Extension of a High Resolution Lagrangian Method to Consider the Real Gas Effect

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Abstract

In the present research a high order Gudonov-type method has been used for the simulation of very high pressure flow fields, as well as the capturing of strong shocks, which usually occur in explosion of high explosives. The treatment strong shocks and the flow field behind the shocks needs a very high resolution scheme. To resolve accurately the shock and the release waves behind the shock the piece-wise parabolic method (PPM) of Colella [1] was utilized in this research. A major problem which encountered in very high pressure problems is the equation of state which differs completely form the ideal-gas equation of state (EOS). Here, the original PPM is extended for real gas effect consideration.

Keyword*: Strong Shock, Real Gas, Gudonov Method, Numerical Simulation*

1. Introduction

The main objective of the present work has been to develop a numerical algorithm for the simulation of flow filed produced after detonating a high explosive in air as well as in water (for simulating underwater explosion). The flow field contains a strong shock wave followed by a rapid expansion region. In recent years, a number of new shock-capturing schemes, often called high-resolution schemes, have been proposed. Among them are the FCT, MUSCL, ENO, and PPM methods. There are several excellent review articles, which compare these schemes from different point of views. Interested readers referred to those articles, particularly the paper of Yang et al. [2]. After comparing different schemes, they recommended PPM (Piecewise Parabolic Method) as the best in the overall performance. When an explosion occurs in water, the treatment of the air-water interface needs special treatment. The tracking of the interface, which is a troublesome subject in an Eulerian approach, will be handled with no additional work in a Lagrangian framework. Therefore, in the present work a Lagrangian PPM has been employed. The original PPM algorithm was developed for ideal gases [1]. In the present work PPM is extended to treat the real gas effect.

2. Governing Equations

The basic equations consist of conservation laws of mass, momentum, and energy with no molecular diffusion effect (i.e., Euler equations). These equations in one-dimensional Lagrangian coordinate have the following form:

$$\frac{\partial \tau}{\partial t} - \frac{\partial (r^{\alpha} u)}{\partial m} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + r^{\alpha} \frac{\partial p}{\partial m} = 0 \tag{2}$$

$$\frac{\partial E}{\partial t} + \frac{\partial (r^{\alpha} up)}{\partial m} = 0 \tag{3}$$

where, τ, u, p and E are specific volume, velocity, pressure and total energy per unit volume, respectively. t is time, m refers to mass coordinate, and r is distance from the origin. α indicates the geometry of the problem, $\alpha=0$ for slabs, $\alpha=1$ for cylinders, and $\alpha=2$ for spheres. Here, the products of explosion are represented by JWL EOS [3]:

$$p = A\left(1-\frac{\omega}{R_1 V}\right)e^{-R_1 V} + B\left(1-\frac{\omega}{R_2 V}\right)e^{-R_2 V} + \frac{\omega e}{\tau} \tag{4}$$

where A, B, ω, R_1 and R_2 are empirical constants. $V=\tau/\tau_0$, $\tau_0=\rho_0^{-1}$ and ρ_0 is the initial density of the charge. e is internal energy. Mie-Gruneisen EOS with a third order fit to Hugoniot is utilized for water:

$$p = \frac{\rho_0 C^2 \mu\left[1+\left(1-\frac{\gamma_0}{2}\right)\mu-\frac{a}{2}\mu^2\right]}{\left[1-(S_1-1)\mu-S_2\frac{\mu^2}{\mu+1}-S_3\frac{\mu^3}{(\mu+1)^2}\right]^2} + (\gamma_0+a\mu)e \tag{5}$$

C, a, γ_0, S_1, S_2 and S_3 are empirical constants. μ is defined as: $\mu=\rho/\rho_0-1$.

3. Numerical Method

The Lagrangian PPM consists of three steps: (A) the interpolation of distributions of the dependent variables (τ, u, p) in mass coordinate, (B) solving appropriate Riemann problems to calculate the time-averaged pressures and velocities at the edge of the cells, and (C) updating the conserved quantities (τ, u, p). The original PPM algorithm was developed for ideal gases [1]. In the present work PPM is extended to treat the real gas effect. PPM is a Flux Difference Splitting Method. Since the splitting is based on the characteristics, the sound speed in the material under investigation should be determined in each time step. For non-gaseous materials as well as high pressure gases the ideal-gas assumption is not valid. Thus, the thermodynamic concepts should be utilized to find a formula for the speed of sound. Using thermodynamics relationships can be shown that [4]:

$$c^2 = \tau^2(pp_e - p_\tau)$$

where c is the speed of sound. Partial derivatives of pressure can be calculated explicitly from the equation of state. The equations of states utilized in the present work are chosen such that no approximation is required for the sound speed calculation.

4. Results

The developed code has been examined versus several test problems such as the standard shock tube problem (i.e., Sod problem), the explosion of a planar charge of Comp.B (a strong high explosive) and the problem of underwater explosion. The results (efficiency of the calculation and the accuracy) are very promising.

5. References

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