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Some Validation of Nonlinear $k-\varepsilon$ Models on Predicting Noncircular Duct Flows

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Abstract

Nonlinear relationship between Reynolds stresses and the rate of strain for nonlinear $k-\varepsilon$ turbulence models is validated theoretically by using the boundary layer assumptions against the turbulence-driven secondary flows in noncircular ducts and then the prediction performance for several nonlinear models is evaluated numerically through the application to the turbulent flow in a square duct.

Keyword: *nonlinear model, turbulence-driven secondary flow, validation, noncircular duct*

1. Introduction

It has been well known that the transverse mean flow exists within noncircular ducts even when the flow is fully developed. This transverse mean flow is due to the anisotropy of the turbulent normal stresses. Although this secondary flow amounts to a few percent of the axial velocity, it exists a great influence on the flow field, i.e. turbulence-driven secondary motion redistributes the kinetic energy, influences the axial (streamwise) velocity and thereby affects the wall shear stress. Thus, many experimental and numerical studies have been conducted on turbulent flows in noncircular straight ducts [1-7].

At present, several nonlinear $k-\varepsilon$ models are proposed to predict turbulent driven secondary flow. However, these nonlinear models are still not evaluated theoretically and validated numerically each other.

2. Theoretical Evaluation of Nonlinear models

In the present paper, nonlinear relationship between Reynolds stresses and the rate of strain for nonlinear $k-\varepsilon$ turbulence models which allow for the prediction of turbulence-driven secondary flows is evaluated theoretically by using the boundary layer assumptions against the turbulent flows in a noncircular square duct shown in Fig. 1. Three nonlinear $k-\varepsilon$ turbulence models by Speziale (SP model)[6], Shih et al. (SZL model)[8] and Myong and Kasagi (MK model)[9] are considered. Note that, in particular the SZL model is now implemented in a commercial CFD code.

The nonlinear relationship between Reynolds stresses and the rate of strain for these nonlinear $k-\varepsilon$ turbulence models are generally expressed as follows:

$$\begin{aligned}\overline{u_i u_j} = \frac{2}{3}k\delta_{ij} - \nu_t S_{ij} + C_1\nu_t\frac{k}{\varepsilon}(S_{ik}S_{kj} - \frac{1}{3}S_{kl}S_{kl}\delta_{ij}) + C_2\nu_t\frac{k}{\varepsilon}(\Omega_{ik}S_{kj} + \Omega_{jk}S_{ki}) \\ + C_3\nu_t\frac{k}{\varepsilon}(\Omega_{ik}\Omega_{jk} - \frac{1}{3}\Omega_{kl}\Omega_{kl}\delta_{ij})\end{aligned} \quad (1)$$

$$\nu_t = C_\mu\frac{k^2}{\varepsilon}, \quad S_{ij} = \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}, \quad \Omega_{ij} = \frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \quad (2)$$

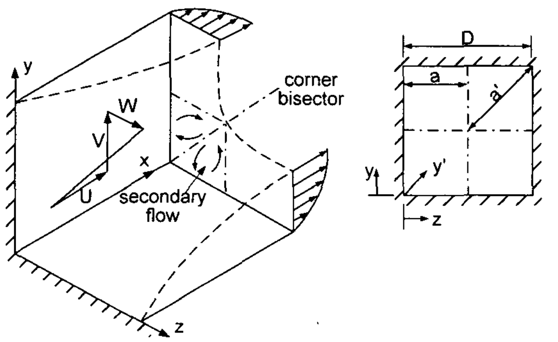

Fig. 1 Coordinate system and pertinent variables in a straight square duct.

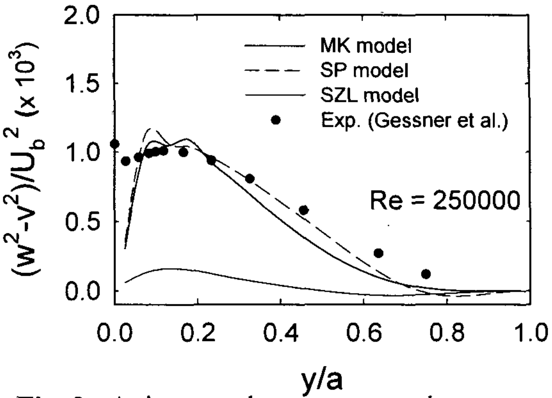

Fig. 2 Anisotropy between secondary normal stresses along the wall bisector.

For turbulent flows in a square duct shown in Fig. 1, we can simplify each component of Reynolds stresses of Eq. (1) by using the boundary layer approximations. Here, we briefly discuss the resulting forms of Reynolds stresses in the nonlinear model, comparing with those of the previous representative algebraic stress models of Demuren and Rodi (hereafter DR) [5] and Launder and Ying (LY) [4]. First, the expressions for the primary Reynolds shear stresses $\overline{uv}$ and $\overline{uw}$ happen to be identical to those for eddy diffusivity formulation in the common isotropic $k-\varepsilon$ model, and also the same forms as those of both DR and LY models. However, they are obtained only by boundary layer approximation in the nonlinear model, but by drastic assumptions in the latter models. Second, in the nonlinear models both the separation of secondary normal Reynolds stresses $\overline{w^2}-\overline{v^2}$ and the secondary Reynolds shear stress $\overline{vw}$, known to play an important role in the secondary flow generation, are in general similar to those of DR model which is known to be superior to any other existing models. In the DR model, however, they are obtained by using several ad-hoc assumptions with unknown validity. In contrast, the nonlinear model has obtained these forms only by the boundary layer assumption. In particular, the nonlinear model also confirms validity two different mechanisms that contribute to the generation of secondary shear stress $\overline{vw}$ identified by Perkins [1]; the first mechanism is associated with the gradients of secondary velocities, and its contributions to $\overline{vw}$ can be represented in terms of an isotropic eddy diffusivity, while the second one is associated with the primary velocity gradients because of its close relations with the distortion of the primary stress field in the corner. Third, both secondary normal Reynolds stresses $\overline{v^2}$ and $\overline{w^2}$ in the nonlinear model have the gradient terms of primary velocity in both y and z directions. On the contrary, most of the algebraic stress models including the DR and LY models have only one directional gradient term of primary velocity for each secondary normal Reynolds stress, which is clearly unreasonable on the physical ground.

In contrast to the MK and SZL models, however, the SP model in general cannot reproduce properly the experimental evidence for the relationship among the normal Reynolds stresses. For example, for two-dimensional boundary layer flows where only $\partial U/\partial y$ term plays an important role in Eq. (1), it can be seen that both MK and SZL models reproduce correctly the relationship with $\overline{u^2} > \overline{w^2} > \overline{v^2}$, while the SP model does not with $\overline{w^2} > \overline{u^2} = \overline{v^2}$.

3. Application of Nonlinear $k-\varepsilon$ models to square duct flow

The nonlinear models are applied to the fully developed turbulent flow in a square duct in order to validate quantitatively their prediction performance. Only the 1/4 square duct is modeled using flow symmetry to reduce the size of the computational model. The simulation is conducted by the author's MOSA3D code[10] which uses finite volume method, SIMPLE algorithm, body-fitted and non-staggered grid systems. The computational grid is distributed uniformly with 20 x 20 in y and z directions, respectively. Since the present study simulates the fully developed flow, only two grid cells are used in the main (axial) flow direction. The conventional wall function method is used and the distance of the first grid points from the wall is in the range of 110~140 as the nondimensional wall unit (y^+ or z^+).

Typical predicted quantities such as mean axial velocity, wall shear stress, turbulent kinetic energy and Reynolds stresses are compared with available experimental data [2, 3] to underpin knowledge about the solution quality obtained from these nonlinear models. Figure 2 shows the predicted and measured distributions of separation of secondary normal Reynolds stresses $\overline{w^2} - \overline{v^2}$ along the wall bisector. The MK and SP models work fairly well along the wall bisector except the near-wall region where the discrepancy is due to the adoption of the conventional wall function method as mentioned above. The SZL model's prediction is, however, too small by a factor of 10.

The prediction by each model for the separation of secondary normal stresses and the secondary Reynolds shear stress is strongly responsible for its prediction for the secondary motion. Although not shown here, each model shows qualitatively similar behavior, but quantitatively large difference on the prediction performance for the secondary motion: the maximum secondary velocity is estimated to be about 1.3 % of the bulk mean velocity by both MK and SP models, while about 0.15 % by the SZL model. Compared with about 1.8% by the experiment [2, 3], the SP model is found to underpredict quantitatively by a factor of 10. Thus, the SZL model cannot predict properly the experimentally well-known variations for local wall shear stress, axial mean velocity and primary Reynolds stress due to the secondary motion, although it predicts the secondary motion.

4. Conclusion

The following conclusions can be made from the present study.

(1) Both MK and SP models are capable of predicting both qualitatively and quantitatively fairly well local wall shear stress, axial mean velocity, primary Reynolds stress and turbulent kinetic energy including the turbulence-driven secondary flow in norcircular ducts, while the SZL model adopted in a commercial code is not with the prediction level of secondary flows one order less than that of the experiment.

(2) In contrast to the MK and SZL models, the SP model in general cannot reproduce properly the experimental evidence for the relationship among the normal Reynolds stresses.

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