THE 5^{th} ASIAN COMPUTATIONAL FLUID DYNAMICS
BUSAN, KOREA, OCTOBER 27~30, 2003

Comparative Study of $k - \omega$ Turbulence Models for Transonic Separated Flows

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A number of two-equation turbulence models based on the ω equation have been proposed. Wilcox[1] analyzed the $k - \omega$ model and consequently set up the closure coefficients of the model. His model gives excellent results for zero gradient boundary layer problems. Despite of the favorable numerical aspects, it has a strong freestream dependence from which general $k - \epsilon$ models do not suffer. Menter[2] therefore combined the $k - \omega$ model and the $k - \epsilon$ model into a single model. This model has been called the Shear-Stress Transport(SST) model. Recently, the non-linearity in the eddy viscosity[3] has been reconsidered in view of realizability that cannot be satisfied with any linear eddy viscosity model based on the Boussinesq's assumption. It was found that the non-linear eddy viscosity models, denoted by WD+ model, improve the performance of the turbulence models for flows in presence of adverse pressure gradients, especially in Shock-Wave/Boundary-Layer Interaction(SWBLI)[4].

The DADI method is applied to find steady-state solutions. For $k - \omega$ turbulence equations, the source vector must be implicitly treated since it results in the stiffness problem of time marching methods. Albeit the implicit treatment of the source term greatly improves the robustness involved with the positivity of the turbulence variables, the ω equation cannot preserve the positivity constantly. This causes unphysically low level of ω resulting in larger values of eddy viscosity. Following Zheng and Liu[5], a lower limit for ω is imposed for every iteration after the turbulence equations are solved. The multigrid method for the turbulence equations requires careful approach because of their high stiffness that results from the non-linear source terms. The gradient-related terms of the source vector are calculated only on the finest grid and restricted as the frozen values to coarser grids. The eddy viscosity is also restricted in the same way. Freezing the eddy viscosity helps the robust convergence especially when the freestream values are used for the initial conditions. No-slip conditions are applied and the value of k is set to zero at the wall. Since the specific dissipation rate ω is theoretically infinite at the wall, the boundary value of ω is only specified. It is noted that this boundary conditions using the distance to the first cell center may cause grid dependence. In this study, a modified ω value at the wall is used in order to reduce the grid dependence.

To verify the implementation of the turbulent models, the turbulent boundary layer over a flat plate is first presented. The freestream conditions are M_∞=0.2 and $Re = 10^7$ based on the plate length. Four sets of grids with different normal wall distances are made in order to perform a grid convergence study. Using the modified ω boundary values can alleviate the grid spacing limitation of the turbulence models. The important role of the modification is to increase the value of ω as the grid spacing increases. Figure 1 shows the skin friction coefficient distribution for the relatively coarse grids and velocity profiles at X/C=0.5 for $\Delta y = 8 \times 10^{-6}$ and 12×10^{-6} when the modified ω value is used. As shown, the grid limitation is reduced for $k - \omega$ SST and WD+ models. The modification, however, does not work for the original $k - \omega$ model. In the velocity profiles all models give quite a good agreement with the laws of the wall even when the grid points are three within the viscous sub-layer.

The present method is applied to the turbulent transonic flows past the RAE2822 airfoil. For a C-type computational grid, the farfield boundary is located at 20-chord length away. The grid consists of 383 grid points in the flow direction and 65 in the normal direction. Case 1 represents the subcritical unseparated flows. As shown in Fig. 2, the computed results of all three models agree well with

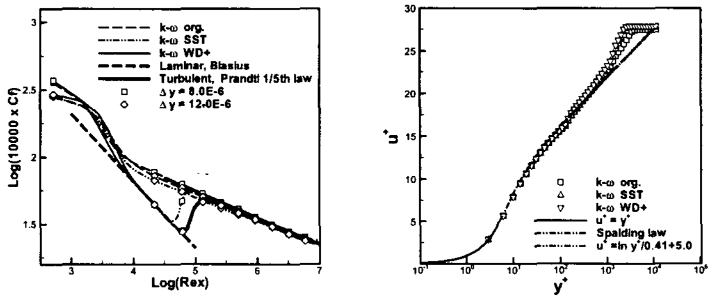

Figure 1. Solution with turbulence model using modified ω at wall.

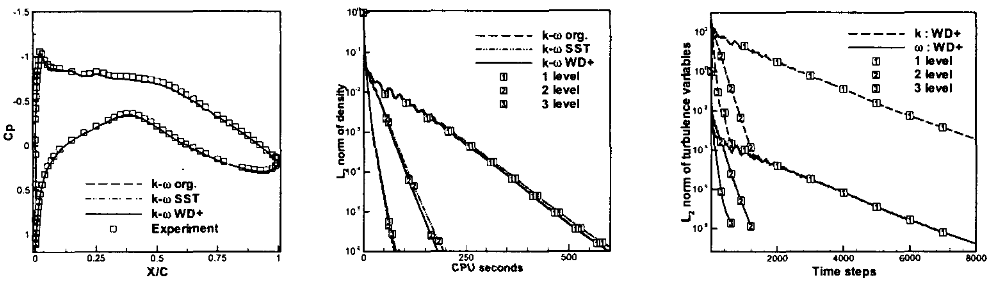

Figure 2. Wall pressure coefficient and convergence histories for Case 1 (RAE2822).

experimental data. In all test cases, the solutions are converged to below 0.01% C_L and 0.1% C_D. As the multigrid level increases, the number of multigrid cycles decreases. When the L_2-norm is reduced to 6 orders of magnitude, the present multigrid reduces the iteration and the computing time to a factor of 18 and 7.5, respectively. Convergence histories of the k and ω equations are displayed for the $k - \omega$ WD+ model. The convergence rate of each level is nearly the same to that of density residual.

For Case 10 with the shock-wave/boundary-layer interaction, the SST and WD+ models in Fig. 3 predict the shock aft of the measured location but give better agreement with experiment than the original $k - \omega$ model. All models predict the separation with the same extent but different position. The velocity profiles on the upper surface in Fig. 4 show that the original $k - \omega$ model gives the smallest separation bubble. The $k - \omega$ WD+ model gives the best agreement with experimental profiles. The results show that the non-linear eddy viscosity model for the $k - \omega$ models is superior

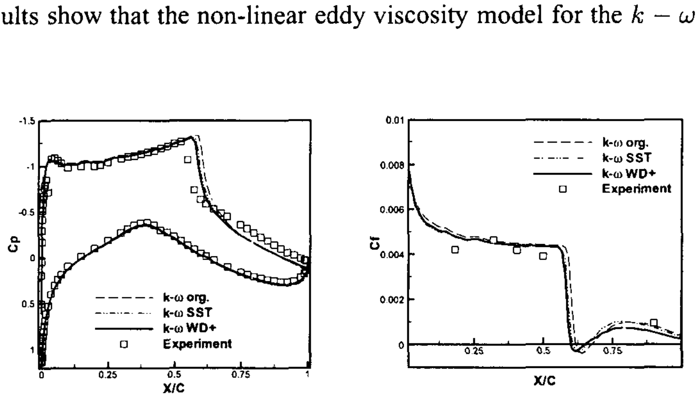

Figure 3. Wall coefficients with turbulence model for Case 10 (RAE2822).

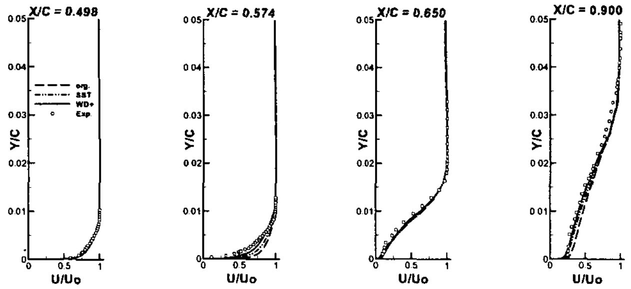

Figure 4. Velocity profiles on upper surface for Case 10 (RAE2822).

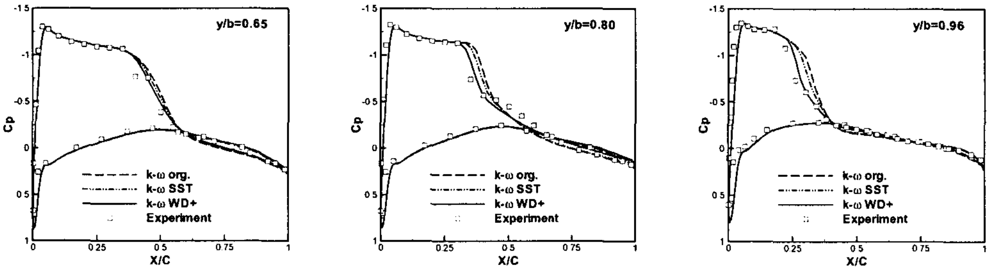

Figure 5. Wall pressure coefficients over ONERA-M6 wing at α=5.06 deg.

to the linear model in the adverse pressure gradient region.

The last test case is the transonic flow past the ONERA-M6 wing. To compare the results for the attached and separated cases, the flow with the angle of attack of 3.06 deg and Mach number of 0.8395 is selected for the nearly attached flow and α=5.06 deg and M_∞=0.8447 for the separated flow. The freestream Reynolds number is 11.7×10^6. The grid consists of 149(streamwise)×49(normal)×33 grid points. In Fig. 5, all three models produce nearly the same pressure distributions at α=3.06 deg(not displayed) but an apparent difference at α=5.06 deg is shown in the separated region. The $k-\omega$ WD+ model gives the best agreement with experiment. In addition, region of reduced pressure recovery is certainly appeared for the WD+ model. The difference between the WD+ and SST models is noticeable for this 3D case compared to that of the RAE2822 Case 10 with separation. Although stricter confirmation is required, the difference may reflect different definition of the turbulent eddy viscosity and the closure coefficients between the WD+ and SST models.

As a conclusion, numerical results for the unseparated and separated transonic flows showed that the WD+ model using the weakly non-linear eddy viscosity is in better overall agreement with experiment. Numerical results also showed that the present freezing and limiting strategies help fast convergence and robust computation of the $k-\omega$ turbulence equations.

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