-j}\lambda,\ \text{for}\ j=\rho,\cdots\cdots,n. \quad (9)$$

**Lemma 1:** From the filter (2) and (3), the state is given by

$$x=\xi+\Omega^T\theta+\varepsilon \quad (10)$$

with $\varepsilon\in R^n$, which satisfies

$$\dot{\varepsilon}=A_0\,\varepsilon+\Psi(y)W. \quad (11)$$

Furthermore, let $P\in R^{n\times n}$ be positive definite, satisfying

$$PA_0+A_0^TP=-6I \quad (12)$$

and let

$$V_\varepsilon=\frac{1}{2}\varepsilon^TP\varepsilon, \quad (13)$$

$$\Delta(y,y_r)=\frac{\Psi(y)-\Psi(y_r)}{y-y_r}, \quad (14)$$

it can be shown that

$$\dot{V}_\varepsilon\le-2\|\varepsilon\|^2+\frac{1}{4}z_1^4\|P\Delta\|^4+\frac{1}{4}\|W\|^4+\frac{1}{2}\|P\Psi(y_r)\|^2\|W\|^2 \quad (15)$$

**Proof:** A direct evaluation

$$\dot{\varepsilon}=\dot{x}-\dot{\Omega}^T\theta-\dot{\xi} \quad (16)$$

given (13). From (11) and (12), it is obtained that

$$\dot{V}_\varepsilon=\frac{1}{2}(\dot{\varepsilon}^TP\varepsilon+\varepsilon P\dot{\varepsilon}^T) \quad (17)$$

$$=-3\|\varepsilon\|^2+\varepsilon^TP\Psi(y)W$$

$$=-3\|\varepsilon\|^2+z_1\varepsilon^TP\Delta(y,y_r)W+\varepsilon^TP\Psi(y_r)W \quad (18)$$

$$\le-2\|\varepsilon\|^2+\frac{1}{2}z_1^2\|P\Delta(y,y_r)W\|^2+\frac{1}{2}\|P\Psi(y_r)\|^2\|W\|^2$$

$$\le-2\|\varepsilon\|^2+\frac{1}{4}z_1^4\|P\Delta(y,y_r)\|^4+\frac{1}{4}\|W\|^4+\frac{1}{2}\|P\Psi(y_r)\|^2\|W\|^2.$$

### 3.2 Adaptive control design

We start the robust adaptive control design with the K-filter in Section 3.1 and the assumptions introduced in Section 2. The approach to the control design is backstepping with tuning functions, which means that the design will start from the output dynamics and will complete in $\rho$ steps.

Let us define a number of notations:

$$z_1=y-y_r,\ z_{\rho+1}=0,\ z_i=v_{\rho,i}-\hat{\varsigma}\,y_r^{i-1}-\alpha_{i-1},\ i=2,\cdots\cdots,\rho,$$

$$\tilde{\varsigma}=\varsigma-\hat{\varsigma},\ \tilde{\theta}_i=\theta-\hat{\theta}_i,\ \tilde{W}=W^*-\hat{W},\ i=1,\cdots\cdots,\rho,$$

$$\omega=[v_{\rho,2},\cdots\cdots,v_{n,2},\Phi_{(1)}+\Xi_{(2)}]^T,\ \bar{\lambda}_i=[\lambda_1,\cdots\cdots,\lambda_i]^T$$

$$\bar{\omega}=[0,v_{\rho+1,2},\cdots\cdots,v_{n,2},\Phi_{(1)}+\Xi_{(2)}]^T,\ \bar{y}_i=[y_1,\dot{y}_r,\cdots\cdots,y_r^{(i)}]^T,$$

$$\mathrm{X}_i=[\xi^T,vec(\Xi)^T,\hat{\varsigma},\bar{\lambda}_i^T,\bar{y}_i^T]^T,\ \sigma_{j,i}=\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\Gamma\frac{\partial\alpha_{i-1}}{\partial y}\omega$$

where $\alpha_i$, $i=1,\cdots\cdots,\rho$, are stabilizing functions to be decided in the adaptive control design, $\varsigma=1/b_\rho$, $\hat{\rho}$ is an estimate of $\rho$, $\hat{\theta}_i$ are the estimate of $\theta$ the subscript $i$th row of a matrix, and $vec(\cdot)$ denotes a vector obtained by arranging the columns of a matrix in one column. We set $\alpha_1=\hat{\varsigma}\bar{\alpha}_1$.

**Stepl:** We start with the dynamics of the tracking error $z_1$. The derivative of $z_1$ is

$$\dot{z}_1=x_2+\phi_{0,1}(y)+\Psi_{(1)}(y)W-\dot{y}_r$$

$$=\omega_0+b_\rho v_{\rho,2}+\bar{\omega}^T\theta+\varepsilon_2+\Psi_{(1)}(y)W-\dot{y}_r \quad (19)$$

where $v_{\rho,2}=z_2+\hat{\varsigma}\dot{y}_r+\alpha_1$ and $\alpha_1=\hat{\varsigma}\bar{\alpha}_1$, $\varsigma=1/b_\rho$, we rewrite (19) in the form

$$\dot{z}_1=\bar{\alpha}_1+\omega_0+\bar{\omega}^T\theta+\varepsilon_2+\Psi_{(1)}(y)W-b_\rho(\dot{y}_r+\bar{\alpha}_1)\tilde{\varsigma}+b_\rho z_2 \quad (20)$$

Then our choice of the stabilizing function is

$$\bar{\alpha}_1=-c_1z_1-d_1z_1-\omega_0-\bar{\omega}^T\hat{\theta}-\Psi_{(1)}(y)\hat{W}-\sum_{i=1}^{\rho}\frac{z_1^3}{8d_i}\|P\Delta(y,y_r)\|^4 \quad (21)$$

where $\delta$ is constant design parameter.

Substituting (21) into (20) results in

$$\dot{z}_1=-c_1z_1-d_1z_1+\hat{b}_\rho z_2+\varepsilon_2+\Psi_{(1)}(y)\left[W-\hat{W}\right]+\left[\omega-\hat{\varsigma}(\dot{y}_r+\bar{\alpha}_1)e_1\right]^T\tilde{\theta}-b_\rho(y+\bar{\alpha}_1)\hat{\varsigma}-\sum_{i=1}^{\rho}\frac{z_1^3}{8d_i}\|P\,\Delta(y,y_r)\|^4. \quad (22)$$

Two positive constants $c_1$ and $d_1$ are introduced for uniformity with subsequent steps in which $d_i$ is a coefficient of a nonlinear damping term counter acting $\varepsilon_2$.

The choice of the update law for $\hat{\theta}$ is postponed until the last step and we only select the first tuning function

$$\tau_1 = \left[ \omega - \hat{\varsigma}(\dot{y}_r + \overline{\alpha}_1)\, e_1 \right] z_1 . \tag{23}$$

However, the update law for $\hat{\varsigma}$ is designed at the first step as

$$\dot{\hat{\varsigma}} = -\gamma\, sgn(b_\rho)(\dot{y}_r + \overline{\alpha}_1) z_1 ,\ \gamma > 0 . \tag{24}$$

**Step** $i$, $i = 2, \cdots\cdots, \rho$: It can be shown that

$$\alpha_{i-1} = \alpha_{i-1}(X_{i-1}, y, \hat{\theta}, \hat{V}),\ i = 2, \cdots\cdots, \rho \tag{25}$$

The stabilizing functions $\alpha_i$ for $i = 2, \cdots\cdots, \rho$, are designed as

$$\begin{aligned} \alpha_i = & -z_{i-1} - c_i z_i - d_i\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i + k_i v_{\rho,1} + \frac{\partial \alpha_{i-1}}{\partial X_{i-1}} \dot{X} \\ & + \frac{\partial \alpha_{i-1}}{\partial y}(\omega_0 + \omega^T \hat{\theta}) + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \frac{\partial \alpha_{i-1}}{\partial \hat{V}} \dot{\hat{V}} \\ & + \dot{\hat{\varsigma}}\, y_r^{(i-1)} + \frac{\partial \alpha_{i-1}}{\partial y} \Psi_{(1)}(y) \hat{W} \end{aligned} \tag{26}$$

It can be shown from (27) that, for $i = 2, \cdots\cdots, \rho$

$$\begin{aligned} \dot{z}_i = & -z_{i-1} - c_i z_i - d_i\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i + z_{i+1} - \frac{\partial \alpha_{i-1}}{\partial y} \omega^T \tilde{\theta} - \frac{\partial \alpha_{i-1}}{\partial y} \varepsilon_2 \\ & + \sum_{j=i+1}^{\rho} \sigma_{i,j} z_j - \frac{\partial \alpha_{i-1}}{\partial y} \Psi_{(1)}(y)\left[W - \hat{W}\right] - \sum_{j=2}^{i-1} \sigma_{j,i} z_j \end{aligned} . \tag{27}$$

The tuning functions are defined by

$$\tau_i = \tau_{i-1} - \frac{\partial \alpha_1}{\partial y} \omega z_i ,\ i = 2, \cdots\cdots, \rho . \tag{28}$$

Based on the tuning functions, the adaptive law is designed as

$$\dot{\hat{\theta}} = \Gamma \tau_\rho . \tag{29}$$

Finally, at step $\rho$ the control is designed as

$$u = \frac{1}{\sigma(y)}(\alpha_\rho - v_{\rho,\rho+1} + \hat{\varsigma} y_r^{(\rho)}) . \tag{30}$$

## 4.Stabilityanalysis

Define a Lyapunov function candidate

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{|b_\rho|}{2\gamma} \tilde{\varsigma}^2 + \sum_{i=1}^{\rho} \frac{1}{4d_i} \varepsilon^T P \varepsilon \tag{31}$$

Then from (20), (21), (23) and (24), the time derivative of $V_1$ is

$$\begin{aligned} \dot{V}_1 = & \hat{b}_\rho z_1 z_2 - c_1 z_1^2 - d_1 (z_1 - \frac{1}{2d_1} \varepsilon_2)^2 + \tilde{\theta}^T \Gamma^{-1} (\Gamma \tau_1 - \dot{\hat{\theta}}) \\ & + z_1 \Psi_{(1)}(y)\left[W - \hat{W}\right] - \sum_{i=2}^{\rho} \frac{1}{4d_i} \varepsilon_2^2 \\ & - \sum_{i=2}^{\rho} \frac{1}{4d_i} (\varepsilon_1^2 + \varepsilon_3^2 + \cdots + \varepsilon_n^2) + \sum_{i=1}^{\rho} \frac{1}{2d_i} (\varepsilon^T P \Psi(y) W) \\ & - \sum_{i=1}^{\rho} \frac{z_1^4}{8d_i} \|P\Delta(y, y_r)\|^4 - \sum_{i=1}^{\rho} \frac{5}{4d_i} \|\varepsilon\|^2 . \end{aligned} \tag{32}$$

Using (32), the derivative of the Lyapunov function

$$V_2 = V_1 + \frac{1}{2} z_2^2 \tag{33}$$

is expressed as

$$\begin{aligned} \dot{V}_2 = & -c_1 z_1^2 - c_2 z_2^2 - d_1 (z_1 - \frac{1}{2d_1} \varepsilon_2)^2 - d_2 (\frac{\partial \alpha_1}{\partial y} z_2 + \frac{1}{2d_2} \varepsilon_2)^2 \\ & + (\Gamma \tau_2 - \dot{\hat{\theta}})(\tilde{\theta}^T \Gamma^{-1} + \frac{\partial \alpha_1}{\partial \hat{\theta}} z_2) - \sum_{i=3}^{\rho} \frac{1}{4d_i} \varepsilon_2^2 \\ & + z_1 \Psi_{(1)}(y)\left[W - \hat{W}\right] + \sum_{i=1}^{\rho} \frac{1}{2d_i} (\varepsilon^T P \Psi(y) W) \\ & - \sum_{i=2}^{\rho} \frac{1}{4d_i} (\varepsilon_1^2 + \varepsilon_3^2 + \cdots + \varepsilon_n^2) - \sum_{i=1}^{\rho} \frac{z_1^4}{8d_i} \|P\Delta(y, y_r)\|^4 + z_2 z_3 \\ & - z_2 \frac{\partial \alpha_1}{\partial y} \Psi_{(1)}(y)\left[W - \hat{W}\right] - \sum_{i=1}^{\rho} \frac{5}{4d_i} \|\varepsilon\|^2 . \end{aligned} \tag{34}$$

where $W - \hat{W} = \tilde{V} \eta(z_1)$.

In the $\rho$ step, let

$$V_\rho = V_{\rho-1} + \frac{1}{2} z_\rho^2 + \frac{1}{2} \hat{V}^T Q^{-1} \hat{V} \tag{35}$$

The time derivative of $V_\rho$ is

$$\begin{aligned} \dot{V}_\rho = & -\sum_{i=1}^{\rho} c_i z_i - \sum_{i=1}^{\rho} d_i (z_i \frac{\partial \alpha_{i-1}}{\partial y} + \frac{1}{2d_i} \varepsilon_2)^2 - \sum_{i=1}^{\rho} \frac{z_1^4}{8d_i} \|P\Delta(y, y_r)\|^4 \\ & - \sum_{i=2}^{\rho} \frac{1}{4d_i} (\varepsilon_1^2 + \varepsilon_3^2 + \cdots + \varepsilon_n^2) + \sum_{i=1}^{\rho} \frac{1}{2d_i} (\varepsilon^T P \Psi(y) W) \\ & - \sum_{i=2}^{\rho} z_i \frac{\partial \alpha_{i-1}}{\partial y} \Psi_{(1)}(y) \tilde{V} \eta(z_1) - \sum_{i=1}^{\rho} \frac{5}{4d_i} \|\varepsilon\|^2 \\ & + z_1 \Psi_{(1)}(y) \tilde{V} \eta(z_1) - \tilde{V}^T Q^{-1} \dot{\hat{V}} \end{aligned} \tag{36}$$

From (18), we have

$$\begin{aligned} \dot{V}_\rho \le & -\sum_{i=1}^{\rho} c_i z_i - \sum_{i=1}^{\rho} d_i (z_i \frac{\partial \alpha_{i-1}}{\partial y} + \frac{1}{2d_i} \varepsilon_2)^2 \\ & - \sum_{i=2}^{\rho} \frac{1}{4d_i} (\varepsilon_1^2 + \varepsilon_3^2 + \cdots + \varepsilon_n^2) + \sum_{i=1}^{\rho} z_i \frac{\partial \alpha_{i-1}}{\partial y} \Psi_{(1)}(y) \tilde{W}_n \\ & + \sum_{i=1}^{\rho} \frac{\partial \alpha_{i-1}}{\partial y} \Psi_{(1)}(y) \hat{W}_n (|z_i| - \frac{z_i^2}{\delta}) - \tilde{W}_n^T Q^{-1} \dot{\hat{W}}_n \\ & + \sum_{i=1}^{\rho} \frac{1}{4d_i} (\frac{1}{2} \|W\|^4 + \|P \Psi(y_r)\|^2 \|W\|^2) , \end{aligned} \tag{37}$$

$$\dot{V}_\rho \le -\sum_{i=1}^{\rho} c_i z_i + \beta , \tag{38}$$

where

$$\beta(\|W\|) = \sum_{i=1}^{\rho} \frac{1}{4d_i} (\frac{1}{2} \|W\|^4 + \|P \Psi(y_r)\|^2 \|W\|^2) . \tag{39}$$

$$\dot{\hat{V}} = Q \left( \sum_{i=2}^{\rho} z_i \frac{\partial \alpha_{i-1}}{\partial y} \Psi_{(1)}(y) \eta(z_1) - z_1 \Psi_{(1)}(y) \eta(z_1) \right) \tag{40}$$

Since $\beta(\|W\|)$ is bounded, we can conclude from (38) that $V_\rho$ is bounded, which implies that $z_i$, $\hat{\theta}$, $\hat{\varsigma}$ and $\varepsilon$ are bounded.

## 5.Simulation

Consider the nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 + \theta (e^y - 1) + y w , \\ \dot{x}_2 &= u + w , \\ y &= x_1 , \end{aligned} \tag{41}$$

we have $v_p = v_n = \lambda$. The filters are designed as

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_2\end{bmatrix} = \begin{bmatrix}-k_1 & 1\\ -k_2 & 0\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix} + \begin{bmatrix}k_1\\ k_2\end{bmatrix} y,$$

$$\begin{bmatrix}\dot{\Xi}_1\\ \dot{\Xi}_2\end{bmatrix} = \begin{bmatrix}-k_1 & 1\\ -k_2 & 0\end{bmatrix}\begin{bmatrix}\Xi_1\\ \Xi_2\end{bmatrix} + \begin{bmatrix}e^y - 1\\ 0\end{bmatrix},$$

$$\begin{bmatrix}\dot{\lambda}_1\\ \dot{\lambda}_2\end{bmatrix} = \begin{bmatrix}-k_1 & 1\\ -k_2 & 0\end{bmatrix}\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} u.$$

It can be obtained that

$$\Delta(y, y_r) = \begin{bmatrix}1\\ 0\end{bmatrix}, \quad P = \begin{bmatrix}\frac{1}{k_1} + \frac{k_2}{k_1} & -1\\ -1 & \frac{1}{k_1} + \frac{k_1}{k_2} + \frac{1}{k_1 k_2}\end{bmatrix}.$$

Following the steps presented in Section 4, we have

$$\alpha_1 = -c_1 z_1 - d_1 z_1 - \xi_2 - (\Xi_2 + e^y - 1)\hat{\theta} - y\hat{W}_n\, sat(z_1) - \frac{1}{16}\left(\frac{1}{d_1} + \frac{1}{d_2}\right) z_1^3 \left\|P\Delta(y, y_r)\right\|^4 \tag{42}$$

$$\alpha_2 = -z_1 - c_2 z_2 - d_2\left(\frac{\partial \alpha_1}{\partial y}\right)^2 z_2 + k_2 \lambda_1 + \frac{\partial \alpha_1}{\partial \xi_2}\dot{\xi}_2 + \frac{\partial \alpha_1}{\partial \Xi_2}\dot{\Xi}_2 + \frac{\partial \alpha_1}{\partial y}\left[\xi_2 + \lambda_2 + (\Xi_2 + e^y - 1)\hat{\theta}\right] + \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_1}{\partial \hat{\theta}}\Gamma\tau_2 + \frac{\partial \alpha_1}{\partial \hat{W}_n}\dot{\hat{W}}_n - \frac{\partial \alpha_1}{\partial y} y\hat{W}_n\, sat\left(z_2 \frac{\partial \alpha_1}{\partial y}\right), \tag{43}$$

and $u = \alpha_2 + \ddot{y}_r$.

The adaptive law is obtained as

$$\dot{\hat{\theta}} = \left(z_1 - \frac{\partial \alpha_1}{\partial y} z_2\right)(\Xi_2 + e^y - 1) - (\dot{y}_r + \alpha_1)e_1 z_1 \tag{44}$$

$$\dot{\hat{V}} = Q\left(z_2 \frac{\partial \alpha_1}{\partial y} y\eta(z_1) - z_1 y\eta(z_1)\right) \tag{45}$$

In the simulation study, we set $\theta = 0.5$. The desired trajectory was set as $k_1 = 3$, $k_1 = 2$ $c_1 = c_2 = d_1 = d_2 = \Gamma = Q = 1$. The desired trajectory was set as $y_r = 1 - cos t$. The disturbance in the simulation was set to $w(t) = 0.2\ cos 10t$.

Fig. 1, 2 show the system output together with the desired trajectory and the control input, respectively.

## 6.Conclusions

In this paper we have presented a solution to robust adaptive control of nonlinear output feedback systems perturbed by disturbance with unknown bounds. The proposed control design has removed the requirements of the bound of uncertain parameters and the bound of the disturbances in nonlinear robust adaptive control. The proposed algorithm ensures that the tracking error converges to an arbitrarily small neighborhood of the origin, while maintaining the boundedness of all other variables.

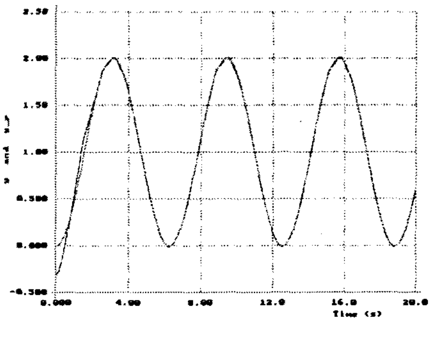

Fig. 1. Reference and Output under disturbance

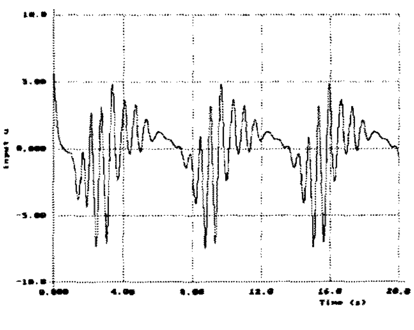

Fig. 2. Control input under disturbance ($\delta = 0.05$).

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